In proving “convergence in probability” we often use Chebychev’s Inequality, as in the following example.

**Ex:** Flip a fair coin \( n \) times, independently. Let \( Y_n \) be equal to the number of heads minus \( n/2 \). Does \( Y_n \) converge?

The following result is a generalization of the previous example.

### 5.3 The Weak Law of Large Numbers

The Weak Law of Large Numbers (WLLN) is an important special case of convergence in probability. Consider \( X_1, X_2, \ldots \) IID, with \( E(X_i) = \mu \), and \( \text{var} X_i = \sigma^2 < \infty \) for all \( i \). The sample mean sequence \( M_n = \frac{X_1 + X_2 + \cdots + X_n}{n} \) converges to \( \mu \) in probability.

**Proof:** (Use the Chebychev Inequality on \( M_n \).)

**Ex:** Polling: We want to estimate the fraction of the population that will vote for XYZ. Let \( X_i \) be equal to 1 if the \( i^{th} \) person votes in favor of XYZ, and 0 otherwise. How many people should we poll, to make sure our error will be less than 0.01 with 95% probability? (Answer: with Chebychev Inequality, we get \( n = 50,000 \). However, this is too conservative. Using the Central Limit Theorem, we will get that a poll over a much smaller number of people will suffice.)