2.7.3 Iterated expectation

Using the total expectation theorem lets us compute the expectation of a random variable iteratively: To compute $E(X)$, first determine $E(X|Y)$, then use:

$$E(X) = E[E(X|Y)]$$

The outer expectation is over the marginal distribution of Y. This follows from the total expectation theorem, because it is simply a restatement of:

$$E(X) = \sum_y E(X|Y = y)p_Y(y) == E[E(X|Y)]$$

(recall that $E(X|Y)$ is a random variable, taking values $E(X|Y = y)$ with probability $p_Y(y)$.)

Ex: The joint PMF of the random variables X and Y takes the values $[3/12, 1/12, 1/6, 1/6, 1/6, 1/6]$ at the points $[(-1, 2), (1, 2), (1, 1), (2, 1), (-1, -1), (1, -1)]$, respectively. Compute $E(X)$ using iterated expectations.

Ex: Consider three rolls of a fair die. Let X be the total number of 6's, and Y be the total number of 1's. Note that $E(X) = 1/2$. Confirm this result by computing $E(X|Y)$ and then $E(X)$ using iterated expectations.

2.8 Independence

The results developed here will be based on the independence of events we covered in before. Two events A and B are independent if $\mathrm{P}(A \cap B) = \mathrm{P}(A, B) = \mathrm{P}(A)\mathrm{P}(B)$.

2.9 Independence of a R.V. from an Event

Definition 7 *The random variable X is independent of the event A if*

$$\mathrm{P}(\{X = x\} \cap A) = \mathrm{P}(X = x)\mathrm{P}(A) = p_X(x)\mathrm{P}(A)$$

for all x.

Ex: Consider two tosses of a coin. Let X be the number of heads and let A be the event that the number of heads is even. Show that X is NOT independent of A.