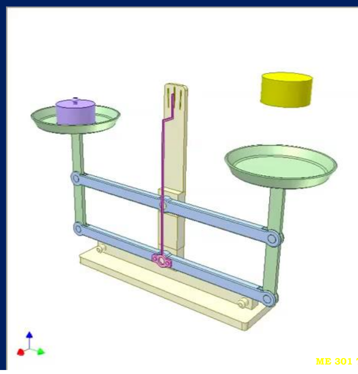
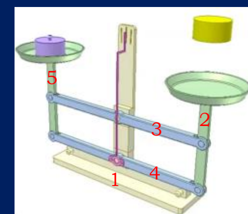
Degree of Freedom of Mechanisms



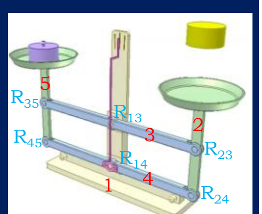
Degree of Freedom of Mechanisms

Equal arm balance:
$\lambda = 3$ (co-planar)
$\ell = 5$



Degree of Freedom of Mechanisms

Equal arm balance:
$\lambda = 3$ (co-planar)
$\ell = 5$
j = 6
$\sum_{i=1}^{6} f_i = 6$ (6R)



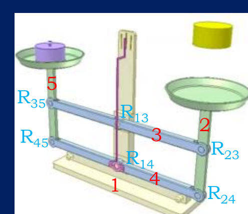
Degree of Freedom of Mechanisms

Equal arm balance:
$\lambda = 3$ (co-planar)
$\ell = 5$
j = 6
$\sum_{i=1}^{6} f_i = 6$ (6R)

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$$F = 3(5 - 6 - 1) + 6$$

$$F = 0!$$



The equal arm balance is in *permanent critical form.* It will not work for any arbitrary dimensions however here it has special dimensions which lets it work:

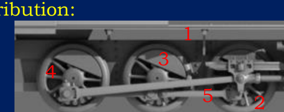
$|R_{35}R_{45}| = |R_{13}R_{14}| = |R_{23}R_{24}|$ and
$|R_{13}R_{35}| = |R_{45}R_{14}|$ (=) $|R_{13}R_{23}| = |R_{14}R_{24}|$
(=) is for equal arm balance.

Degree of Freedom of Mechanisms

Exceptions: Mechanism in Permanent Critical Form

Degree of Freedom of Mechanisms

Exceptions: Mechanism in Permanent Critical Form
Steam locomotive drive distribution:
$\lambda = 3$ (co-planar)
$\ell = 5$



Degree of Freedom of Mechanisms

Exceptions: Mechanism in Permanent Critical Form

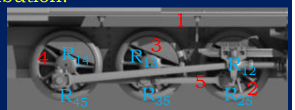
Steam locomotive drive distribution:

λ = 3 (co-planar)

ℓ = 5

j = 6

$\sum_{i=1}^{6} f_i$ = 6 (6R)



Degree of Freedom of Mechanisms

Exceptions: Mechanism in Permanent Critical Form

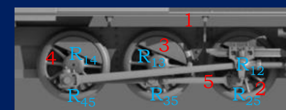
Steam locomotive drive distribution:

λ = 3 (co-planar)

ℓ = 5

j = 6

$\sum_{i=1}^{6} f_i$ = 6 (6R)



$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(4 - 6 - 1) + 6$

$F = 0!$

The parallelogram linkage is in permanent critical form. It will not work for *any arbitrary* dimensions however here it has special dimensions which lets it work:

$|R_{14}R_{45}| = |R_{13}R_{35}| = |R_{12}R_{25}|$ and

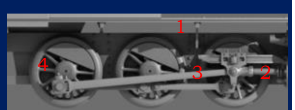
$|R_{13}R_{14}| = |R_{35}R_{45}|$, $|R_{13}R_{12}| = |R_{35}R_{25}|$

Degree of Freedom of Mechanisms

Steam locomotive power arm:

λ = 3 (co-planar)

ℓ = 4



Degree of Freedom of Mechanisms

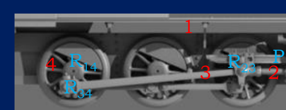
Steam locomotive power arm:

λ = 3 (co-planar)

ℓ = 4

j = 4

$\sum_{i=1}^{4} f_i$ = 4 (3R+P)



Degree of Freedom of Mechanisms

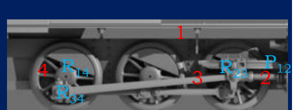
Steam locomotive power arm:

λ = 3 (co-planar)

ℓ = 4

j = 4

$\sum_{i=1}^{4} f_i$ = 4 (3R+P)



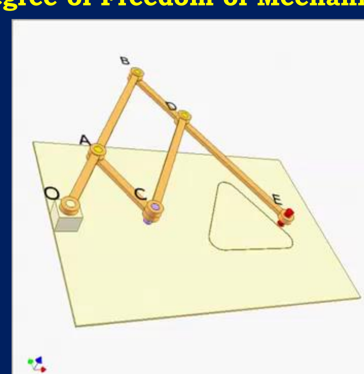
$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(4 - 4 - 1) + 4$

$F = 1$

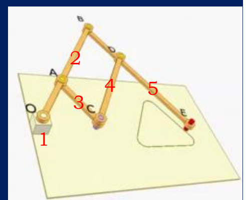
This is just a slider-crank mechanism like an internal combustion engine.

Degree of Freedom of Mechanisms



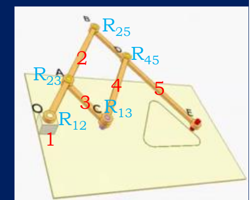
Degree of Freedom of Mechanisms

Pantograph mechanism:
λ = 3 (co-planar)
$\ell = 5$



Degree of Freedom of Mechanisms

Pantograph mechanism:
λ = 3 (co-planar)
$\ell = 5$
j = 5
$\sum_{i=1}^{5} f_i = 5$ (5R)

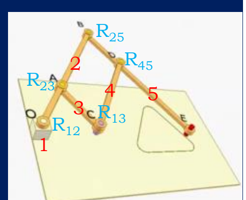


Degree of Freedom of Mechanisms

Pantograph mechanism:
λ = 3 (co-planar)
$\ell = 5$
j = 5
$\sum_{i=1}^{5} f_i = 5$ (5R)

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

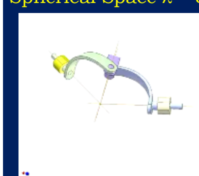
$$F = 3(5 - 5 - 1) + 5$$
$$F = 2$$



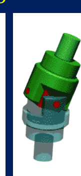
Point E can trace any curve on plane (i.e arbitrary say x and y) so that point C can follow the same curve scaled down.

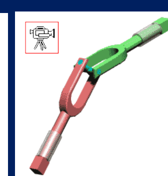
Special Spaces

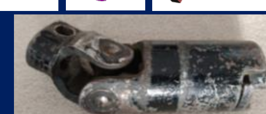
Spherical Space λ = 3



Spherical Four-Bar





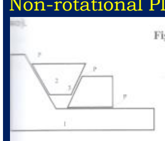


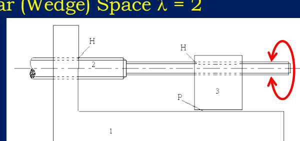
Cardan (Universal) "*Joint*"s

Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Special Spaces

Non-rotational Planar (Wedge) Space λ = 2





Mechanisms with Screws

Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132
and https://solomon.desertcart.com/products/50863575-multi-ware-car-scissor-lift-car-jack-1-ton

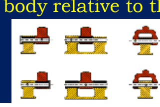
Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

The general degree of freedom equation cares only the topological characteristics of the mechanism (e.g. number of links, number and type of joints *but* not the dimensions) and the degree of freedom of the space the bodies are in.

- A joint should permit relative motion of a body with respect to other.
  If there is no relative motion permitted, it is *not* a joint kinematically and those parts of the body should be accounted as a single link.
- What identifies the joint type is the relative motion of one body relative to the other.



Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

- The degree of freedom of the space of all the links in the mechanism (i.e. λ) should be the same. If not the mechanism should be separated.

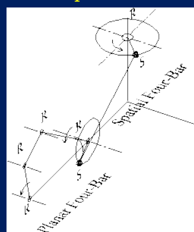


Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

- The degree of freedom of *ordinary* gear pair, $f_i = 1$ because it is rolling without slipping.
- The *regular* gear pair (GP*) is in permanent critical form because of special dimensional requirements:
  - The gears are centered at certain special points (generally centers of circles),
  - The distances between these joints are the sum (or difference if internal mesh) of the radii of the gears.

  Therefore GP* is a *workaround* to obtain correct degree of freedom for the gears in permanent critical form.

Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

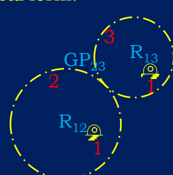
Therefore GP* is a workaround to obtain correct degree of freedom for the gears in permanent critical form.

$\lambda = 3$ (co-planar)

$\ell = 3$

$j = 3$

$\sum_{i=1}^{3} f_i = 3$ (2R+GP)



$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(3 - 3 - 1) + 3$

$F = 0$

Will not work unless R_{12} and R_{13} are centers of gears **AND** $|R_{12}R_{13}| = r_2 + r_3$

Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

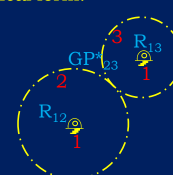
Therefore GP* is a workaround to obtain correct degree of freedom for the gears in permanent critical form.

$\lambda = 3$ (co-planar)

$\ell = 3$

$j = 3$

$\sum_{i=1}^{3} f_i = 4$ (2R+GP*)



$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(3 - 3 - 1) + 4$

$F = 1$

Permanent critical form because R_{12} and R_{13} are centers of gears **AND** $|R_{12}R_{13}| = r_2 + r_3$

Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

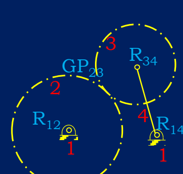
Force-closed gear pair:

$\lambda = 3$ (co-planar)

$\ell = 4$

$j = 4$

$\sum_{i=1}^{4} f_i = 4$ (3R+GP)



$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(4 - 4 - 1) + 4$

$F = 1$

Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

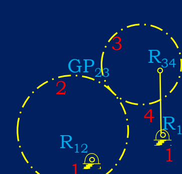
Force-closed gear pair:

$\lambda = 3$ (co-planar)

$\ell = 4$

$j = 4$

$\sum_{i=1}^{4} f_i = 4$ (3R+GP)



$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(4 - 4 - 1) + 4$

$F = 1$

Constrained – Unconstrained Mechanisms

Constrained mechanism may mean:

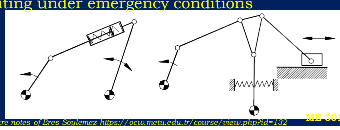
1. F = 1
2. F > 1 and number of actuators = F

Unconstrained (underactuated) mechanisms have number of inputs < F therefore their motion cannot be determined kinematically. Forces determine the motion.

Typical applications:

Car differential (F = 2 only drive is engine so that on straight roads two wheel share the same speed but during turns inner wheel is slower)

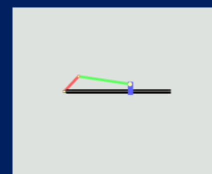
Force limiting under emergency conditions

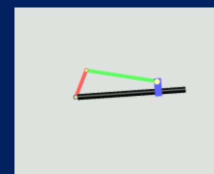


Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Kinematic Inversion

Let the fixed link move and fix another link with the purpose of obtaining another mechanism.





Kinematic Inversion

Let the fixed link move and fix another link with the purpose of obtaining another mechanism.

Grübler's Equation

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

F = 1 (Single degree of freedom / constrained mechanism)

λ = 3 : Planar mechanism

ℓ: Number of links of the mechanism

j: Number of joints of the mechanism

f_i = 1 (only revolute and prismatic joints so $\sum_{i=1}^{j} f_i$ = j)

$1 = 3(\ell - j - 1) + j$

$\boxed{3\ell - 2j = 4}$: Grübler's (Gruebler's) Equation

Grübler's Equation

$3\ell - 2j = 4$

Consequences:

- ℓ must be even

  $3\ell = 2j + 4$, $2j + 4$ must be even whatever j is. For 3ℓ to be even ℓ must be even!

- $\ell_2 \geq 4$

  $\ell = \ell_2 + \ell_3 + \ell_4 + \cdots + \ell_n$

  Number of kinematic elements in a mechanism = $2j$

  $2j = 2\ell_2 + 3\ell_3 + 4\ell_4 + \cdots + n\ell_n$

  Substituting into Grübler's equation:

  $3(\ell_2 + \ell_3 + \ell_4 + \cdots + \ell_n) - (2\ell_2 + 3\ell_3 + 4\ell_4 + \cdots + n\ell_n) = 4$

  $\ell_2 - [\ell_4 + 2\ell_5 \ldots + (n-3)\ell_n] = 4$

  $\ell_4 + 2\ell_5 \ldots + (n-3)\ell_n = P \geq 0$

  For $P = 0$, $\ell_2 = 4$, for $P > 0$, $\ell_2 > 4$

Grübler's Equation

$3\ell - 2j = 4$

Consequences (cont'ed):

- Number of kinematic elements in one link cannot exceed half the number of links in the mechanism, $\ell_k, k \leq \ell/2$

  Consider type (a) link with maximum possible kinematic elements on it.

  Type (b) links (ternary) links are connected to type (a) links.

  Type (c) links (binary) connect type (b) links.

  Type (a) link has i kinematic elements on it and there are i type (b) links, (i-1) type (c) links.

  Total number of links in the mechanism for this case is

  $\ell = 1 + i + (i-1) = 2i$

  or

  $i = \frac{\ell}{2}$

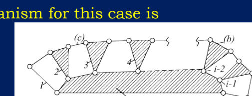


Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Enumeration of Kinematic Chains and Mechanisms

Enumeration: A complete, ordered listing of all the items in a collection (*Wikipedia*)

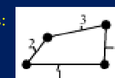
1. the act or process of making or stating a list of things one after another also: the list itself
2. the act or process of counting something or a count made of something

Determine all possible kinematic chains that satisfy certain predetermined criteria.

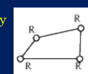
Enumeration of Kinematic Chains and Mechanisms

$3\ell - 2j = 4$

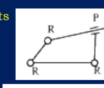
List all planar F = 1 mechanisms with 4 links:



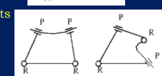
With revolute joints only



With three revolute and one prismatic joints



With two revolute and two prismatic joints



Three prismatic joints is not allowed (why?)

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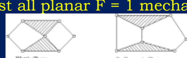
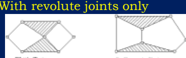
Enumeration of Kinematic Chains and Mechanisms

$3\ell - 2j = 4$

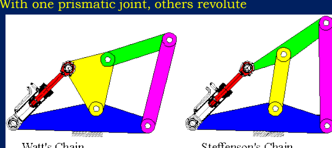
List all planar F = 1 mechanisms with 6 links:



With revolute joints only



With one prismatic joint, others revolute



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