2. Kinematic Analysis

3. Analytic–Closed Form Solution of Loop Closure Equations

Inverted Slider-Crank Mechanism

$$s_{23}e^{i\theta_{12}} + a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = a_1 + a_4 e^{i\theta_{14}}$$

Re: $s_{23}cos\theta_{12} + a_3 sin\theta_{12} = a_1 + a_4 cos\theta_{14}$ (1)

Im: $s_{23}sin\theta_{12} - a_3 cos\theta_{12} = a_4 sin\theta_{14}$ (2)

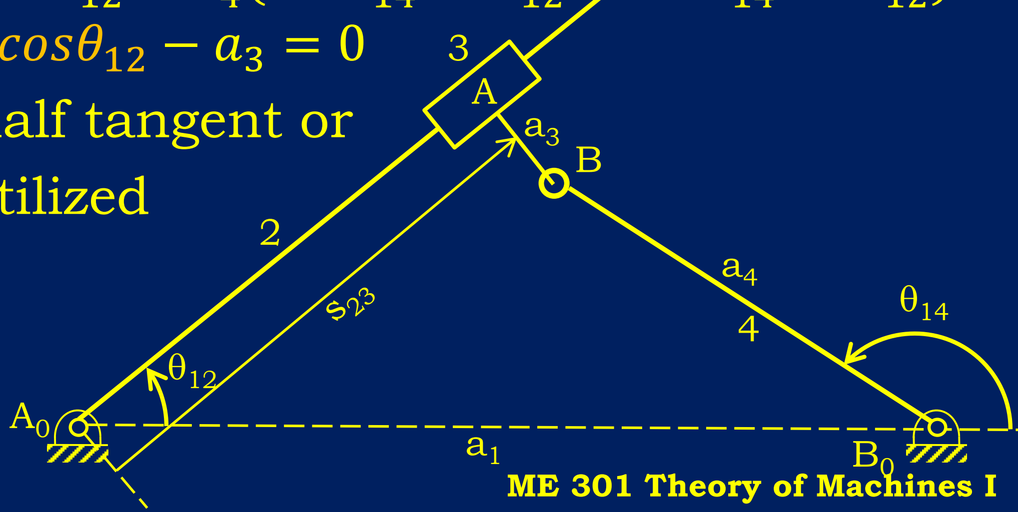
1. Let θ_{14} be the input

b. First solve θ_{12} then s_{23}

$sin\theta_{12}(1) - cos\theta_{12}(2) \rightarrow a_3 = a_1 sin\theta_{12} + a_4(cos\theta_{14}sin\theta_{12} - sin\theta_{14}cos\theta_{12})$

$(a_1 + a_4 cos\theta_{14})sin\theta_{12} - a_4 sin\theta_{14}cos\theta_{12} - a_3 = 0$

To solve this equation either half tangent or phase angle method may be utilized to obtain two closures.



2. Kinematic Analysis

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Inverted Slider-Crank Mechanism

1. Let θ_{14} be the input

    b. First solve θ_{12} then s_{23} (cont'ed)

    In general s_{23} can be obtained either from (1) (except $cos\theta_{12} = 0$) or from (2) (except $sin\theta_{12} = 0$) however while performing full cycle analysis to avoid *false singularities* the following procedure works:

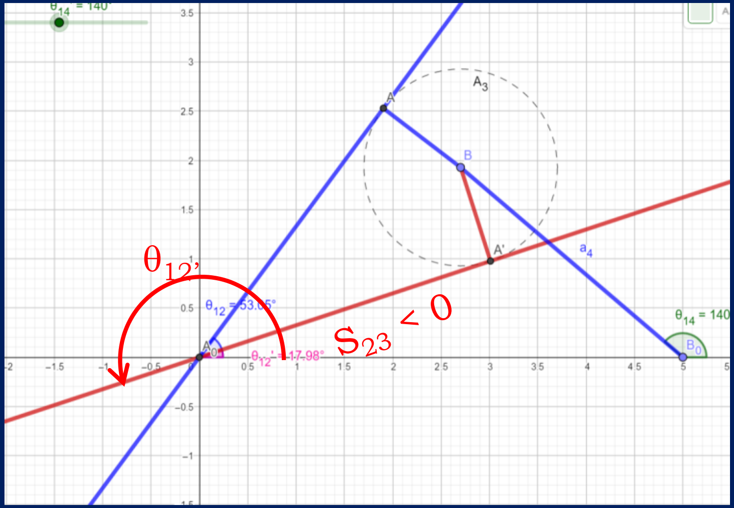
$$s_{23}cos\theta_{12} = a_1 + a_4cos\theta_{14} - a_3sin\theta_{12} = x_{23}$$

$$s_{23}sin\theta_{12} = a_4sin\theta_{14} + a_3cos\theta_{12} = y_{23}$$

Multiply x_{23} by $cos\theta_{12}$ and y_{23} by $sin\theta_{12}$

$$s_{23} = x_{23}cos\theta_{12} + y_{23}sin\theta_{12}$$

which is free of singularities for all values of θ_{12}.



2. Kinematic Analysis

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Inverted Slider-Crank Mechanism

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Im: $s_{23}sin\theta_{12} - a_3 cos\theta_{12} = a_4 sin\theta_{14}$ (2)

2. Let s_{23} be the input

a. First solve θ_{12} then θ_{14}

$$a_4 cos\theta_{14} = s_{23}cos\theta_{12} + a_3 sin\theta_{12} - a_1$$

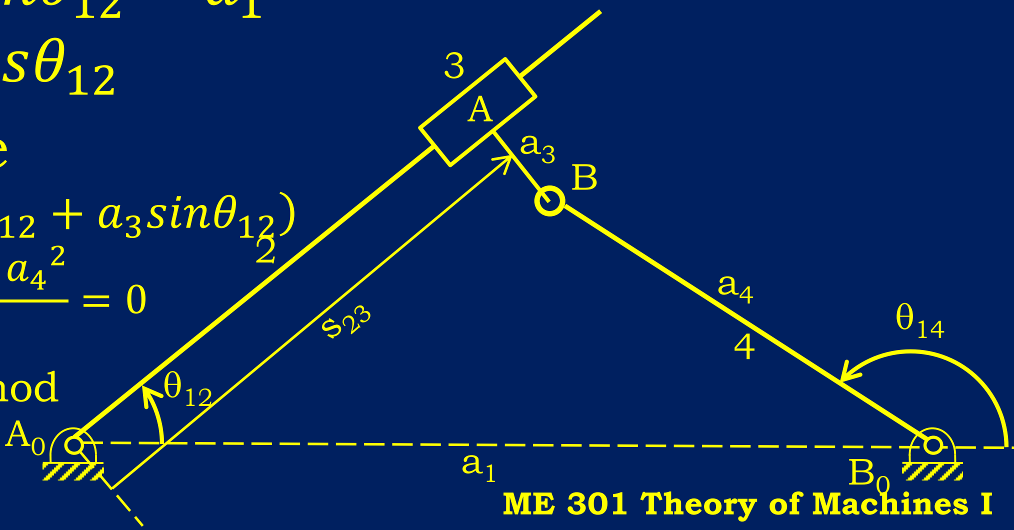
$$a_4 sin\theta_{14} = s_{23}sin\theta_{12} - a_3 cos\theta_{12}$$

Sum of the squares of these

$$a_4{}^2 = s_{23}{}^2 + a_1{}^2 + a_3{}^2 - 2a_1(s_{23}cos\theta_{12} + a_3 sin\theta_{12})$$

$$s_{23}cos\theta_{12} + a_3 sin\theta_{12} - \frac{s_{23}{}^2 + a_1{}^2 + a_3{}^2 - a_4{}^2}{2a_1} = 0$$

Use half-tangent or phase angle method to solve and get two closures.



2. Kinematic Analysis

3. Analytic–Closed Form Solution of Loop Closure Equations

Inverted Slider-Crank Mechanism

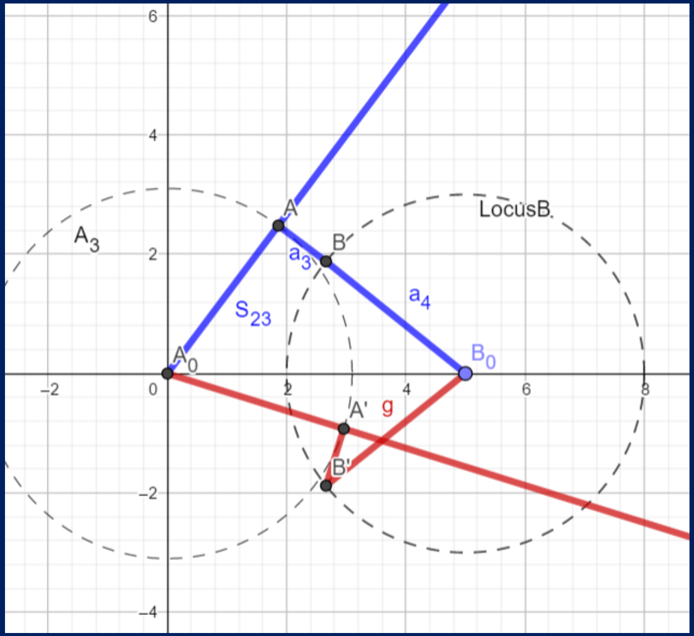
2. Let s_{23} be the input

   a. First solve θ_{12} then θ_{14}(cont'ed)

$$a_4 cos\theta_{14} = s_{23} cos\theta_{12} + a_3 sin\theta_{12} - a_1 = x_{14}$$

$$a_4 sin\theta_{14} = s_{23} sin\theta_{12} - a_3 cos\theta_{12} = y_{14}$$

$$a_4, \theta_{14} = Pol(x_{14}, y_{14})$$



2. Kinematic Analysis

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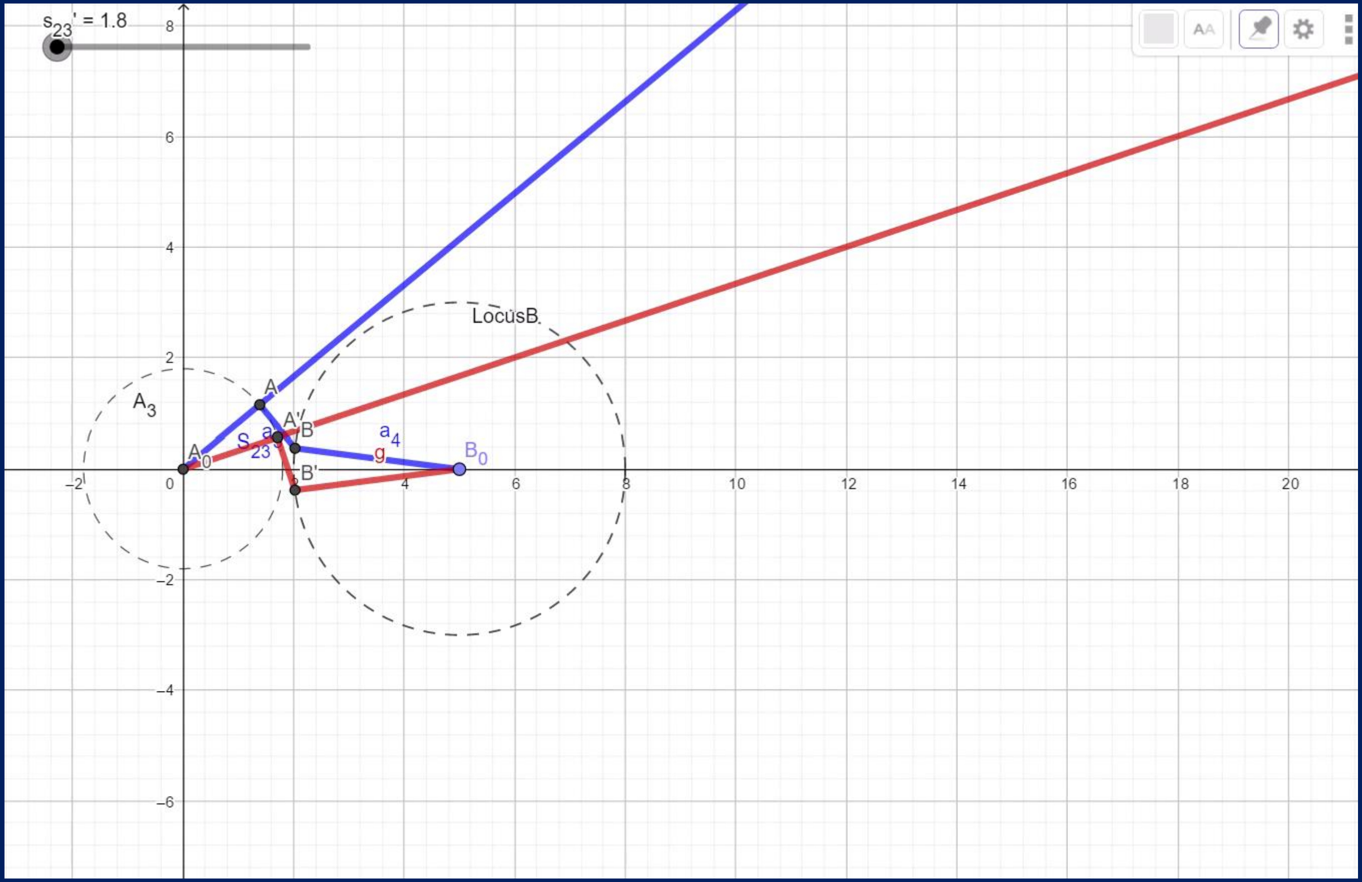
Inverted Slider-Crank Mechanism

2. Let s_{23} be the input



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Im: $s_{23}sin\theta_{12} - a_3 cos\theta_{12} = a_4 sin\theta_{14}$ (2)

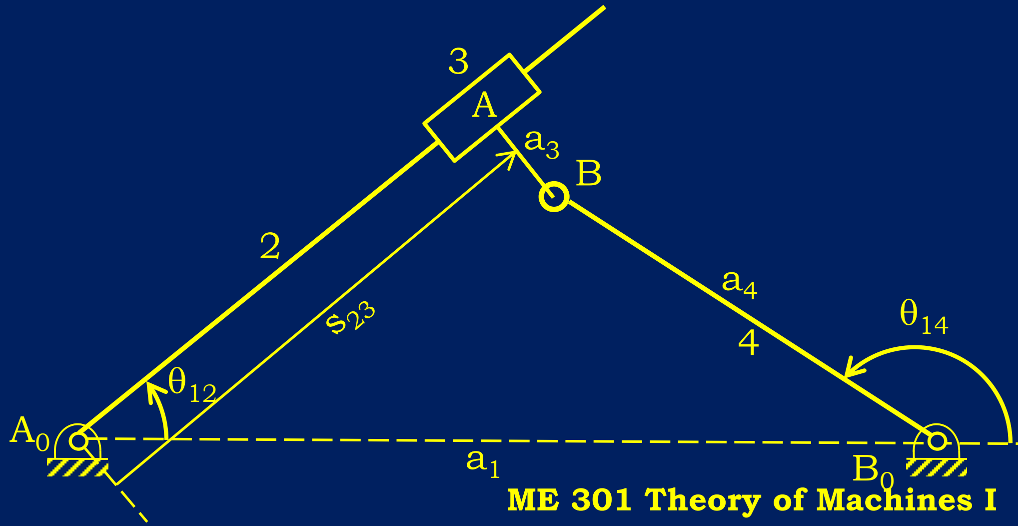
2. Let s_{23} be the input

b. First solve θ_{14} then θ_{12}

Very similar to case a

3. Let θ_{14} be the input

Very similar to 1



2. Kinematic Analysis

3. Analytic–Closed Form Solution of Loop Closure Equations

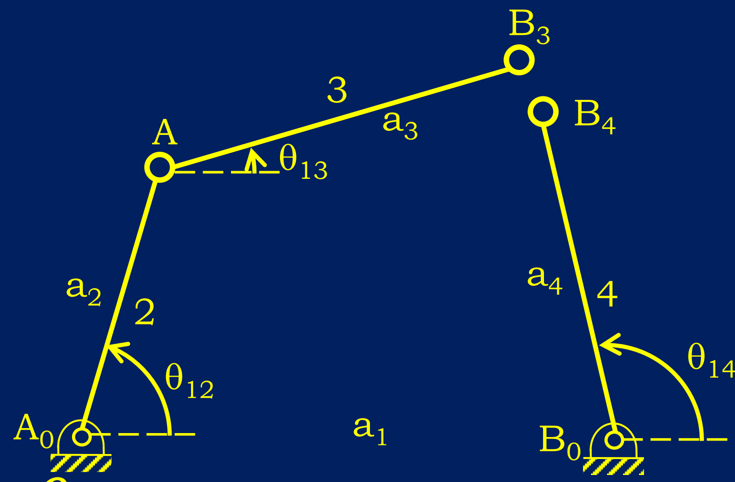
Four-Bar Mechanism

$$\overrightarrow{A_0A} + \overrightarrow{AB_3} = \overrightarrow{A_0B_0} + \overrightarrow{B_0B_4}$$

$$a_2e^{i\theta_{12}} + a_3e^{i\theta_{13}} = a_1 + a_4e^{i\theta_{14}}$$

Re: $a_2cos\theta_{12} + a_3cos\theta_{13} = a_1 + a_4cos\theta_{14}$ (1)

Im: $a_2sin\theta_{12} + a_3sin\theta_{13} = a_4sin\theta_{14}$ (2)



Let θ_{12} be the input, solve θ_{14} then θ_{13}

$$a_3cos\theta_{13} = a_1 + a_4cos\theta_{14} - a_2cos\theta_{12}$$

$$a_3sin\theta_{13} = a_4sin\theta_{14} - a_2sin\theta_{12}$$

Square and add:

$$2a_1a_4cos\theta_{14} - 2a_1a_2cos\theta_{12} - 2a_2a_4(cos\theta_{12}cos\theta_{14} + sin\theta_{12}sin\theta_{14}) + a_1{}^2 + a_2{}^2 - a_3{}^2 + a_4{}^2 = 0$$

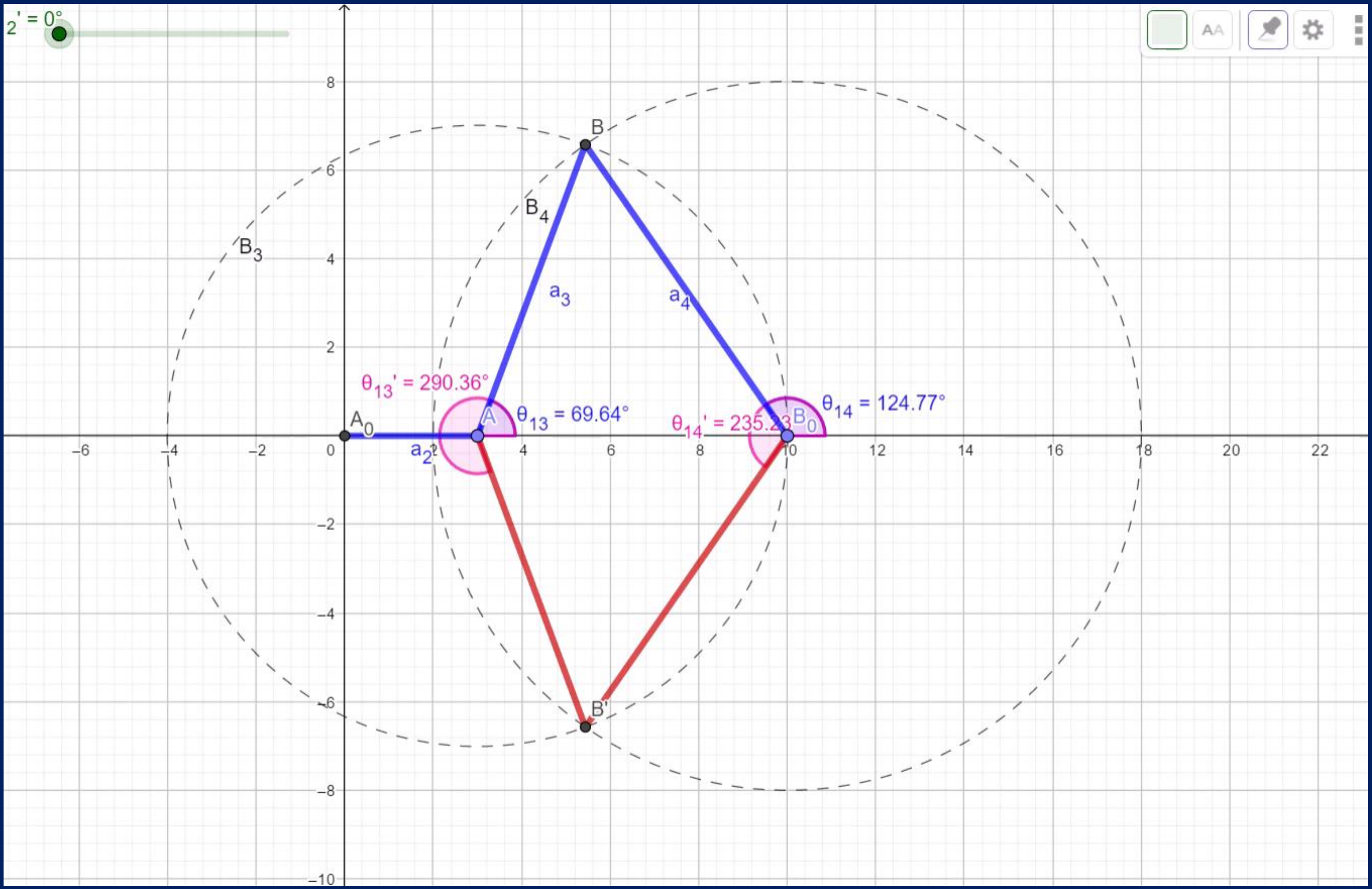
Can be solved by half-tangent or phase angle method.

Then both $cos\theta_{13}$ and $sin\theta_{13}$ can be determined to find unique θ_{13}

2. Kinematic Analysis

3. Analytic–Closed Form Solution of Loop Closure Equations

Four-Bar Mechanism



2. Kinematic Analysis

3. Analytic–Closed Form Solution of Loop Closure Equations

Take Home Message

1. The equation about the first variable to be solved is in one of the following forms:
   a. Quadratic (special case perfect square),
   b. **Only** sine (or cosine) of the angle is known,
   c. The equation is in a form $acos\theta + bsin\theta + c = 0$.

   All these equations have **two** (*very rarely double*) **solutions** which determine the two closures of the mechanism.
2. The equation about the second variable always has a unique solution which depends on the closure selected in 1.
   a. Sine and cosine of the angle are **both** known,
   b. The unknown is linear in terms of knowns.

In general each loop has two closures so for a mechanism with L loops the number of possible closures is 2^L.

2. Kinematic Analysis

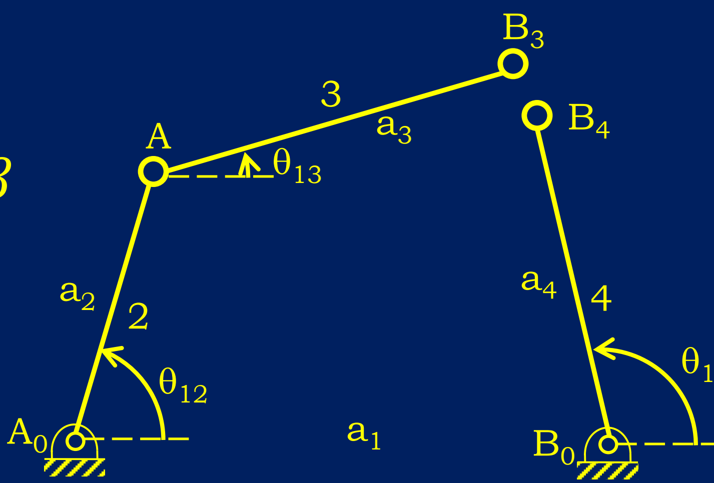
3. Analytic–Closed Form Solution of Loop Closure Equations

Returning back to Four-Bar Mechanism

$$2a_1a_4cos\theta_{14} - 2a_1a_2cos\theta_{12} - 2a_2a_4(cos\theta_{12}cos\theta_{14} + sin\theta_{12}sin\theta_{14}) + {a_1}^2 + {a_2}^2 - {a_3}^2 + {a_4}^2 = 0$$

Define:

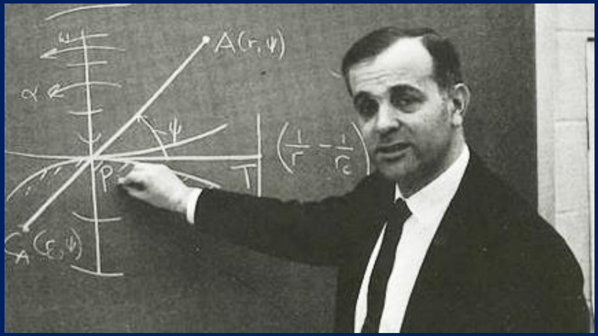
$$K_1 = \frac{a_1}{a_2}, K_2 = \frac{a_1}{a_4}, K_3 = \frac{{a_1}^2 + {a_2}^2 - {a_3}^2 + {a_4}^2}{2a_2a_4}$$



and recall $cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$

$$K_1cos\theta_{14} - K_2cos\theta_{12} + K_3 = cos(\theta_{14} - \theta_{12})$$

Freudenstein's Equation



Ferdinand Freudenstein (1926-2006)