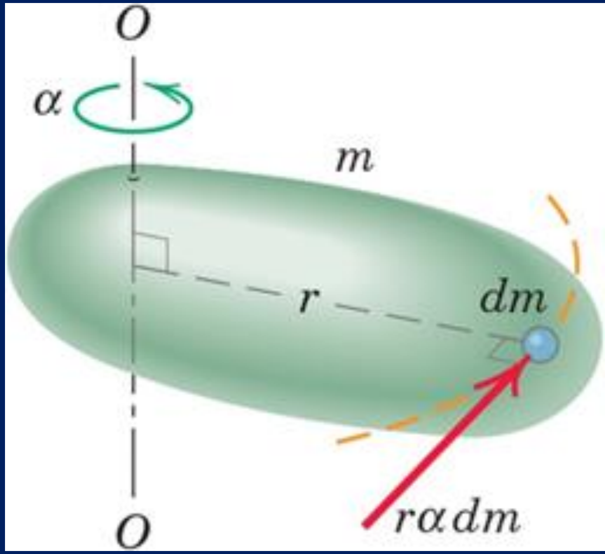
Dynamic Force Analysis

Quick Review of ME 208 Dynamics

B/1 Mass Moments of Inertia about an Axis



$a_t = r\alpha$

$dF = r\alpha dm$

Moment of this force about O-O

$dM = rdF = r^2\alpha dm$

For all particles in the rigid body

$$M = \int_m dM = \int_m r^2\alpha dm = \alpha \int_m r^2 dm$$

Mass moment of inertia of the body about axis O-O is defined as

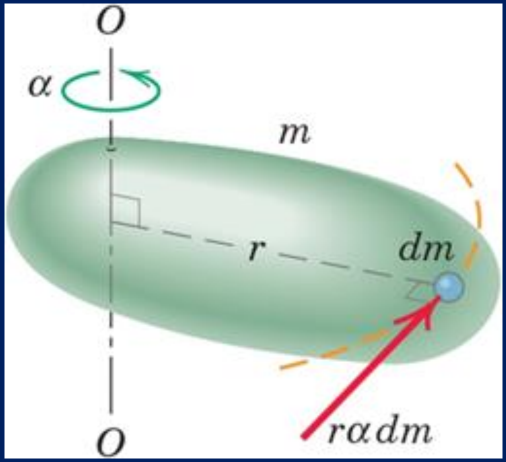
$$I_O \equiv \int_m r^2 dm \quad [kg \cdot m^2]$$

Mass is a resistance to linear acceleration, mass moment of inertia is a resistance to angular acceleration.

Dynamic Force Analysis

Quick Review of ME 208 Dynamics

B/1 Mass Moments of Inertia about an Axis



For discrete system of particles

$$I_O = \sum_{i=1}^{n} {r_i}^2 m_i$$

For constant density (homogeneous) rigid bodies

$$I_O = \rho \int_V r^2 dV$$

Radius of Gyration

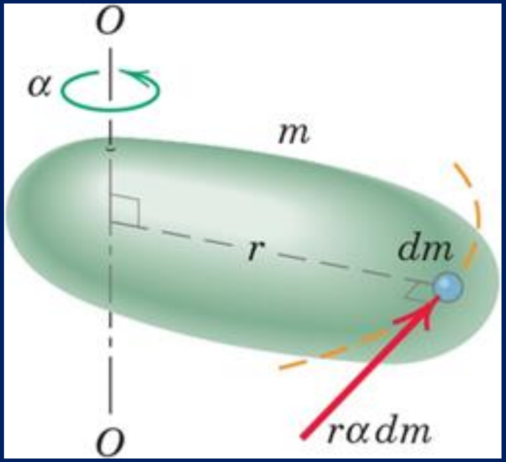
$$k_x \equiv \sqrt{\frac{I_x}{m}}, I_x = {k_x}^2 m$$

Radius of gyration is a measure of mass distribution of a rigid body about the axis. A system with equivalent mass moment of inertia is a very thin ring of same mass and radius k_x.

Dynamic Force Analysis

Quick Review of ME 208 Dynamics

B/1 Mass Moments of Inertia about an Axis



Transfer of Axis (Parallel Axis Theorem)

$$I_x = I_G + m|gX|^2$$

$$k_x{}^2 = k_G{}^2 + |gX|^2$$

Composite Bodies

Mass moment of inertia of a composite body about an axis is the sum of individual mass moments of each part about the same axis (which may be calculated utilizing parallel axis theorem if mass moment of inertia of each part is known about its mass center).

Dynamic Force Analysis

Quick Review of ME 208 Dynamics

A. Direct Application of Newton's Second Law – Force Mass Acceleration Method for a Rigid Body

6/2 General Equations of Motion

$$\sum \vec{F} = \dot{\vec{G}} = m\vec{a}_G$$

$$\sum \vec{M}_G = \dot{\vec{H}}_G = I_G\vec{\alpha}$$

These are known as Euler's first and second laws.

By using statics information one may replace the forces on a rigid body by a single resultant force passing through mass center and a couple moment. The equivalent force causes linear acceleration of the mass center in the direction of force, the couple moment causes angular acceleration.

Dynamic Force Analysis

Quick Review of ME 208 Dynamics

Plane Motion Equations

$$\sum \vec{F} = m\vec{a}_G$$

$$\vec{H}_G = \sum_{i=1}^{n} \vec{\rho}_i \times m_i \dot{\vec{\rho}}_i = \int_m \vec{\rho} \times \dot{\vec{\rho}} dm$$

For a rigid body $|\vec{\rho}_i| = const$ therefore

$$\dot{\vec{\rho}} = \vec{\omega} \times \vec{\rho}$$

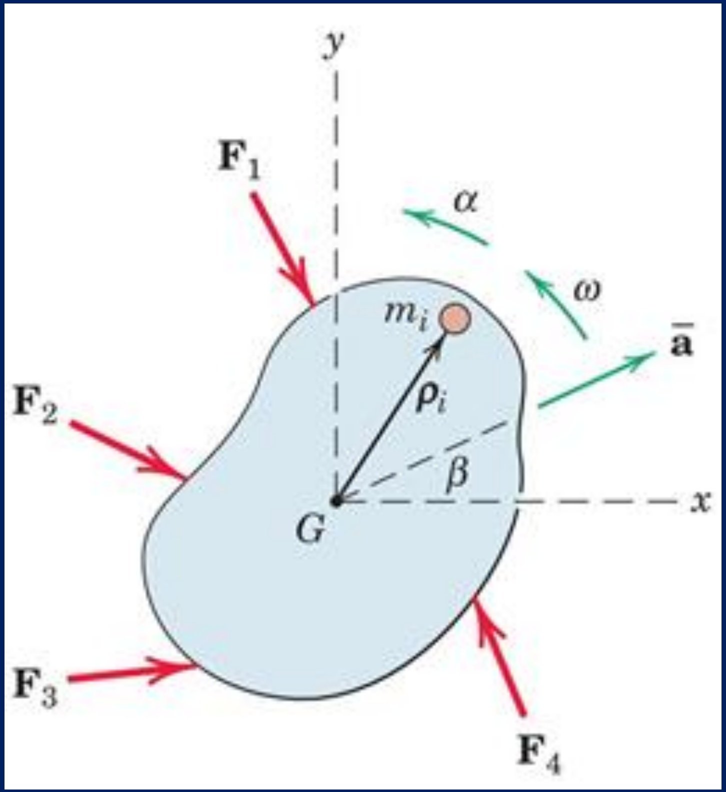
$$\vec{\rho} \times \dot{\vec{\rho}} = \vec{\rho} \times \vec{\omega} \times \vec{\rho} = -\vec{\rho} \times (\vec{\rho} \times \vec{\omega}) = \rho^2 \vec{\omega}$$

$$\vec{H}_G = \int_m \rho^2 \vec{\omega} dm = \vec{\omega} \int_m \rho^2 dm = \vec{\omega} I_G$$

For a rigid body I_G is constant so

$$\dot{\vec{H}}_G = I_G \dot{\vec{\omega}} = I_G \vec{\alpha}$$

$$\sum \vec{F} = m\vec{a}_G , \sum \vec{M}_G = I_G \vec{\alpha}$$



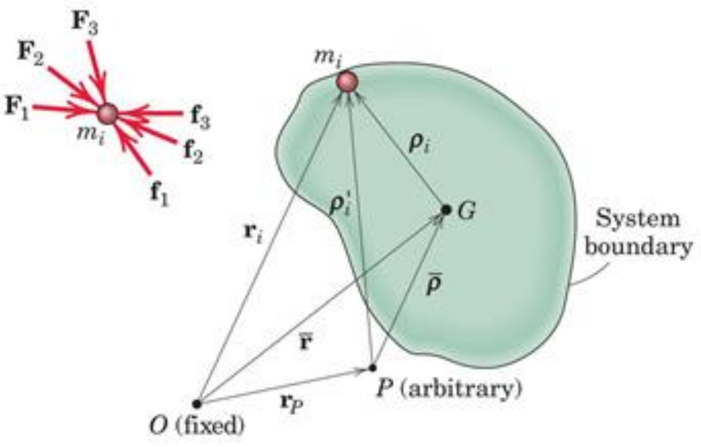
Dynamic Force Analysis

Quick Review of ME 208 Dynamics

Alternative Moment Equation

Sometimes it may be more convenient to take moment about another point rather than the mass center G. In that case

$$\sum \vec{M}_P = \dot{\vec{H}}_G + \vec{\rho}_G \times m\vec{a}_G, \vec{\rho}_G = \overrightarrow{PG}$$



This equation can be written as

$$\sum M_P = I_G\alpha + \textit{Moment of } m\vec{a}_G \textit{ about } P \quad (1)$$

If point P is a fixed point (like the axis of rotation) then

$$\sum M_O = I_G\alpha + \textit{Moment of } m\vec{a}_G \textit{ about } P, a_{G_t} = |OG|\alpha,$$

$$\textit{Moment of } m\vec{a}_G \textit{ about } P = |OG|^2\alpha$$

$$\sum M_O = I_G\alpha + |OG|^2 m\alpha = (I_G + m|OG|^2)\alpha = I_O\alpha$$

Dynamic Force Analysis

D'Alembert's Principle for a Rigid Body

Newton's second law:

$$\sum \vec{F} = m\vec{a}_G$$

D'Alembert states that a body resists the unbalanced force on it with a fictitious force called the *inertia force* and the sum is zero:

$$\sum \vec{F} + \vec{F}^i = \vec{0}$$

$$\vec{F}^i = -m\vec{a}_G$$

Similarly Euler's second law can be expressed as:

$$\sum \vec{M}_G + \vec{T}^i = \vec{0}$$

$$\vec{T}^i = -I_G\vec{\alpha}$$

Dynamic Force Analysis

D'Alembert's Principle for a Rigid Body

Similarly Euler's second law can be expressed as:

$$\sum \vec{M}_x + \vec{T}^i = \vec{0}$$

When moment is taken about a point other than the mass center, G, the moment of $-m\vec{a}_G$ (on the right hand side of the equation) is also included therefore is same as Eq. (1). Please remember, still

$$\vec{T}^i = -I_G\vec{\alpha}$$

However, for fixed axis rotation (say about O) inertia force may be moved to rotation axis and its moment is included in I_O as:

$$\sum \vec{M}_O - I_O\vec{\alpha} = \vec{0}$$

Dynamic force analysis is also known as *kinetostatics* and *dynamic equilibrium.*