

Decisions made by engineers, managers, corporation presidents, and individuals are commonly the result of choosing one alternative over another.

- Decisions often reflect a person's educated choice of how to best invest funds; **capital**
- Amount of capital is usually restricted (how to invest to **add value**?)
- Engineers play a major role in capital investment decisions based on their analysis

Fundamentally, **engineering economy** involves **formulating**, **estimating** and **evaluating** the economic outcomes when alternatives to accomplish a defined purpose are available.

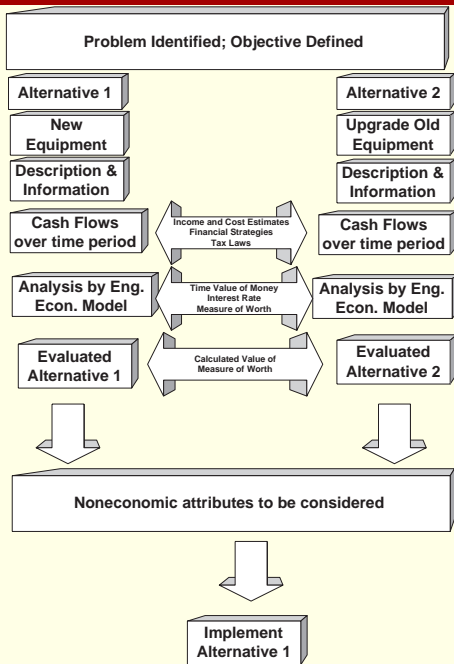
## Engineering Economy

A collection of mathematical techniques that simplify economic comparison and assist people in making decisions

- Since decisions affect what will be done, the time frame of engineering economy is primarily **the future**
- Numbers used in engineering economy analysis are **best estimates of what is expected to occur**
- Estimates often involve three essential elements:
  - **cash flows**
  - **time of occurrence**
  - **interest rates**

## Sensitivity Analysis

Stochastic nature of estimates will likely make the observed value in future differ from the estimate made now. Sensitivity analysis is performed during the engineering economic study to determine how the decision might change based on varying estimates.



## Cash Flows

The estimated **inflows** (revenues) and **outflows** (costs) of money

## Analysis

Computations considering the time value of money on cash flows to obtain a measure of worth

## Time Value of Money

Cash flows occur over a substantial period of time. How can we treat the same amount of money available at different times?

## Which would you prefer?

- \$100 cash today **OR**
- Assurance of receiving \$100 a year from now (**You will receive it for sure!**)

## You want the \$100 now! Because:

- You can use it for an extra year
- If you had no current use for it, you could let some use it

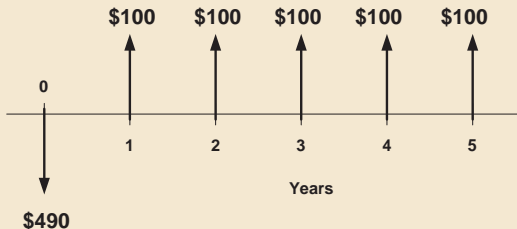
## Interest

- In **Financial World** money itself is a commodity
- Like any other goods/commodities that are bought and sold **money costs money**
- The charge for using money is called **INTEREST**
  - Cost of money is measured by **interest rate** a percentage that is periodically applied and added to an amount of money over specified length of time

## We want \$100 now because:

there is **TIME VALUE OF MONEY** in the form of willingness of banks, businesses, and people to pay interest for its use.

## What is the value of this investment?

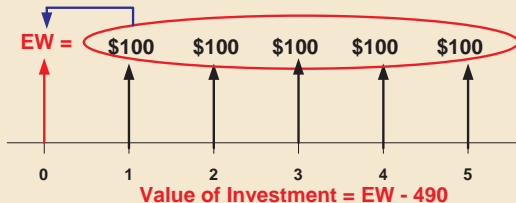


- $500 - 490 = \$10$  ?
- **NO!!! DUE TO TIME VALUE OF MONEY**

## Economic Equivalence

The observation that money has a **time value** leads us to:

- How do we compare various cash flows dispersed over the time horizon?
- **Economic equivalence** exists between cash flows that have the same economic effect and therefore be traded for one another in financial marketplace.
- Economic Equivalence refers to the fact that: **any cash flow - whether a single payment or a series of payments - can be converted to an equivalent cash flow at any point in time.**
- And that equivalence depends on the **interest rate.**



## Interest

- 1 Manifestation of time value of money
- 2 Computationally, Ending amount of money - Beginning amount of money
- 3 Two perspectives to an amount of interest
  - 1 **Interest Paid**: when money is borrowed and repaid as a larger amount
  - 2 **Interest Earned**: when money is invested and obtained back as a larger value
- 4 Interest Rate (%) =  $\frac{\text{interest accrued per unit time}}{\text{original amount}} * 100$
- 5 The time unit of the rate is the **interest period**.
- 6 Interest period of the interest rate should always be stated (1 % per month). Otherwise assume 1 year interest period.

## Example

A person borrows \$10,000 on May 1 and must repay a total of \$11,000 exactly one year later. Determine the interest amount and the interest rate paid.

## Solution

- Interest = \$11,000 – \$10,000 = \$1,000
- Interest Rate =  $\frac{\$1,000}{\$10,000} = 10\%$  per year



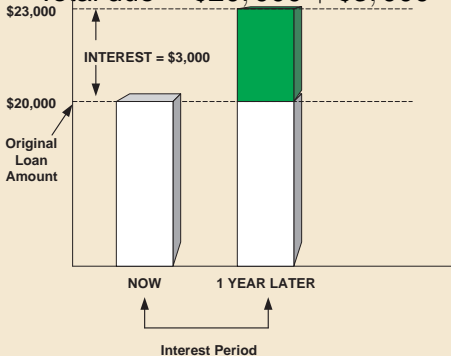
## Example

321 Consulting Company plans to borrow \$20,000 from a bank for 1 year at 15 % interest rate to buy new office equipments. Compute the interest and the total amount due after 1 year.

## Solution

- Interest =  $\$20,000 * 0.15 = \$3,000$

- Total due =  $\$20,000 + \$3,000 = \$23,000$



## Note That

The total amount due may also be computed as  
 $\text{Principal} * (1 + \text{interest rate}) =$   
 $\$20,000 * (1 + 0.15) = 23,000$

- 1 For more than one interest period, the terms **simple interest** and **compound interest** become important
- 2 **Simple interest** is calculated using the principal only, ignoring any interest accrued in preceding interest periods.
- 3  $\text{Interest} = (\text{Principal})(\# \text{ of periods})(\text{interest rate})$

## Example

321 Credit Union loaned money to an engineer. The loan is for \$1,000 for 3 years at 5% per year simple interest. How much money will the engineer repay at the end of 3 years?

## Solution

- Interest per year =  $1000(0.05) = \$50$
- Total Interest =  $1000(3)(0.05) = \$150$
- The amount due after 3 years =  $1000 + 150 = 1150$
- OR:  $P(1+i * n) = 1000(1+3*0.05) = 1000(1.15)$

## Simple Interest Calculations

End of Year	Amount Borrowed	Interest	Amount Owed	Amount Paid
0	\$1000			
1	-	$1000 * 0.05 = 50$	1050	-
2	-	$1000 * 0.05 = 50$	1100	-
3	-	$1000 * 0.05 = 50$	1150	1150

- For **Compound interest rate**, the interest is accrued on the principal **plus** the total amount of interest accumulated in all previous periods.
- Interest = (principal+all accumulated interest)(interest rate)
- Therefore, compound interest means interest on top of interest (**interest earned earns interest**)

## Example

Same as previous example but the loan is at 5% per year compound interest.

## Compound Interest Calculations

End of Year	Amount Borrowed	Interest	Amount Owed	Amount Paid
0	\$1000			
1	-	$1000 * 0.05 = 50$	1050	-
2	-	$1050 * 0.05 = 52.5$	1102.5	-
3	-	$1102.5 * 0.05 = 55.13$	1157.63	1157.63

## Note on Calculating Total Amount Due

- Year 1:  $1,000(1.05) = 1,050$
- Year 2:  $1,050(1.05) = 1,000(1.05)(1.05) = 1102,5$
- Year 3:  $1102,5(1.05) = 1,000(1.05)(1.05)(1.05) = 1157.63$

Therefore, in general the formula form:

$$\text{Total due} = \text{Principal}(1 + \text{interest rate})^{\text{number of years}}$$

### In Class Work 1

Calculate the interest and payment amounts for a \$5,000 loan which will be repaid in 5 years at 10% interest rate under the following plans:

- 1 Simple interest, pay all at end
- 2 Compound interest, pay all at end
- 3 Simple interest paid annually, principal paid at end
- 4 Compound interest and portion of principal (\$1,000) repaid annually

## Plan 1

End of Year	Interest for Year	Owed at End of Year	End of Year Payment	Owed after Payment
0				\$5,000
1	$5,000 * 0.1 = 500$	5,500	-	5,500
2	$5,000 * 0.1 = 500$	6,000	-	6,000
3	$5,000 * 0.1 = 500$	6,500	-	6,500
4	$5,000 * 0.1 = 500$	7,000	-	7,000
5	$5,000 * 0.1 = 500$	7,500	<u>7,500</u>	
Total			7,500	

## Plan 2

End of Year	Interest for Year	Owed at End of Year	End of Year Payment	Owed after Paid
0				\$5,000
1	$5,000 * 0.1 = 500$	5,500	-	5,500
2	$5,500 * 0.1 = 550$	6,050	-	6,050
3	$6,050 * 0.1 = 605$	6,655	-	6,655
4	$6,655 * 0.1 = 665,5$	7320,5	-	7320,5
5	$7320,5 * 0.1 = 732.05$	8052.55	<u>8052.55</u>	
Total			8052.55	

### Plan 3

End of Year	Interest for Year	Owed at End of Year	End of Year Payment	Owed after Payment
0				\$5,000
1	$5,000 * 0.1 = 500$	5,500	500	5,000
2	$5,000 * 0.1 = 500$	5,500	500	5,000
3	$5,000 * 0.1 = 500$	5,500	500	5,000
4	$5,000 * 0.1 = 500$	5,500	500	5,000
5	$5,000 * 0.1 = 500$	5,500	<u>5,500</u>	
Total			<u>7,500</u>	



## Plan 4

End of Year	Interest for Year	Owed at End of Year	End of Year Payment	Owed after Payment
0				\$5,000
1	$5,000 * 0.1 = 500$	5,500	1,500	4,000
2	$4,000 * 0.1 = 400$	4,400	1,400	3,000
3	$3,000 * 0.1 = 300$	3,300	1,300	2,000
4	$2,000 * 0.1 = 200$	2,200	1,200	1,000
5	$1,000 * 0.1 = 100$	1,100	1,100	
Total			<u>6,500</u>	

- $P$ : value or amount of money at present or time 0.
  - Present Worth (PW)
  - Present Value (PV)
  - Net Present Value (NPV)
- $F$ : value or amount of money at some future time.
  - Future Worth (FW)
  - Future Value (FV)
- $A$ : series of consecutive, equal, end-of-period amounts of money.
- $n$ : number of interest periods
- $i$ : interest rate or rate of return per time period
  - **compound** unless otherwise stated
  - expressed as percent per interest period
- $t$ : time stated in periods

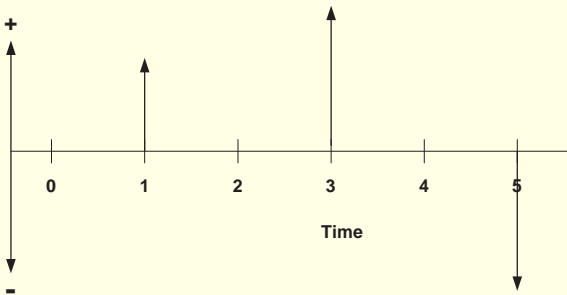
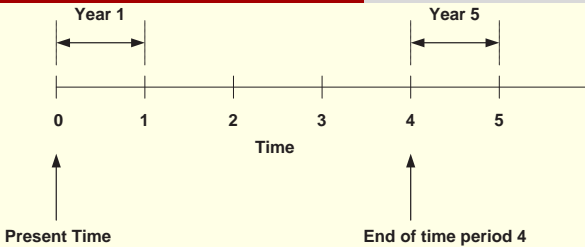
## Example

321 Credit Union loaned money to an engineer. The loan is for \$1,000 for 3 years at 5% per year. How much money will the engineer repay at the end of 3 years? Define the engineering economy symbols involved?

## Solution

- $P = \$1,000$
- $i = 5\%$
- $n = 3$  years
- $F = ?$  at the end of year 3

- Cash Flows: The estimated **inflows** (revenues) and **outflows** (costs) of money
  - + : inflows (revenues, salvage values, tax savings, ...)
  - - : outflows (First cost of assets, operating costs, ...)
- Net Cash flow = Cash Inflows - Cash Outflows
- **End-of-period convention**: All cash flows occur at the end of the **interest period**.
- Cash Flow Diagram: Graphical representation of cash flows drawn on a time scale
  - includes what is known, what is estimated, what is needed
  - time  $t = 0$ : the present,  $t = 1$  is the end of time period 1
  - Vertical arrow pointing up indicates cash inflow (+).
  - Vertical arrow pointing down indicates cash outflow flow (-)
  - The perspective must be determined before placing signs on the cash flows

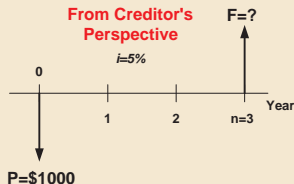
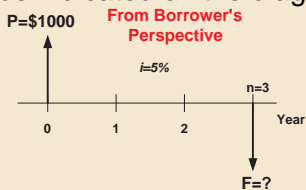


## Example

321 Credit Union loaned money to an engineer. The loan is for \$1,000 for 3 years at 5% per year. How much money will the engineer repay at the end of 3 years? Construct the cash flow diagram?

## Solution

Interest rate should be indicated on the diagram:



- For any investment to be profitable, a fair **rate of return** or **return on investment** must be realized
- The reasonable rate of return is called **Minimum Attractive Rate of Return (MARR)**
- $MARR >$  rate of a safe investment
- MARR is not a rate calculated, decided by managers
- **Cost of capital**: the interest paid to raise capital
  - Equity Financing
  - Debt Financing
  - Weighted average of Equity and Debt financing used gives us the cost of capital of the project
- For an accepted project:  $ROR > MARR >$  cost of capital