1.3 Probabilistic Models

A probabilistic model is a mathematical description of an uncertain situation. A probability model consists of an experiment, a sample space, and a probability law.

1.3.1 Experiment

Every probabilistic model involves an underlying process called the experiment.

Ex: Consider the underlying experiments in the two classic probability puzzles: The girl's sibling, and the 3-door problem.

1.3.2 Sample Space

The set of all possible results (OUTCOMES) of an experiment is called the SAMPLE SPACE (Ω) of the experiment.

Ex: List the sample spaces corresponding to the following experiments:

- Experiment 1: Toss a coin and look at the outcome.
  $\Omega =$
- Experiment 2: Toss a coin until you get "Heads".
  $\Omega =$
- Experiment 3: Throw a dart into a circular region of radius r, and check how far it fell from the center.
- Experiment 4: Pick a point (x, y) on the unit square.
- Experiment 5: A family has two children.
- Experiment 6: I select a door, one of the three doors is concealing a prize.

Definition 2 *An event is a subset of the sample space* Ω.

- Ω: certain event, $\emptyset$: impossible event
- TRIAL: single performance of an experiment
- An event A is said to have OCCURRED if the outcome of the trial is in A.
- A given physical situation may be modeled in many different ways. The sample space should be chosen appropriately with regard to the intended goal of modeling.
- Sequential models: tree-based sequental description
  Ex: Consider two rounds of the double-and-quarter game and list all possible outcomes. Consider three tosses of a coin and write all possible outcomes.

1.3.3 Probability Law

The probability law assigns to every event A a nonnegative number $P(A)$ called the probability of event A.

$P(A)$ reflects our knowledge or belief about A. It is often intended as

a model for the frequency with which the experiment produces a value in A when repeated many times independently.

Ex:

Probability Axioms

1. (Nonnegativity) $P(A) \geq 0$ for every event A
2. (Additivity) If A and B are two disjoint events, then

$$P(A \cup B) = P(A) + P(B).$$

   More generally, if the sample space has an infinite number of elements and $A_1, A_2, \ldots$ is a sequence of disjoint events, then

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots.$$

3. (Normalization) $P(\Omega) = 1$

1.3.4 Properties of Probability Laws

(a) $P(\emptyset) = 0$

(b) $P(A^c) = 1 - P(A)$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(d) $A \subset B \Rightarrow P(A) \leq P(B)$

(e) $P(A \cup B) \leq P(A) + P(B)$ $(P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i))$

(f) $P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C)$