There seems to be three different groups of students:

- A group around 6
- A group around 12
- A group around 16
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Assume that you hit a football lying on the ground.

It’s initial speed is $v_0$ making an angle $\alpha$ with the ground.

Choose the origin of time such that $t_i = 0$ and origin of coordinate axis such that $\vec{x}(0) = 0$
Trajectory of a Football

\[ \vec{x}(t) = \vec{v}_0 t + \frac{1}{2} gt^2 \hat{z} \]
\[ z(t) = v_{0z} t + \frac{1}{2} gt^2 \]
\[ z(t) = 0 \text{ when } t = 0 \text{ (initial time) and at } t = -\frac{2v_{0z}}{g} \text{ (when the ball hits the ground)} \]
Trajectory of a Football

- The flight time of the ball is $t = -\frac{2v_0 z}{g}$.
- The only acceleration is along the $z$ axis.
- $-\frac{v_0 z}{g}$ is the time it take for the $z$ component of the velocity to become zero, i.e. the time it takes to reach maximum height.
- The time it takes to fall down is the same (in the absence of air friction).
Trajectory of a Football

- Assume that $x$ and $y$ axis are chosen such that $\vec{v} = -v_0 \sin \alpha \hat{z} + v_0 \cos \alpha \hat{x}$
- $v_y(t) = 0$, $y(t) = 0$ for all times
- $x(t) = v_0 x t$
- Range is the distance the ball covers during its flight, i.e. $R = |x(t_f)|$

$$R = v_{0x} \left( -\frac{2v_0 z}{g} \right)$$
$$= v_0 \cos \alpha \left( -\frac{2(-v_0 \sin \alpha)}{g} \right)$$
$$= \frac{v_0^2 \sin 2\alpha}{g}$$
### Trajectory of a Football

\[ R = \frac{v_0^2 \sin(2\alpha)}{g} \]

- \( \sin 2\alpha \) has maximum value of 1 when \( \alpha = 45^\circ \)
- Increasing \( v_0 \) by a factor of 2 increases the range by 4.
- If \( \alpha_1 + \alpha_2 = \frac{\pi}{2} \), their ranges are the same.
Trajectory

- Trajectory is a relationship between the components of the position of a particle that does not involve time.
- When the particle is at the horizontal distance $x$, the time that has passed is $t(x) = \frac{x}{v_{0x}}$.
- The $z$ coordinate of the particle at that time is

$$z(x) = v_{0z} t(x) + \frac{1}{2} g t(x)^2$$

$$= v_{0z} \left( \frac{x}{v_{0x}} \right) + \frac{1}{2} g \left( \frac{x}{v_{0x}} \right)^2$$

$$= \frac{g}{2v_0^2 \cos^2 \alpha} x(x - R) \quad (29)$$
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\]
**Example**

Q: What is the distance $|OP|$?
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To find the point $P$, we will use the fact that point $P$ is both on the parabola describing the trajectory, and also on the line that describes the hill.
Example

Q: What is the distance $|OP|$?
First choose a coordinate axis.
Example

**Q:** What is the distance $|OP|$?

First choose a coordinate axis.

The initial velocity and acceleration in these new coordinate axes are:

\[ \vec{v}_0 = v_0 \cos(\theta - \alpha) \hat{x} + v_0 \sin(\theta - \alpha) \hat{y} \]

\[ \vec{a} = g \cos \alpha (-\hat{y}) + g \sin \alpha (-\hat{x}) \]
**Example**

- **Q:** What is the distance $|OP|$?
- The velocity at time $t$ can be obtained as

\[
\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t') \, dt' = \vec{v}_0 + t\vec{a}
\]

\[
= [v_0 \cos(\theta - \alpha) - gt \sin \alpha] \, \hat{x} + [v_0 \sin(\theta - \alpha) - gt \cos \alpha] \, \hat{y}
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Example

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The position at time $t$ is

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')dt'$$
Q: What is the distance $|OP|$?

The position at time $t$ is

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')dt'$$

$$= \left[ v_0 t \cos(\theta - \alpha) - \frac{1}{2}gt^2 \sin \alpha \right] \hat{x}$$

$$+ \left[ v_0 t \sin(\theta - \alpha) - \frac{1}{2}gt^2 \cos \alpha \right] \hat{y}$$
Example

**Q:** What is the distance $|OP|$?

Hence, if the object reaches the point $P$ at time $t_0$,

$$x_0 = v_0 t_0 \cos(\theta - \alpha) - \frac{1}{2} g t_0^2 \sin \alpha$$

$$y_0 = v_0 t_0 \sin(\theta - \alpha) - \frac{1}{2} g t_0^2 \cos \alpha$$

(30)
Example

**Q:** What is the distance $|OP|$?

At point $P$, $y_0 = 0$

$$v_0 t_0 \sin(\theta - \alpha) - \frac{1}{2} g t_0^2 \cos \alpha = 0 \quad (30)$$

which has solutions $t_0 = 0$ or

$$t_0 = \frac{2v_0 \sin(\theta - \alpha)}{g \cos \alpha}$$
Example

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$t_0 = 0$ is the beginning of motion. The second solution is the solution we are looking for.
Example

Q: What is the distance $|OP|$?

The distance $|OP| = x_0$.

Using $t_0 = \frac{2v_0 \sin(\theta - \alpha)}{g \cos \alpha}$

$$x_0 = v_0 \left(\frac{2v_0 \sin(\theta - \alpha)}{g \cos \alpha}\right) \sin(\theta - \alpha)$$

$$- \frac{1}{2} g \left(\frac{2v_0 \sin(\theta - \alpha)}{g \cos \alpha}\right)^2 \cos \alpha$$

(30)
Relative Motion

- From the definition of vector addition \( \vec{r} = \vec{R} + \vec{r}' \).
- The displacement of the point \( P \) in a time \( \Delta t \) is
  \[
  \Delta \vec{r} = \Delta \vec{R} + \Delta \vec{r}'
  \]  
  (31)
- The velocities in the two reference frames are related by \( \vec{v} = \vec{v}' + \vec{V} \).
Relative Motion

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\Delta \vec{r} = \frac{\Delta \vec{R}}{\Delta t} + \frac{\Delta \vec{r}'}{\Delta t} \quad (31)
\]

The velocities in the two reference frames are related by \( \vec{v} = \vec{v}' + \vec{V} \).
Relative Motion

ASSUMPTIONS

\[ \Delta \vec{r} \text{ and } \Delta \vec{r}' \text{ are measured in different reference frames. We are assuming that they are equal. Furthermore, we are assuming the } \Delta t \text{ is the same in both reference frames.} \]

\[ \Delta \vec{r} = \Delta \vec{r}' + \Delta \vec{R} \quad (31) \]

\[ \vec{v} = \vec{v}' + \vec{V} \]

From the definition of vector addition, the displacement of the point \( P \) in a

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Example

Question 3.78

Raindrops make an angle $\theta$ with the vertical when viewed through a moving train window. If the speed of the train is $\vec{v}_T$, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?

Solution:

Let $\vec{v}_R = -v\hat{z}$ be the speed of the raindrops in the reference frame of Earth, $\vec{v}_E$ be the velocity of the Earth relative to the train, i.e. $\vec{v}_E = -\vec{v}_T$. 
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\[
\begin{align*}
\vec{v}_E & \quad \vec{v}_R \\
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Let $\vec{v}_{RT}$ be the velocity of the raindrops relative to train.
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- It is given that $\vec{v}_{RT}$ makes $\theta$ radians with respect to the vertical.
- From the figure, it is seen that

$$\tan \theta = \frac{v_E}{v_R} \implies v_R = v_T \cot \theta \quad (32)$$
Reference Frames

- event: position+time
  - A reference frame is a coordinate axis (to measure the position of an event)
  - And a clock at each point of space (to measure the time of an event)
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Dynamics-Newton’s Laws of Motion

1\textsuperscript{st} Law: In an inertial reference frame, in the absence of any external influences, the velocity of an object is constant.
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This is a definition of an inertial reference frame
Dynamics-Newton’s Laws of Motion
Inertial Reference Frame

- To test if a given reference frame is inertial, consider a test object
  - Eliminate all external influences.
  - Check to see if the object accelerates or not
  - If the object is not accelerating, that reference frame is an inertial reference frame

- Given one inertial reference frame, any other frame that moves at constant velocity relative to the inertial reference frame is inertial:

\[ \vec{v} = \vec{V} + \vec{v}' \]  \hspace{1cm} (33)

- If a given reference frame is an inertial reference frame, all objects obey Newtons 1\textsuperscript{st} law in that frame
Dynamics-Newton’s Laws of Motion

2\textsuperscript{nd} Law: In an inertial reference frame, the acceleration of an object is proportional to the force acting on the object. The proportionality constant is $\frac{1}{m}$ where $m$ is the mass of the object

$$\vec{a} = \frac{\vec{F}}{m} \quad (34)$$

3\textsuperscript{rd} Law: If an object $A$ exerts a force $\vec{F}_{AB}$ on another object $B$, then object $B$ also exerts a force $\vec{F}_{BA}$ on object $A$ whose magnitude is equal to the magnitude of $\vec{F}_{AB}$, but opposite in direction:

$$\vec{F}_{AB} = -\vec{F}_{BA} \quad (35)$$
Dynamics-Newton’s Laws of Motion

2
\textsuperscript{nd} and 3
\textsuperscript{rd} laws define the mass of an object

- By the 3
\textsuperscript{rd} law, the magnitudes of the force acting on the standard mass and the unknown mass are equal:

- Using 2
\textsuperscript{nd} law:

\[ ma = m_s a_s \] (36)

- Accelerations can be measured experimentally. Hence the unknown mass can be obtained as:

\[ m = m_s \frac{a_s}{a} \] (37)
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Dynamics-Newton’s Laws of Motion

- Once the mass is defined, 2\textsuperscript{nd} Law can be considered as the definition of the force.
- Also, if the force is given (by some means), the second law can be used to obtain acceleration.