Ex: X and Y are "jointly uniform" in a circular region of radius r centered at the origin. Compute the marginal PDFs of X and Y, their expectations, and the expectation of the product XY.

3.5.1 Conditioning One R.V. on Another

Let X, Y be two r.v.s with joint PDF $f_{X,Y}(x,y)$. For any y with $f_Y(y) > 0$, the conditional PDF of X given that $Y = y$ is defined by

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}.$$

It is best to view y as a fixed number and consider the conditional CDF $f_{X|Y}(x|y)$ as a function of the single variable x.

It may seem that the conditioning is on an event with zero probability which would contradict the definition of conditional probability. However, the PDF is not a probability. The conditional PDF $f_{X|Y}(a|b)$ describes the concentration of probability when X is within a small neighbourhood of a, given that Y is within a small neighbourhood of b.

$$\begin{aligned} \mathrm{P}(a \le X \le a+\delta_1 | b \le Y \le b+\delta_2) &= \frac{\mathrm{P}(a \le X \le a+\delta_1, b \le Y \le b+\delta_2)}{\mathrm{P}(b \le Y \le b+\delta_2)} \\ &\approx \\ &= \end{aligned}$$

Ex: In the example above where X and Y are jointly uniform on a circular region, compute the conditional PDF $f_{X|Y}(x|y)$.

Ex: The speed of Iron-Man flying past an air traffic radar is modelled as an exponentially distributed random variable with mean 200 km per hour. The radar sensor will measure the speed of any target with an additive error. The error is modelled as a zero-mean normal with a standard deviation equal to one tenth of the speed of the target. What is the joint PDF of the actual speed and the error?

Ex: Harry's magic wand breaks at a random point (location of the point is uniform along the length of the stick, which is 40 cm long). Suppose the piece of the stick that Harry is left with is X cm long. Unfortunately, the next day part of the stick is accidentally burnt while casting a spell. After this accident, the length of the stick is reduced to D, where D is uniformly distributed between $[0, X]$. Find $f_D(d)$.