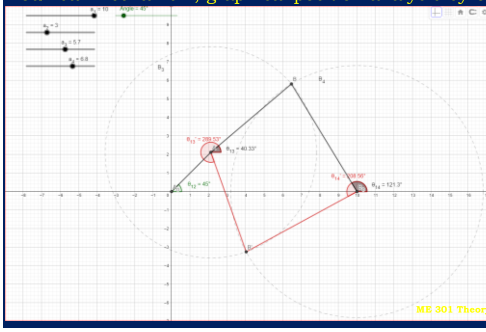
2. Kinematic Analysis

Example Problem

Four-bar mechanism, graphical position analysis by Geogebra



2. Kinematic Analysis

Example Problem

Four-bar mechanism, analytic position analysis by Excel

1. Define the link lengths *and calculate the coefficients of Freudenstein's equation (optional).*
2. Define the sign for closure.

	A	B	C	D
1	Link Lengths			
2	a_1 =	10	ω_{12} =	10
3	a_2 =	3	α_{12} =	0
4	a_3 =	7		
5	a_4 =	8		
6	Closure	-1		
7	K_1	3.3333		
8	K_2	1.25		
9	K_3	2.5833		
10				

2. Kinematic Analysis

Example Problem

Four-bar mechanism, analytic position analysis by Excel

3. Change the names of *fixed parameters.*

a_1 | 10

	A	B	C	D
1	Link Lengths			
2	a_1 =	10	ω_{12} =	10
3	a_2 =	3	α_{12} =	0
4	a_3 =	7		
5	a_4 =	8		
6	Closure	-1		
7	K_1	3.3333		
8	K_2	1.25		
9	K_3	2.5833		

2. Kinematic Analysis

Example Problem

Four-bar mechanism, analytic position analysis by Excel

4. Calculate the coefficients of quadratic equation for θ_{14}.

G14 | =(1-K_2)*COS(B14)+K_3-K_1

	A	B	C	D	E	F	G	H	I
1	Link Lengths								
2	a_1 =	10	ω_{12} =	10					
3	a_2 =	3	α_{12} =	0					
4	a_3 =	7							
5	a_4 =	8							
6	Closure	-1							
7	K_1	3.3333							
8	K_2	1.25							
9	K_3	2.5833							
10									
11	POSITION								
12	θ_{12}		θ_{13}		θ_{14}		Coefficients of Quadratic R		
13	deg	rad	deg	rad	deg	rad	A	B	C
14	0	0	55.738	0.9728	124.85	2.179	-1	0	3.6667
15	1	0.0175	55.455	0.9679	124.42	2.1716	-1	-0.035	3.667

2. Kinematic Analysis

Example Problem

Four-bar mechanism, analytic position analysis by Excel

5. Determine the root corresponding to the selected closure for θ_{14}.

J14 | =(-H14+Clo*SQRT(H14^2-4*G14*I14))/(2*G14)

	A	B	C	D	E	F	G	H	I	J
1	Link Lengths									
2	a_1 =	10	ω_{12} =	10						
3	a_2 =	3	α_{12} =	0						
4	a_3 =	7								
5	a_4 =	8								
6	Closure	-1								
7	K_1	3.3333								
8	K_2	1.25								
9	K_3	2.5833								
10										
11	POSITION									
12	θ_{12}		θ_{13}		θ_{14}		Coefficients of Quadratic			Root
13	deg	rad	deg	rad	deg	rad	A	B	C	
14	0	0	55.738	0.9728	124.85	2.179	-1	0	3.6667	1.9149
15	1	0.0175	55.455	0.9679	124.42	2.1716	-1	-0.035	3.667	1.8976
16	2	0.0349	55.174	0.963	124	2.1642	-0.9998	-0.07	3.668	1.8808
17	3	0.0524	54.895	0.9581	123.58	2.1569	-0.9997	-0.105	3.6698	1.8643
18	4	0.0698	54.619	0.9533	123.17	2.1497	-0.9994	-0.14	3.6721	1.8483
19	5	0.0873	54.346	0.9485	122.76	2.1426	-0.999	-0.174	3.6752	1.8327

2. Kinematic Analysis

Example Problem

Four-bar mechanism, analytic position analysis by Excel

6. Determine θ_{14} in radians (first) then convert into degrees.

F14 | =2*ATAN(J14)

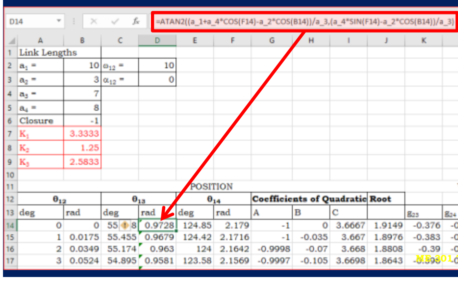
	A	B	C	D	E	F	G	H	I	J
1	Link Lengths									
2	a_1 =	10	ω_{12} =	10						
3	a_2 =	3	α_{12} =	0						
4	a_3 =	7								
5	a_4 =	8								
6	Closure	-1								
7	K_1	3.3333								
8	K_2	1.25								
9	K_3	2.5833								
10										
11	POSITION									
12	θ_{12}		θ_{13}		θ_{14}		Coefficients of Quadratic			Root
13	deg	rad	deg	rad	deg	rad	A	B	C	
14	0	0	55.738	0.9728	124.85	2.179	-1	0	3.6667	1.9149
15	1	0.0175	55.455	0.9679	124.42	2.1716	-1	-0.035	3.667	1.8976
16	2	0.0349	55.174	0.963	124	2.1642	-0.9998	-0.07	3.668	1.8808

2. Kinematic Analysis

Example Problem

Four-bar mechanism, analytic position analysis by Excel

7. Determine θ_{13} in radians (first) then convert into degrees.

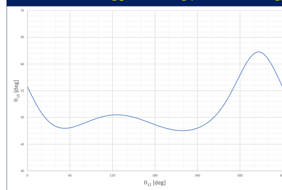


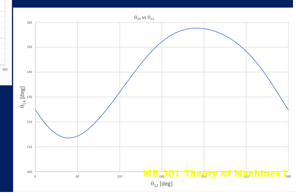
2. Kinematic Analysis

Example Problem

Four-bar mechanism, analytic position analysis by Excel

8. Obtain θ_{13} and θ_{14} versus θ_{12} plots.



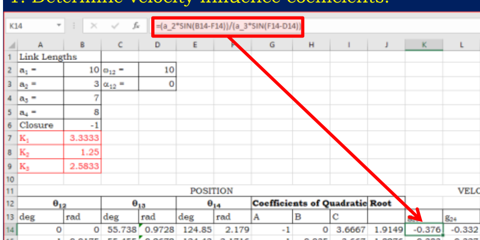


2. Kinematic Analysis

Example Problem

Four-bar mechanism, velocity and acceleration analysis by Excel

1. Determine velocity influence coefficients.

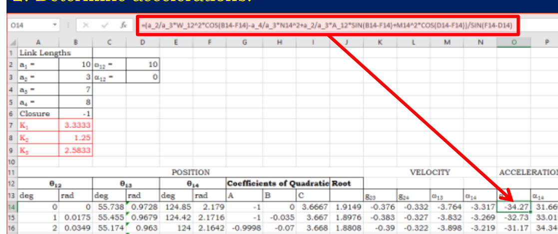


2. Kinematic Analysis

Example Problem

Four-bar mechanism, velocity and acceleration analysis by Excel

2. Determine accelerations.

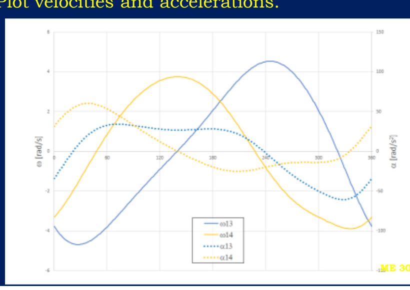


2. Kinematic Analysis

Example Problem

Four-bar mechanism, velocity and acceleration analysis by Excel

3. Plot velocities and accelerations.



2. Kinematic Analysis

Example Problem

(Textbook Prob. 32, p. 208)

a. Determine DOF

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

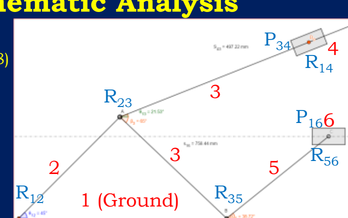
$\lambda = 3$

$\ell = 6$

$j = 7(5R, 2P)$

$$\sum_{i=1}^{7} f_i = 7$$

$$F = 3(6 - 7 - 1) + 7 = 1$$



2. Kinematic Analysis

Example Problem

(Textbook Prob. 32, p. 208)

a. Write LCE(s)

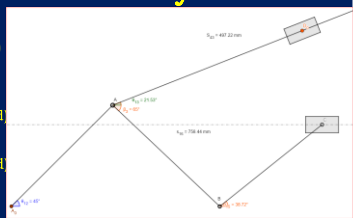
Disconnect A and C

Reconnect A (C disconnected)

$\overrightarrow{A_0A_2} = \overrightarrow{A_0D_O} + \overrightarrow{D_0A_3}$

Reconnect C (A disconnected)

$\overrightarrow{D_0A} + \overrightarrow{AB} + \overrightarrow{BC_5} = \overrightarrow{D_OC_5}$



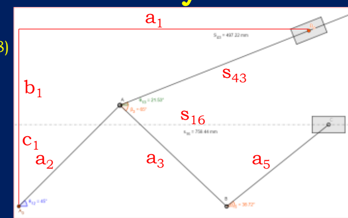
ME 301 Theory of Machines I

2. Kinematic Analysis

Example Problem

(Textbook Prob. 32, p. 208)

a. Write LCE(s)



$\overrightarrow{A_0A_2} = \overrightarrow{A_0D_O} + \overrightarrow{D_0A_3}$

$a_2e^{i\theta_{12}} = a_1 + ib_1 - s_{43}e^{i\theta_{13}}$

$\overrightarrow{D_0A} + \overrightarrow{AB} + \overrightarrow{BC_5} = \overrightarrow{D_OC_5}$

$-s_{43}e^{i\theta_{13}} + a_3e^{i(\theta_{13}-\beta_3)} + a_5e^{i\theta_{15}} = (s_{16} - a_1) - \mathrm{i}(b_1 - c_1)$

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2. Kinematic Analysis

Example Problem

(Textbook Prob. 32, p. 208)

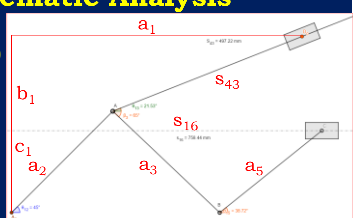
b. Solve LCE(s), θ_{12} input

$a_2e^{i\theta_{12}} = a_1 + ib_1 - s_{43}e^{i\theta_{13}}$

$s_{43}e^{i\theta_{13}} = a_1 + ib_1 - a_2e^{i\theta_{12}}$

$s_{43}cos\theta_{13} = a_1 - a_2cos\theta_{12}$

$s_{43}sin\theta_{13} = b_1 - a_2sin\theta_{12}$



Square and add:

$s_{43}{}^2 = (a_1 - a_2cos\theta_{12})^2 + (b_1 - a_2sin\theta_{12})^2$

$s_{43} = \sigma\sqrt{(a_1 - a_2cos\theta_{12})^2 + (b_1 - a_2sin\theta_{12})^2}$

$\theta_{13} = Pol\left(\frac{a_1 - a_2cos\theta_{12}}{s_{43}}, \frac{b_1 - a_2sin\theta_{12}}{s_{43}}\right)$

Two θ_{13} solutions are 180° apart and two s_{43} are negative of each other!

ME 301 Theory of Machines I

2. Kinematic Analysis

Example Problem

(Textbook Prob. 32, p. 208)

b. Solve LCE(s), θ_{12} input

$-s_{43}e^{i\theta_{13}} + a_3e^{i(\theta_{13}-\beta_3)} + a_5e^{i\theta_{15}} = (s_{16} - a_1) - \mathrm{i}(b_1 - c_1)$

$a_5e^{i\theta_{15}} = (s_{16} - a_1) - \mathrm{i}(b_1 - c_1) + s_{43}e^{i\theta_{13}} - a_3e^{i(\theta_{13}-\beta_3)}$

$a_5cos\theta_{15} = (s_{16} - a_1) + s_{43}cos\theta_{13} - a_3cos(\theta_{13} - \beta_3)$

$a_5sin\theta_{15} = -(b_1 - c_1) + s_{43}sin\theta_{13} - a_3sin(\theta_{13} - \beta_3)$

Square and add:

$a_5{}^2 = [(s_{16} - a_1) + s_{43}cos\theta_{13} - a_3cos(\theta_{13} - \beta_3)]^2 + [-(b_1 - c_1) + s_{43}sin\theta_{13} - a_3sin(\theta_{13} - \beta_3)]^2$

Quadratic in s_{16}

$\theta_{15} = Pol\left(\frac{(s_{16} - a_1) + s_{43}cos\theta_{13} - a_3cos(\theta_{13} - \beta_3)}{a_5}, \frac{-(b_1 - c_1) + s_{43}sin\theta_{13} - a_3sin(\theta_{13} - \beta_3)}{a_5}\right)$

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2. Kinematic Analysis

Example Problem

(Textbook Prob. 32, p. 208)

c. Obtain velocity loop equations, θ_{12} input

$s_{43}cos\theta_{13} = a_1 - a_2cos\theta_{12}$

$s_{43}sin\theta_{13} = b_1 - a_2sin\theta_{12}$

$\dot{s}_{43}cos\theta_{13} - \dot{\theta}_{13}s_{43}sin\theta_{13} = \dot{\theta}_{12}a_2sin\theta_{12}$

$\dot{s}_{43}sin\theta_{13} + \dot{\theta}_{13}s_{43}cos\theta_{13} = -\dot{\theta}_{12}a_2cos\theta_{12}$

$$\begin{bmatrix} cos\theta_{13} & -s_{43}sin\theta_{13} \\ sin\theta_{13} & s_{43}cos\theta_{13} \end{bmatrix}\begin{bmatrix} \dot{s}_{43} \\ \dot{\theta}_{13} \end{bmatrix} = \begin{bmatrix} a_2sin\theta_{12} \\ -a_2cos\theta_{12} \end{bmatrix}\dot{\theta}_{12}$$

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2. Kinematic Analysis

Example Problem

(Textbook Prob. 32, p. 208)

c. Obtain velocity loop equations, θ_{12} input (cont'ed)

$a_5cos\theta_{15} = (s_{16} - a_1) + s_{43}cos\theta_{13} - a_3cos(\theta_{13} - \beta_3)$

$a_5sin\theta_{15} = -(b_1 - c_1) + s_{43}sin\theta_{13} - a_3sin(\theta_{13} - \beta_3)$

$-\dot{\theta}_{15}\, a_5sin\theta_{15} = \dot{s}_{16} + \dot{s}_{43}cos\theta_{13} - \dot{\theta}_{13}s_{43}sin\theta_{13} + \dot{\theta}_{13}a_3sin(\theta_{13} - \beta_3)$

$\dot{\theta}_{15}\, a_5cos\theta_{15} = \dot{s}_{43}sin\theta_{13} + \dot{\theta}_{13}s_{43}cos\theta_{13} - \dot{\theta}_{13}a_3cos(\theta_{13} - \beta_3)$

$$\begin{bmatrix} a_5sin\theta_{15} & 1 \\ -a_5cos\theta_{15} & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta}_{15} \\ \dot{s}_{16} \end{bmatrix} = -\begin{bmatrix} \dot{s}_{43}cos\theta_{13} - \dot{\theta}_{13}s_{43}sin\theta_{13} + \dot{\theta}_{13}a_3sin(\theta_{13} - \beta_3) \\ \dot{s}_{43}sin\theta_{13} + \dot{\theta}_{13}s_{43}cos\theta_{13} - \dot{\theta}_{13}a_3cos(\theta_{13} - \beta_3) \end{bmatrix}$$

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