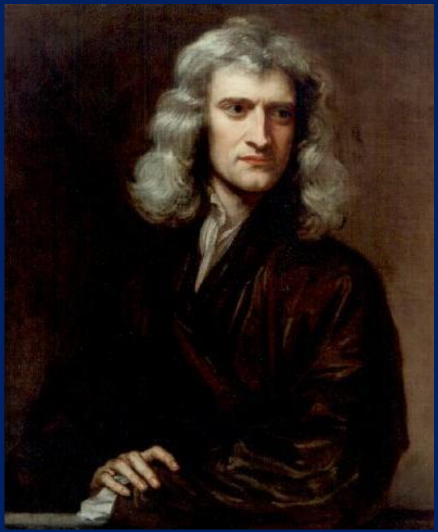
4. Force Analysis

Two Problems of Dynamics

1. Given the forces, find the resulting *motion* (forward dynamics, Wittenbauer's first problem, dynamic motion analysis)
2. Given the motion, determine the forces to produce this motion (inverse dynamics, Wittenbauer's second problem, dynamic force analysis, kinetostatics)

   Special Case: The system is stationary (or moving very slowly therefore *inertia forces* are small compared to other forces), determine the forces for equilibrium (static or quasi-static force analysis).

4. Force Analysis

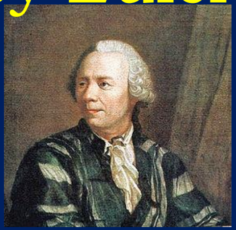


Newton's Laws of Motion

1. $\sum \vec{F} = \vec{0}, \vec{a} = \vec{0}$
2. $\sum \vec{F} = \frac{d}{dt}(m\vec{v}) \rightarrow \sum \vec{F} = m\vec{a}$
3. Action-reaction principle

Law of Universal Gravitation

All valid for particles, generalized for rigid bodies by Euler:



1. $\sum \vec{F} = m\vec{a}_G$
2. $\sum M_G = I_G \alpha$

Please recall from dynamics that Euler's second law can be written for points other than the center of mass, G!

$$\sum M_P = I_G \alpha + \text{Moment of } m\vec{a} \text{ about } P$$

4. Force Analysis

Forces in Machine Systems

Please remember a *generalized force* includes torque as well.

Forces *can be* classified as follows:

1. **(Joint) Reaction Forces:** The joint reaction forces are in the direction(s) where the joint restricts the motion.
2. **External Forces:** The driving forces and torques, load forces and moments (through which the machine does work), weights of links, forces due to non-rigid members.
3. **Friction Forces:** Forces that resist the motion. Two very common friction laws are dry (Coulomb) and viscous friction although others exist as well.

4. Force Analysis

Methods of Force Analysis

1. Vector Mechanics.
2. Analytic Mechanics.

Static Force Analysis

Static force analysis is exact when the machine is stationary and yields accurate results when the machine is moving *slowly* against *heavy* loads (i.e. inertia forces are "small" compared to other forces).

Laws of Static Equilibrium:

$$\sum \vec{F} = \vec{0}$$

$$\sum M_{.} = 0$$

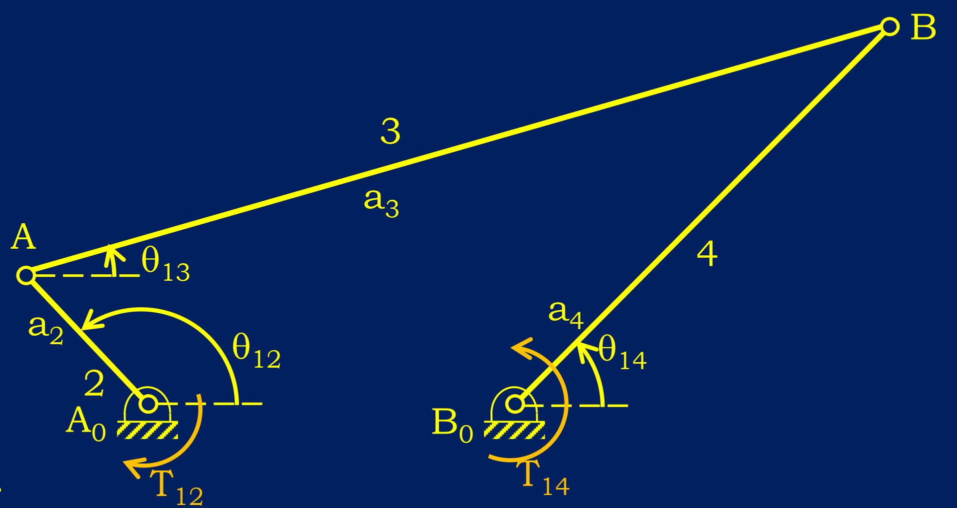
3 independent equations for each rigid body, 2 independent equations for each particle in plane.

Static Force Analysis

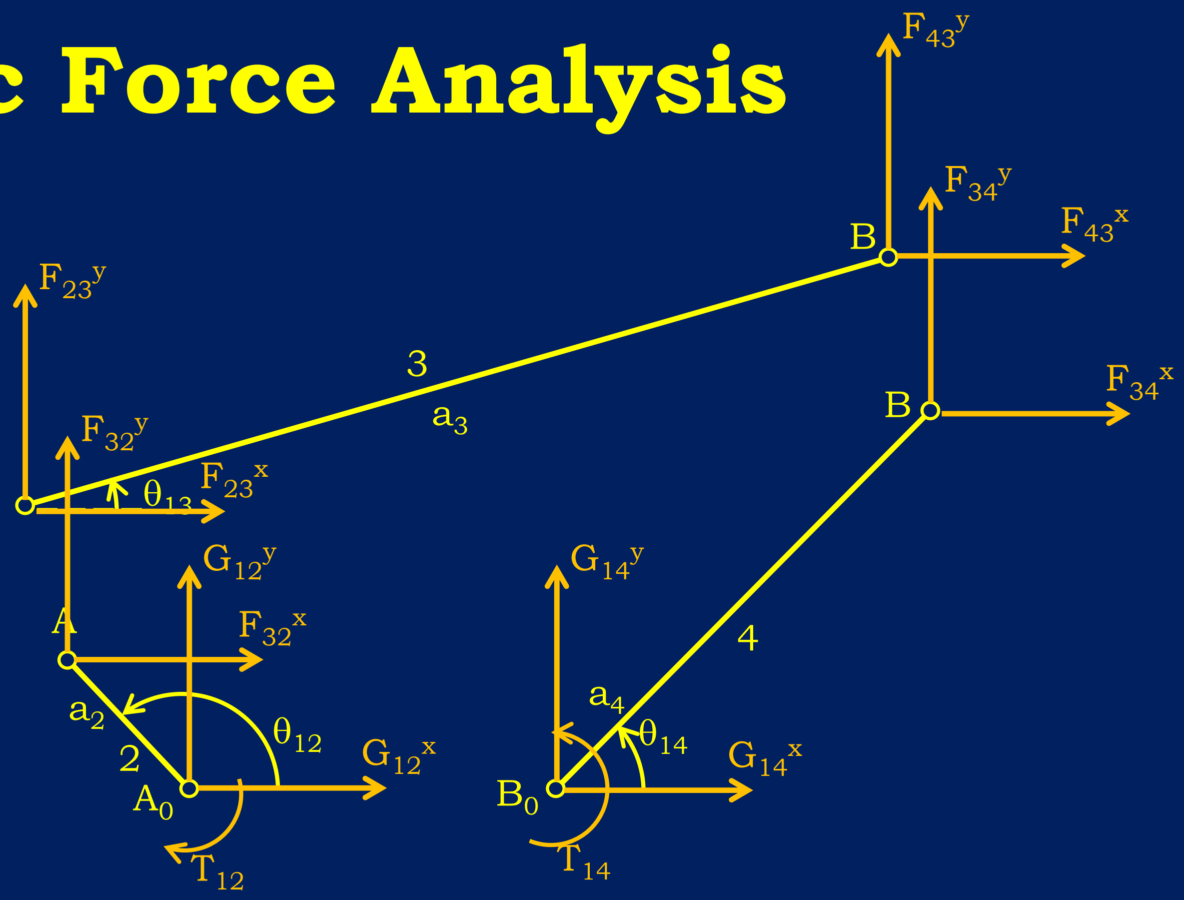
Typical Problem Statement:

Given T_{14} determine T_{12} for static equilibrium for *every* position (i.e. $0 \leq \theta_{12} \leq 2\pi$).

- Utilizing kinematic analysis θ_{13} and θ_{14} corresponding to a given θ_{12} can be evaluated.
- Draw free body diagrams of the moving links keeping position variables (i.e. θ_{12}, θ_{13} and θ_{14}) as parameters.
- Write the equations of equilibrium for free body diagrams, when solved will yield T_{12} for a given T_{14} (*for that position*) for every position of the mechanism.



Static Force Analysis



Free body diagrams

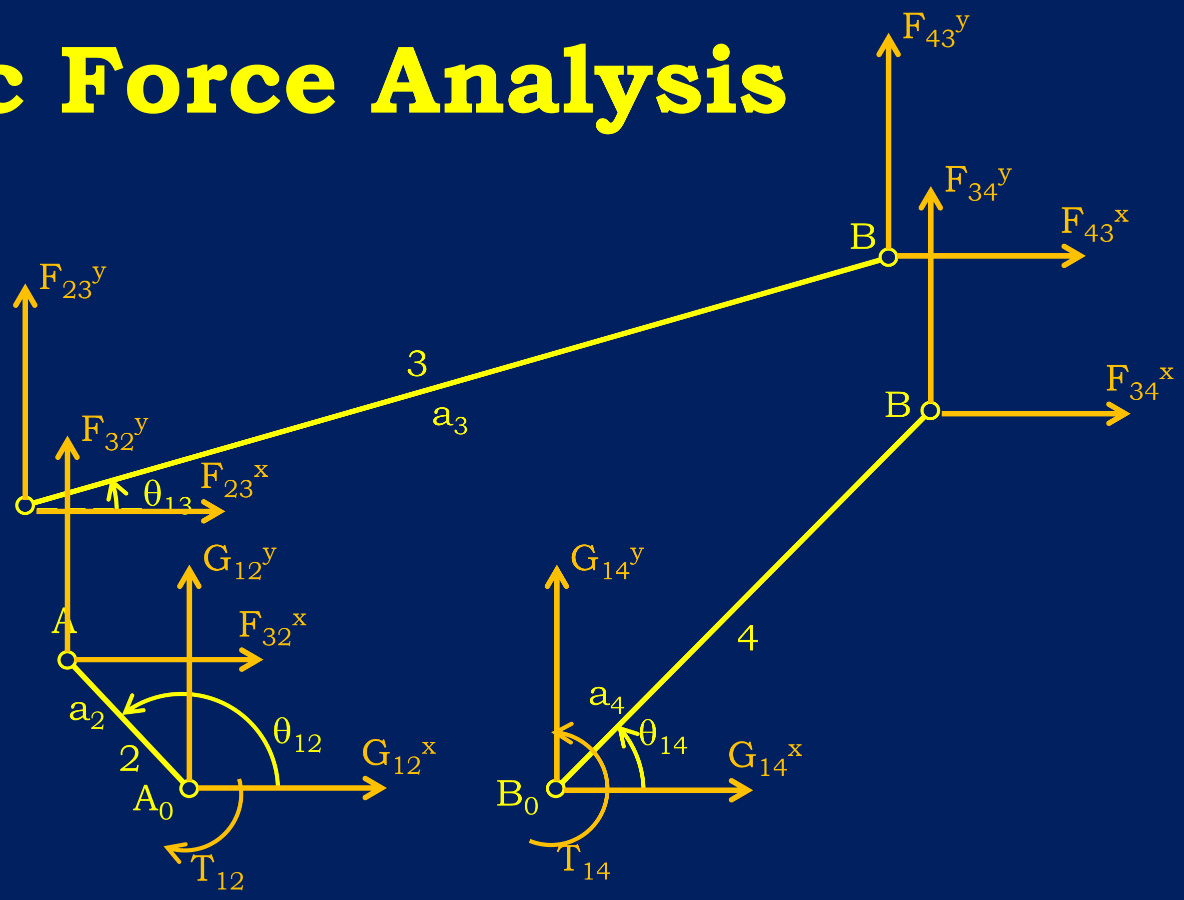
Convention:

G_{1j}: Ground reaction force on link j

F_{ij}: Force of link i on link j

T_{14} known, determine T_{12}, G_{14}^x, G_{14}^y, F_{34}^x, F_{34}^y, F_{43}^x, F_{43}^y, F_{23}^x, F_{23}^y, F_{32}^x, F_{32}^y, G_{12}^x, G_{12}^y (13 unknowns)

Static Force Analysis

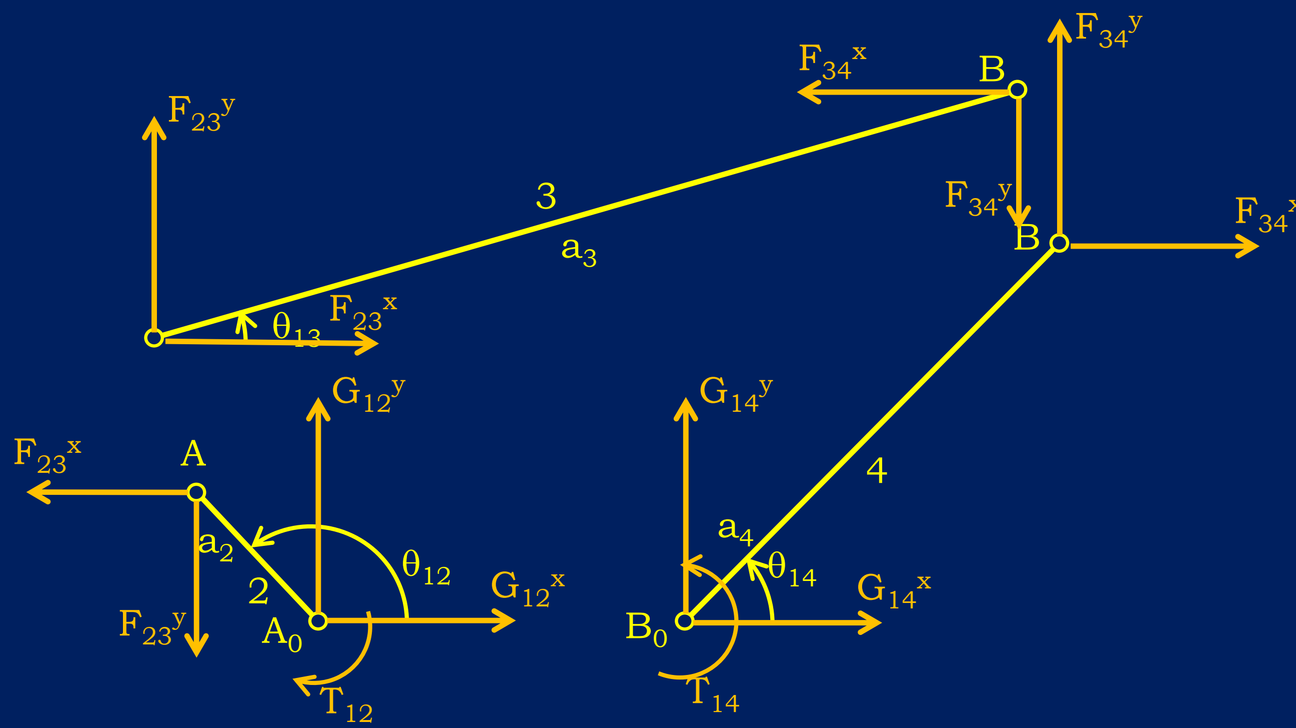


13 unknowns

9 equations of equilibrium + 4 action reaction pairs ($F_{ij}^{x/y} = -F_{ji}^{x/y}$ for joints A and B)

Can be solved *but*…

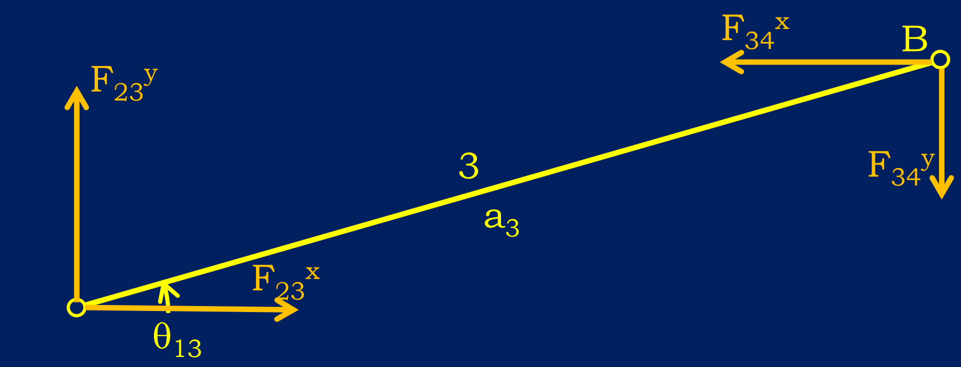
Static Force Analysis



Include action-reaction pairs on FBD

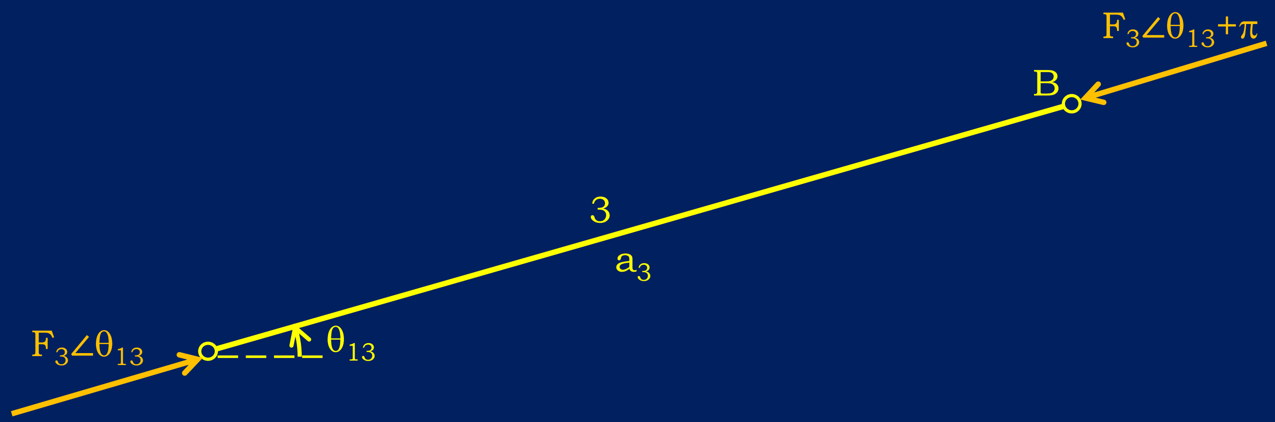
9 equations with 9 unknowns (*4 trivial equations and unknowns eliminated!*)

Static Force Analysis



Link 3:

$$\sum F_x = 0 \rightarrow F_{23}{}^x = F_{34}{}^x$$



$$\sum F_y = 0 \rightarrow F_{23}{}^y = F_{34}{}^y$$

$$\vec{F}_{23} = -\vec{F}_{34}$$

$$\sum M = 0 \rightarrow \vec{F}_{23} \text{ and } \vec{F}_{34} \text{ has to be collinear}$$

Two force member (2F), no equation of equilibrium!

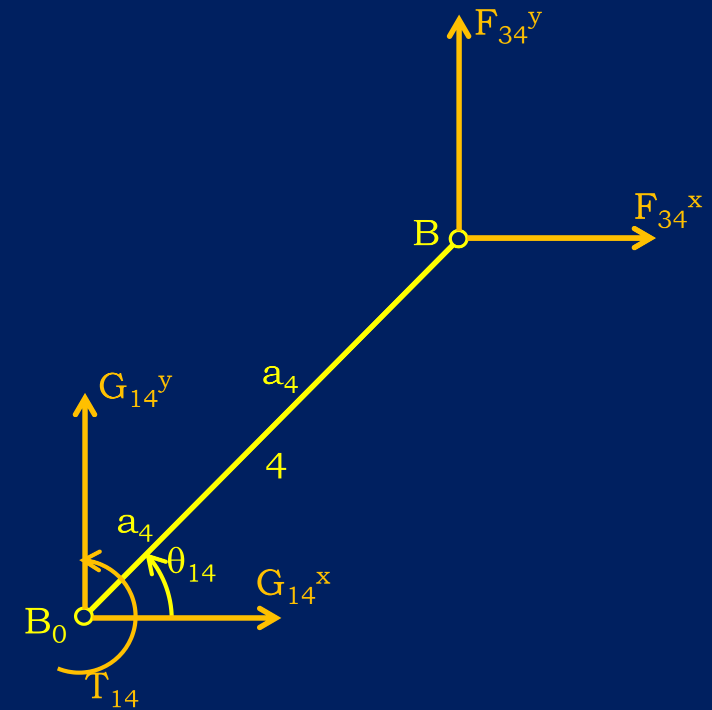
Static Force Analysis

Link 4:

$$\sum F_x = 0 \rightarrow = F_{34}{}^x + G_{14}{}^x = 0$$

$$\sum F_y = 0 \rightarrow = F_{34}{}^y + G_{14}{}^y = 0$$

$$\vec{G}_{14} = -\vec{F}_{34}$$

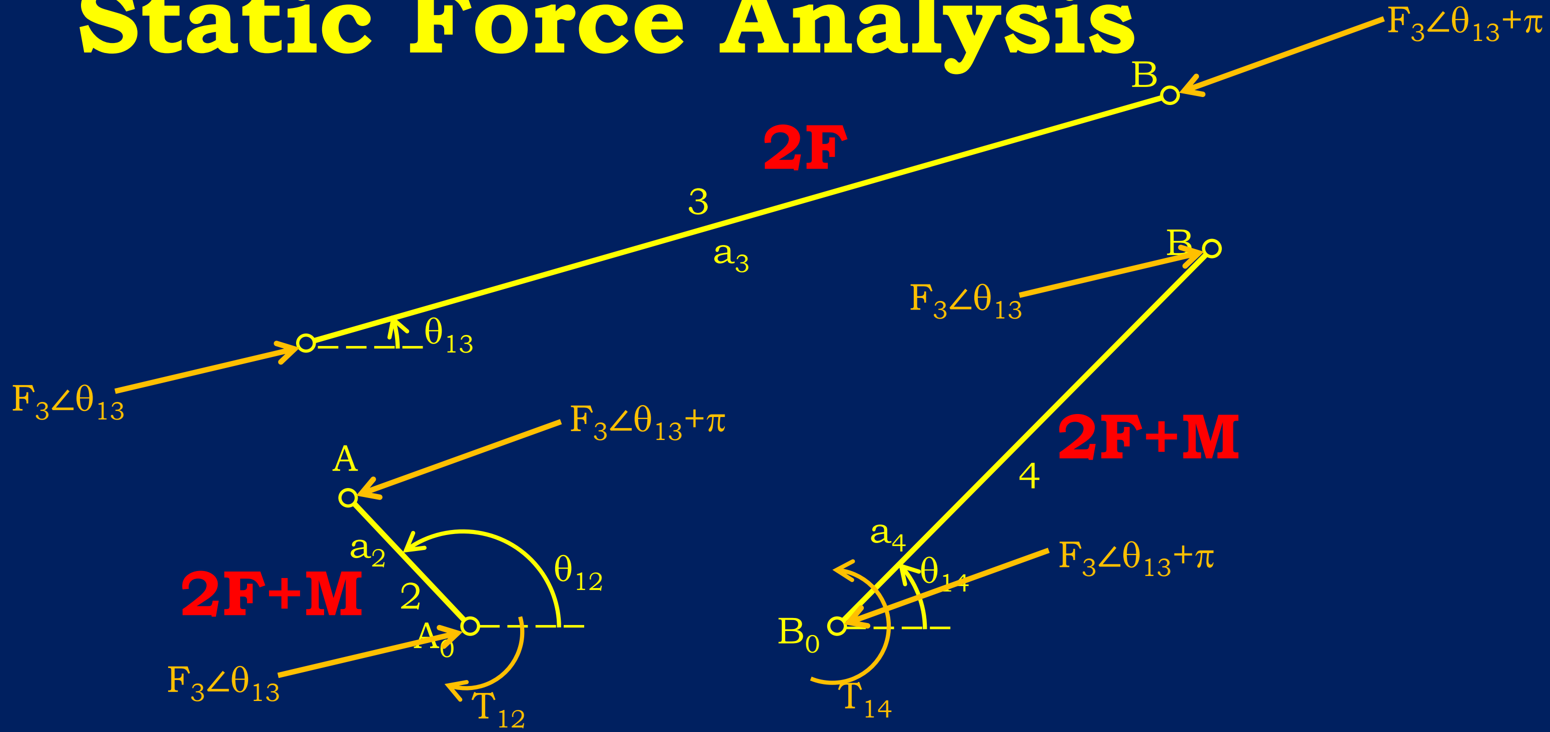


$$\sum M_{B_0} = 0 \rightarrow T_{14} - a_4 sin\theta_{14} F_{34}{}^x + a_4 cos\theta_{14} F_{34}{}^y = 0$$

Two force and a moment member (2F+M), one moment equation only!

Link 2 is a two force and a moment member as well!

Static Force Analysis



Link 4

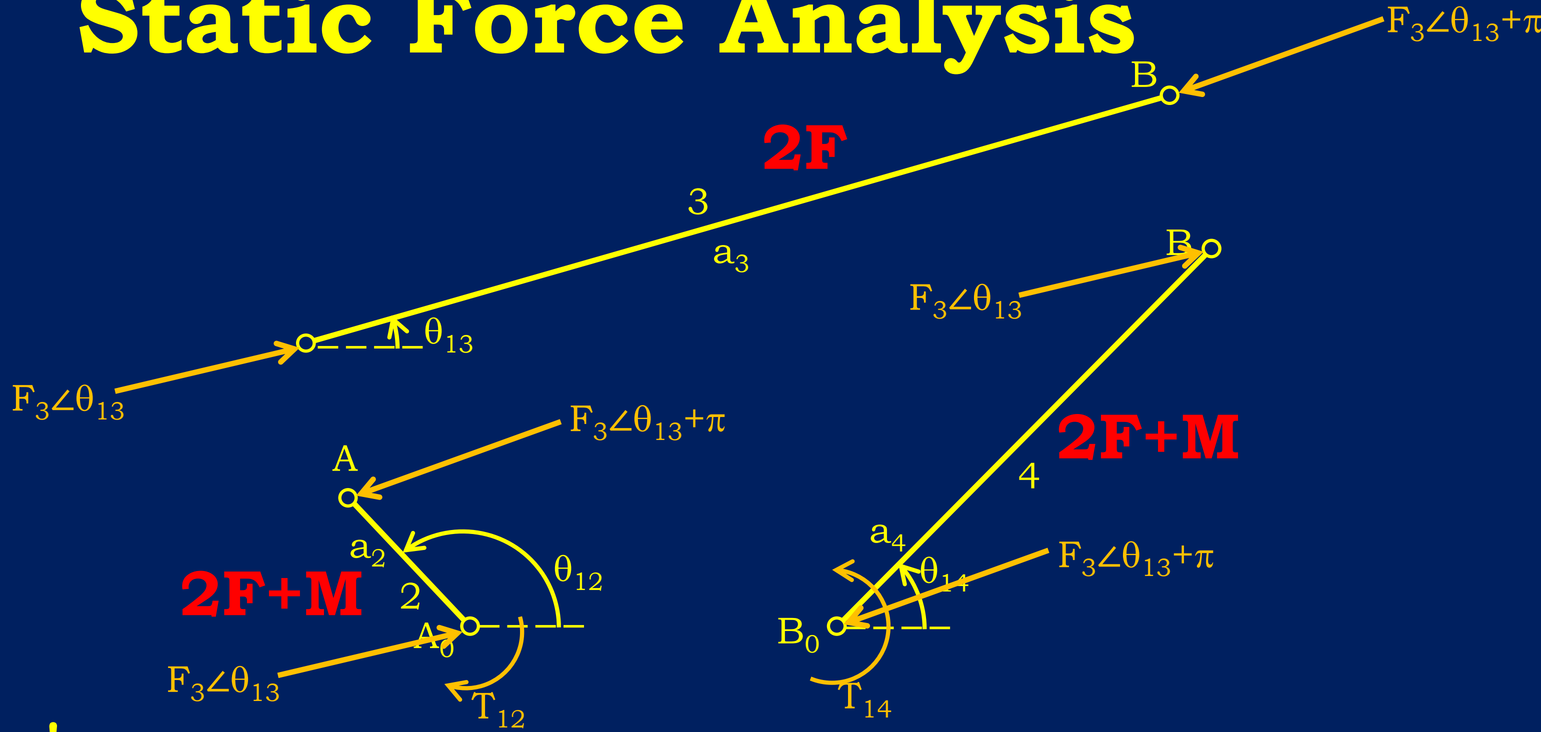
$$\sum M_{B_0} = 0 \rightarrow T_{14} - a_4 cos\theta_{14} F_3 sin\theta_{13} + a_4 sin\theta_{14} F_3 cos\theta_{13} = 0$$

$$T_{14} + a_4 F_3 sin(\theta_{13} - \theta_{14}) = 0$$

$$M = a_i F_j sin\left(\theta_{F_j} - \theta_{a_i}\right)$$

$$F_3 = \frac{T_{14}}{a_4 sin(\theta_{14} - \theta_{13})}$$

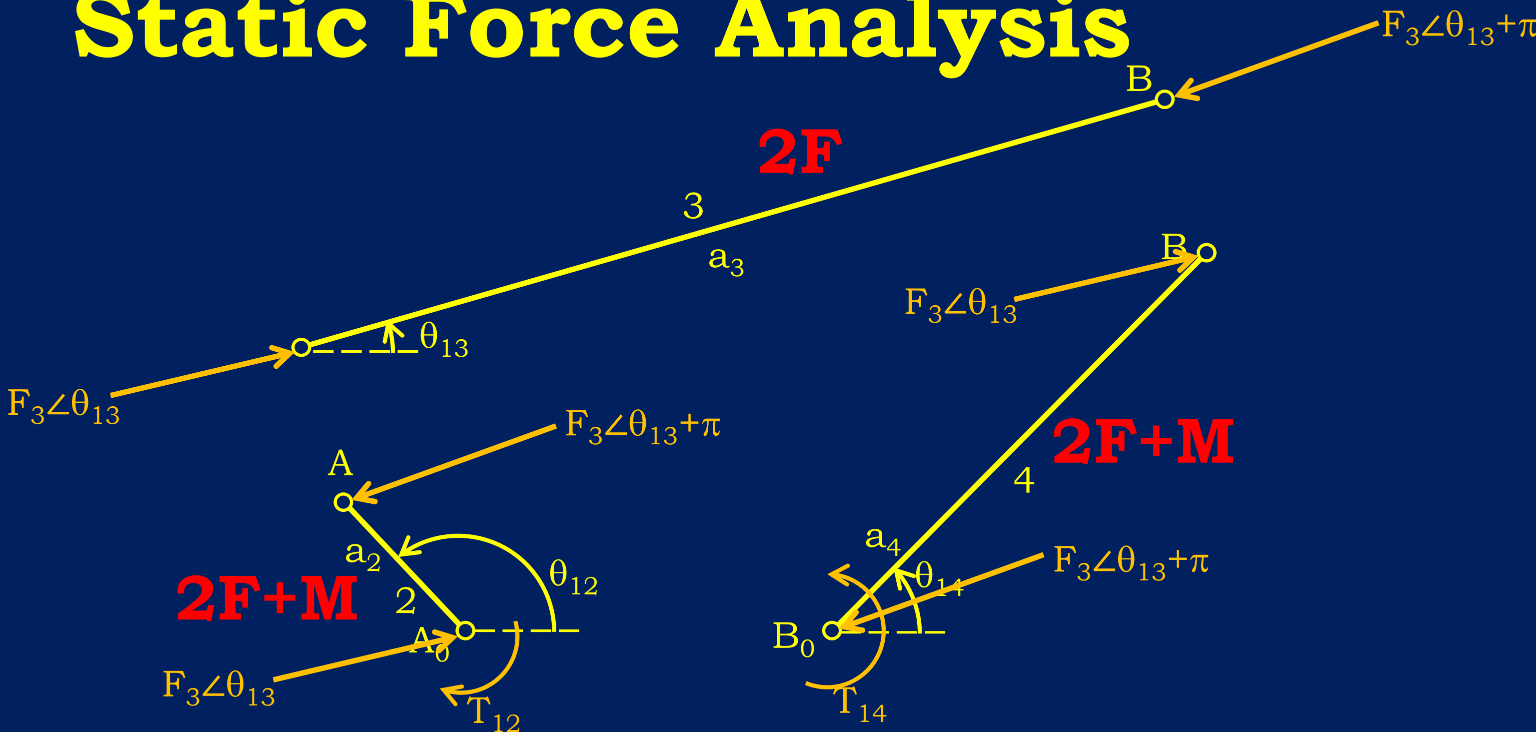
Static Force Analysis



Link 3

No equation!

Static Force Analysis



Link 2

$$\sum M_{A_0} = 0$$

$$\rightarrow -T_{12} - a_2 cos\theta_{12} F_3 sin(\theta_{13} + \pi) + a_2 sin\theta_{12} F_3 cos(\theta_{13} + \pi) = 0$$

$$T_{12} + a_2 F_3 sin[(\theta_{13} + \pi) - \theta_{12}] = 0$$

$$T_{12} = -a_2 F_3 sin[(\theta_{13} + \pi)\theta_{12}]$$

*Only **two** equations of equilibrium!*