Okay, now listen up. Nobody gets in here without answering the following question: A train leaves Philadelphia at 1:00 p.m. It's traveling at 65 miles per hour. Another train leaves Denver at 4:00... Say, you need some paper?

Math phobic's nightmare
More Problems

- You have 10 red socks and 20 brown socks in your drawer. If you reach into it in the dark, how many socks must you take out to be sure of having a pair that matches?
Problem Solving

➢ What is the next letter in the following sequence?

O T T F F S S S
Problem Solving

- **Initial State**
  - Current situation
  - Define the problem

- **Goal State**
  - Desired objective

- **Obstacles**
  - Choices made about limitations
  - Strategy choices
  - Limited resources
Problem Solving

• The process of overcoming obstacles to answer a question or to achieve a goal
Problem Representation

• The importance of determining what information is relevant and what information is irrelevant is the process of problem representation
  – People pay attention to the wrong information
  – People need to focus on the right information
Sample Problem

• A man wanted to enter an exclusive club but did not know the password that was required. He waited by the door and listened. A club member knocked on the door and the doorman said, "twelve." The member replied, "six" and was let in. A second member came to the door and the doorman said, "six." The member replied, "three" and was let in. The man thought he had heard enough and walked up to the door. The doorman said, "ten" and the man replied, "five." But he was not let in.

What should have he said?
The Problem-Solving Cycle

1. Problem Identification
   – We have to recognize that we have a goal or that the solution we had in mind does not work

2. Problem definition and representation
   – We have to define and represent the problem well enough to understand how to solve it
The Problem-Solving Cycle

3. Strategy Formation

- We have to plan a strategy for solving the problem which may involve
  - Analysis – breaking down the whole of a complex problem into manageable elements
  - Synthesis – putting together various elements to arrange them into something useful
  - Divergent thinking – you try to generate a diverse assortment of possible alternative solutions to a problem
  - Convergent thinking – you narrow down the multiple possibilities to converge on a single, best answer
Strategy Formation

• Analysis
  • Breaking into sub goals
  • Study for exam sub goals
    – Read textbook & class notes
    – Identify most relevant topics
    – Create study questions & answers on note cards
    – Learn all concepts on note cards
    – Test self with note cards
    – Recycle through learning and testing until mastery is achieved
Strategy Formation

• Synthesis
  – Organize to aid solution
• Symbols
• Matrixes
• Diagrams

Let L = Lucy, S = Sean, 2L = 3S, S = 10

<table>
<thead>
<tr>
<th></th>
<th>Mango</th>
<th>Peach</th>
<th>Steak</th>
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<td>x</td>
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<td>Henry</td>
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<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
Strategy Formation

• Divergent thinking
  – Generate multiple solutions to problem

• Convergent thinking
  – Narrow down to best answer
The Problem-Solving Cycle

4. Organization of Information
   - You have to organize the available information in a way that enables you to implement the strategy
   - You organize the information strategically, finding a the most suitable representation

5. Resource Allocation
   - We have limited resources (time, money, equipment, space,...)
   - We have to decide how much we want to invest into the problem solving
The Problem-Solving Cycle

6. Monitoring
   – It is necessary to monitor the process of solving the problem to make sure that we are getting closer to the goal
   – We need to reassess what we are doing to be able to compensate for possible flaws

7. Evaluation
   – You need to evaluate your solution after you have finished
   – New problems can be recognized, the problem may be redefined, new strategies may come to light, and new resources may become available
The Problem-Solving Cycle

• **Incubation**
  – Putting the problem aside for a while
  – Problem will be processed subconsciously
  – The benefits of incubation can be enhanced in two ways:
    • Invest enough time in the problem initially
    • Allow sufficient time for incubation to permit the reorganization of information
Types of Problems

• Well-structured problems
  – Clear path to the solution
    • Math problems
    • Anagrams

• Ill-structured problems
  – Dimensions of problem are not specified or easy to infer
    • Finding an apartment
    • Writing a book
Well-Structured Problems

• Problems with clear paths to their solutions
• Computer simulations of well-defined problems
  – Problem space
    • The universe of all possible actions that can be applied to solving a problem
  – Algorithms
    • Sequences of operations that may be used recursively (repeated over and over again)
Well-Structured Problems

• Humans use heuristics
  – Informal, intuitive, speculative strategies that sometimes lead to an effective solution and sometimes do not
  – If we store in long-term memory several simple heuristics that we can apply to a variety of problems, we can lessen the burden of our limited-capacity working memory
Well-Structured Problems

• Heuristics

  1. Means-ends analysis
     • Analyze the problem by viewing the end (the goal to be sought) and then try to decrease the distance between the current position in the problem space and the end goal in that space

  2. Working forward
     • Start at the beginning and try to solve the problem from the start to the finish
Well-Structured Problems

• Heuristics

3. Working backward
   • The problem-solver starts at the end and tries to work backward from there

4. Generate and test
   • The problem-solver generates a list of alternative ways of action, not necessarily in a systematic way, and then notices in turn whether each course of action will work
Ill-Structured Problems

• There is no clear, readily available path to solution
  – ill-structured problems do not have well-defined problem spaces, and problem solvers have difficulty constructing appropriate mental representations for modeling these problems and their solutions
Ill-Structured Problems

• Insight
  – a distinctive and sometimes seemingly sudden understanding of a problem or of a strategy that aids in solving the problem
  – Often, an insight involves reconceptualizing a problem or a strategy for its solution in a totally new way
  – Insight can be involved in solving well-structured problems, but it is more often associated with ill-structured problems
Gestalt View of Insight

- **Wertheimer**
  - Sudden rearrangement of elements creates “insight”
  - Productive thinking goes beyond previously learned associations

- **Kohler**
  - Animal Model of Insight
Obstacles to Problem Solving

- Mental set
- Functional fixedness
- Incorrect or incomplete representation of the problem
- Lack of domain knowledge
Mental Set

• Seeing a problem in a particular way instead of other plausible ways due to experience or context
  – This usually causes you to adopt an ineffective strategy and can prevent problem solving
  – Make assumptions without realizing it
  – Difficult to approach the problem in a new way
Functional Fixedness

• An inability to assign new functions and roles to elements of a problem
  – Two string problem
  – Duncker’s candle problem
Problem I

- Two strings are suspended from the ceiling

- **Goal:** Tie the strings together
- **Problem:** Too far apart to hold one and reach for the other

- **Materials:**
  - Chair
  - piece of paper
  - pair of pliers

- **What is the Solution?**
Problem II

- **Materials:**
  - Box of thumb tacks
  - Candle
  - Matches

- **Objective:** Mount candle on wall to make light

- **What is the Solution?**
Transfer

- **Negative Transfer**
  - Solving prior problem makes it more difficult to solve later problem

- **Positive Transfer**
  - Solving earlier problem helps to solve later problem
Einstellung

- Tendency to solve problems in a particular way when a different approach might have been more productive

- Problems 1-5: B-2C-A
- Problems 6-8: A-C
- Problem 8 can’t be solved by B-2C-A
- 80% who see all 8 problems use B-2C-A
- 1% of controls use B-2C-A
- 64% fail to solve #8 vs. 5% of controls
Gick & Holyoak (1980)

- Give participants one problem to read, with a solution
- Then give them a second problem, which can be solved using a similar solution
The Radiation Problem

Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once with sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through will also be destroyed. At lower intensities the rays are harmless to the healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor with rays without destroying the healthy tissue?
Gick & Holyoak (1980)

• 3 groups of participants
  – Control group that only tried to solve the radiation problem
  – A group previously given the analogous General/Fortress problem & solution
  – A group given the General/Fortress problem and told that its solution would help in solving the radiation problem
Gick & Holyoak (1980) Results

![Graph showing results of Control, Analogy, and Analogy & Hint conditions. The graph indicates that the Analogy & Hint condition has a significantly higher score compared to the Control and Analogy conditions.]
Factors Affecting Use of Analogies

- Similarity
- Number of examples exposed to
  - Gick and Holyoak conducted a study in which the dictator story was just one of three other stories participants heard before radiation problem
  - Only 20% got the problem correct
- Whether schema for problem is activated
  - If the two problems are separated by a delay or if they are presented in different contexts, almost none of the participants use the analogy