5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

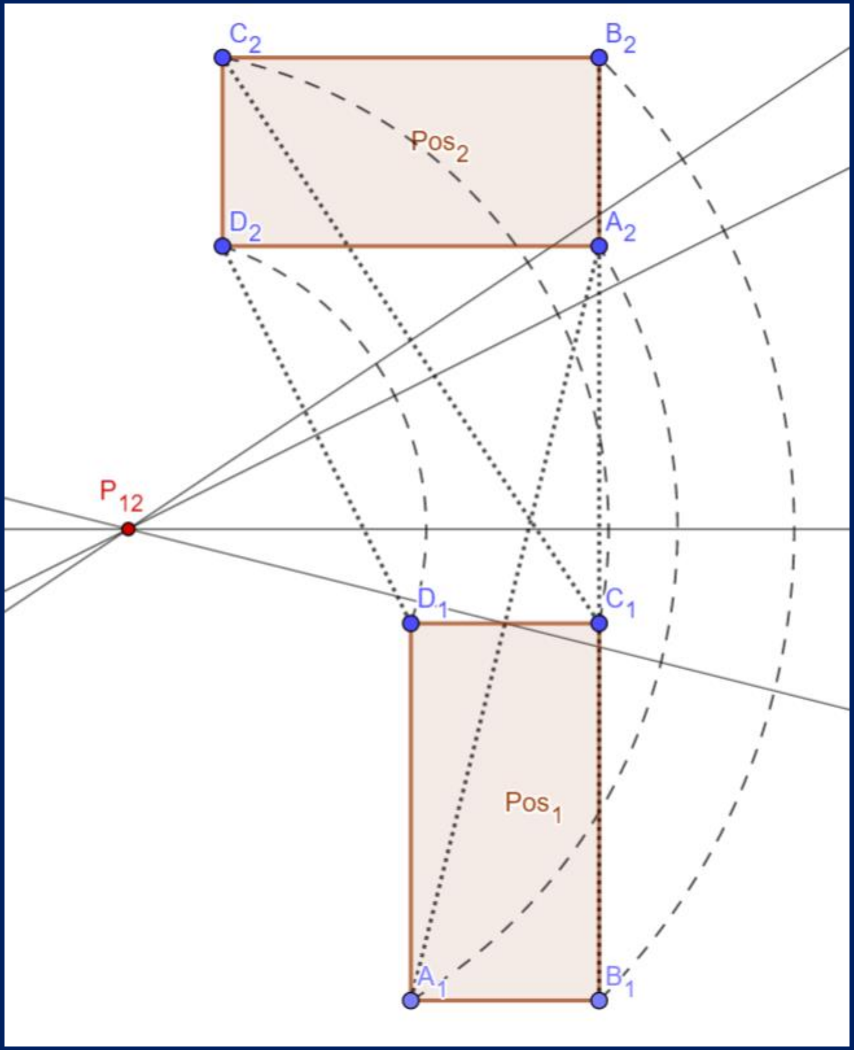
Chasles Theorem: The motion of a rigid body from one position to another in plane motion occurs most simply by a rotation about the pole P_{12} which is located at the perpendicular bisectors of two pairs of *homologous points*.

Please note that when the separation between the two positions of the plane diminishes the homologous points define the velocity and the pole P_{12} boils down to instant center of zero velocity (ICZV).

5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

<u>Chasles Theorem</u>:



5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

Every point on the perpendicular bisector of the homologous points is equidistant to the respective homologous points therefore candidates for the *ground pivots* (fixed revolute joints).

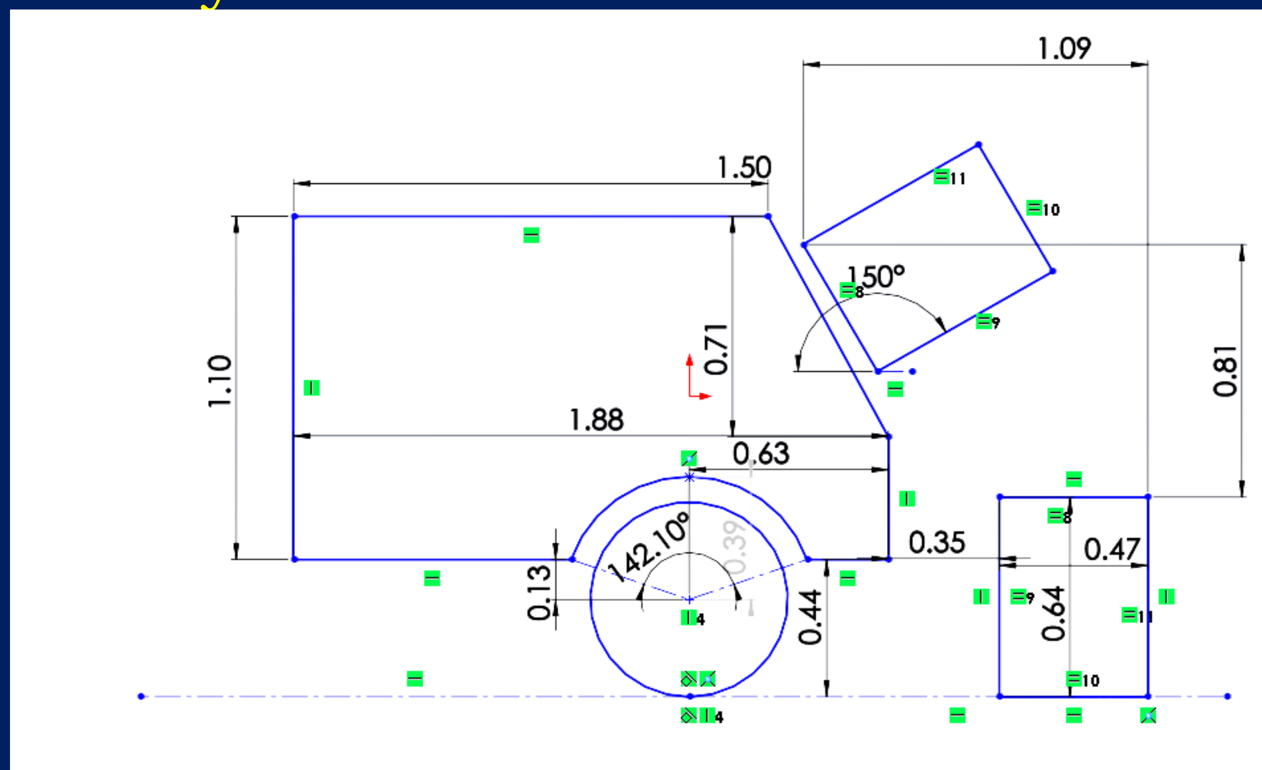
One may select the homologous point anywhere on the moving plane (say in position 1) therefore there are two free parameters in selecting one *moving pivot* (say x, and y). Once moving pivot is selected, its position in the other position of the moving plane (say in position 2) is defined therefore one may draw the perpendicular bisector and select the ground (or fixed) pivot anywhere on it (only one free parameter, say distance from a point). The same procedure is repeated for the other crank therefore there are six free parameters to design the mechanism.

5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

Example: Two positions of a trash bin is given. Design a four bar mechanism that would perform this motion. The moving joints can be anywhere in the boundaries of the trash bin.

SolidWorks



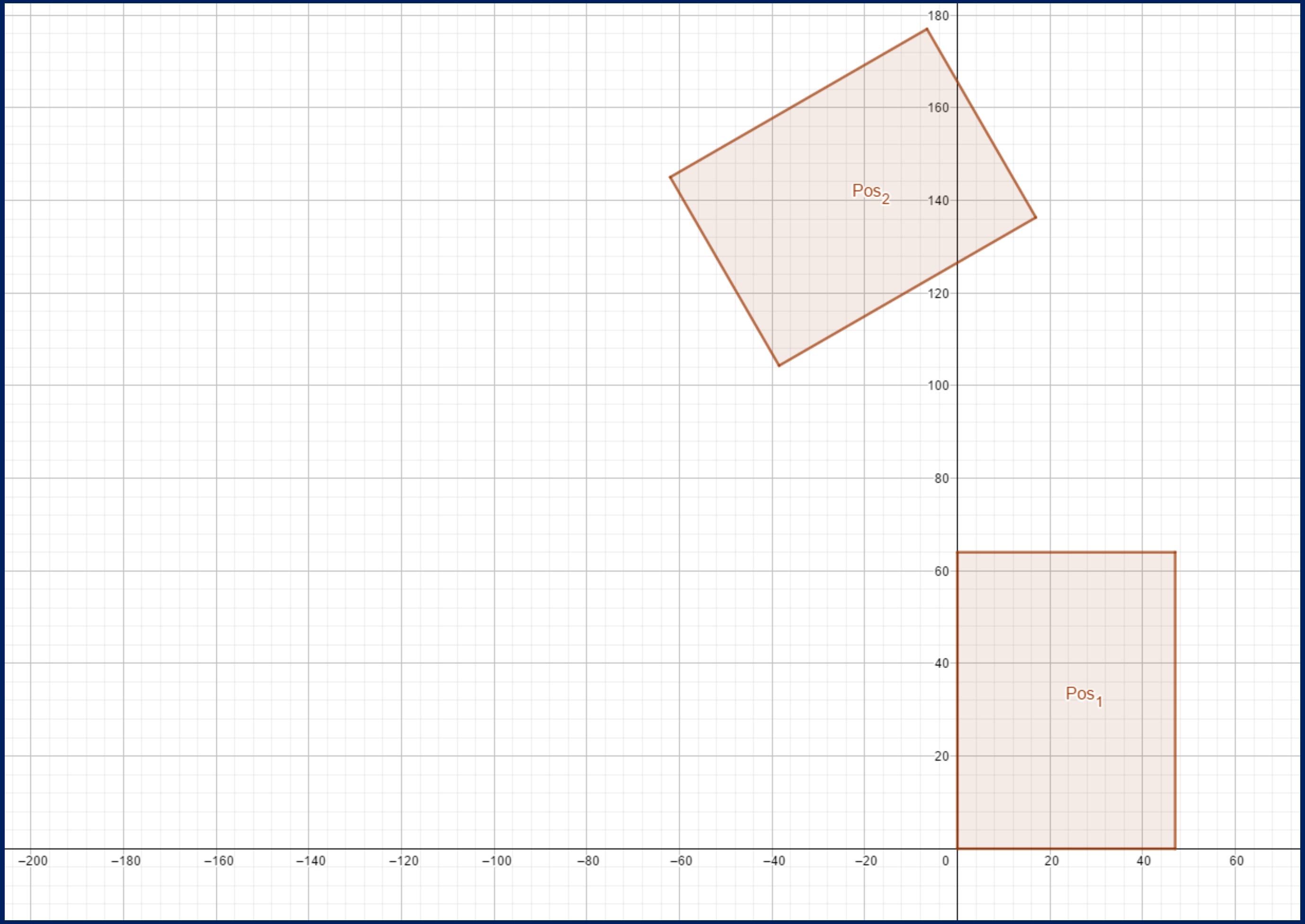
Solution by:

GeoGebra

5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

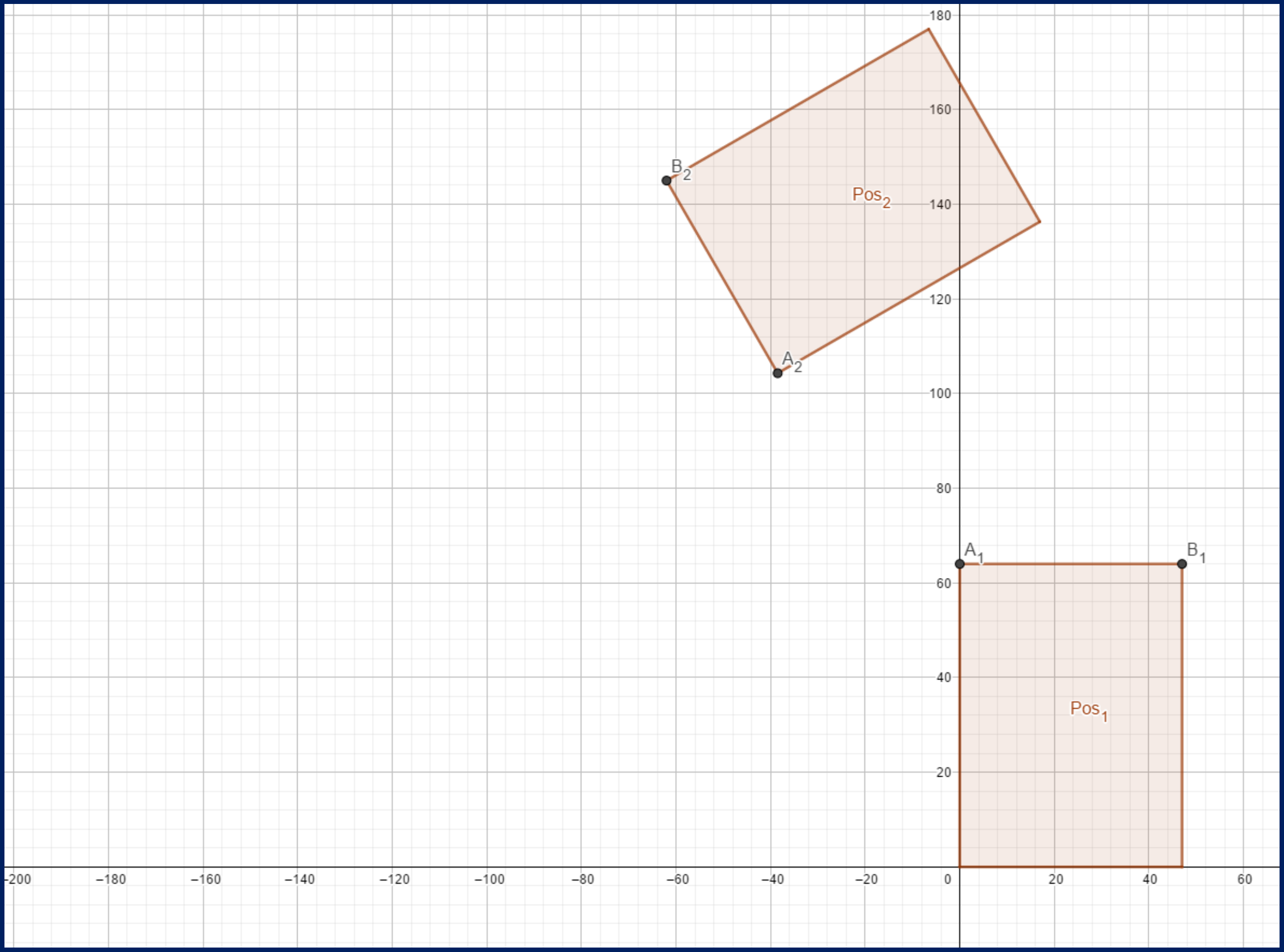
Draw two positions



5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

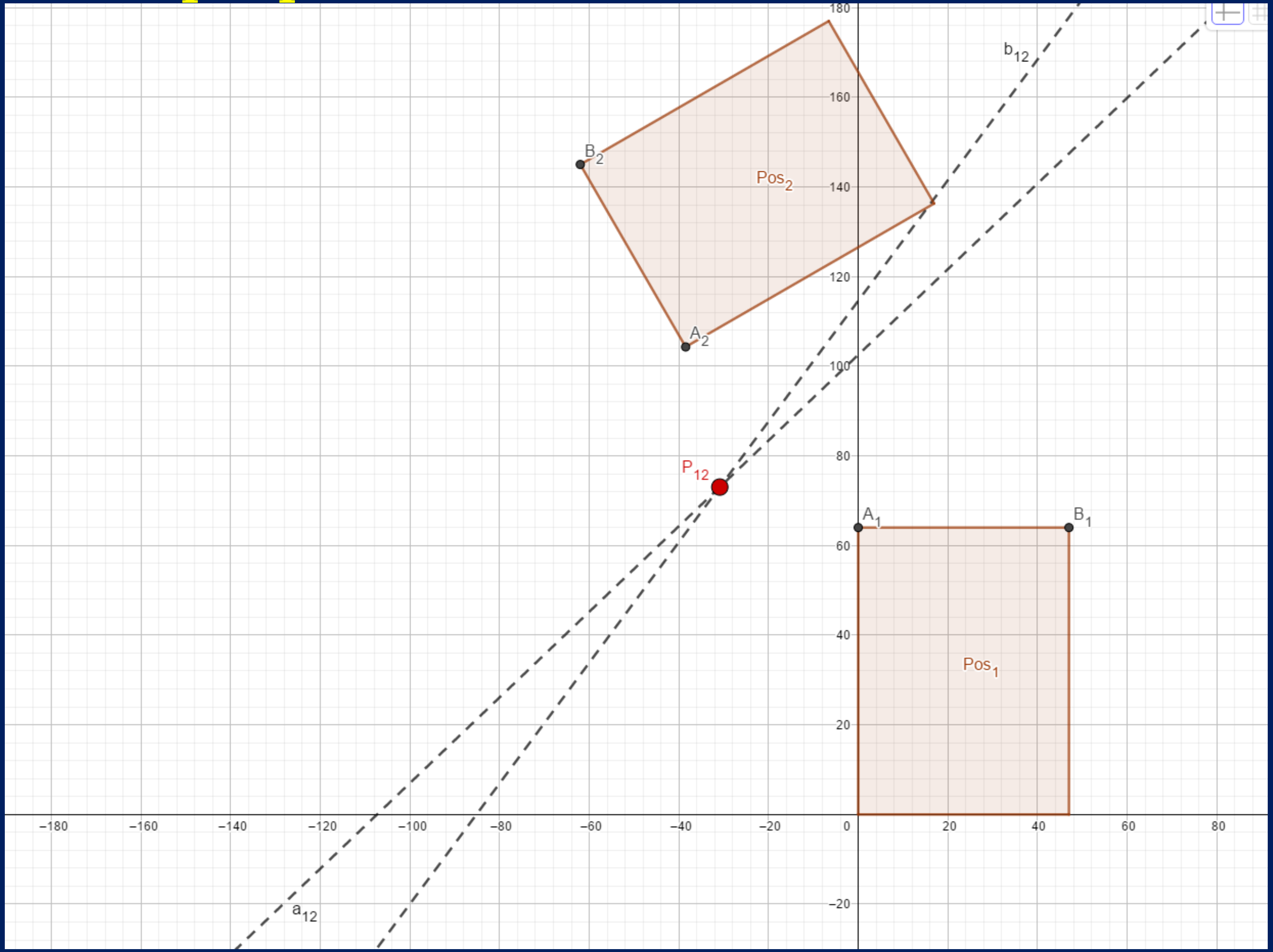
Select two homologous points on two positions



5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

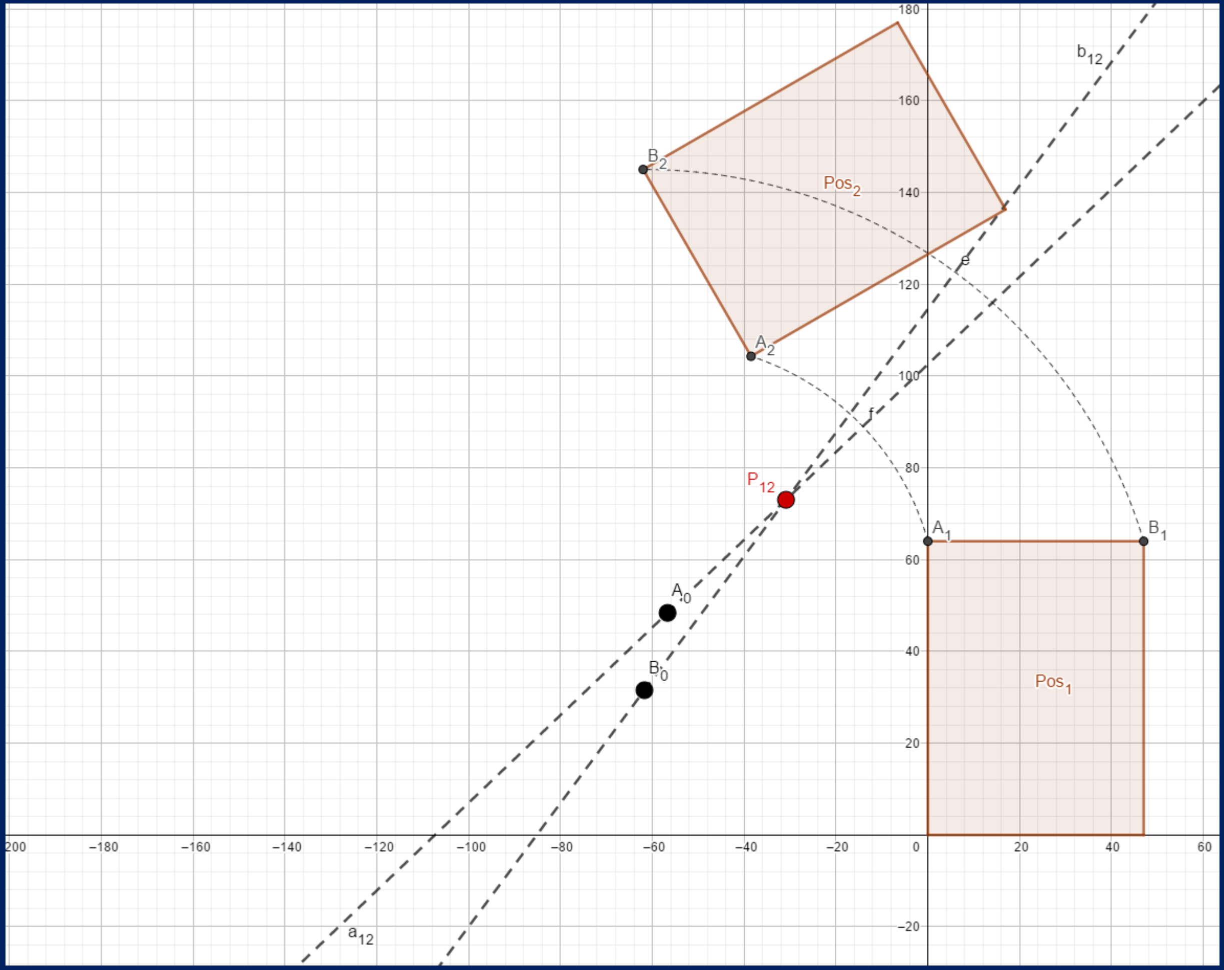
Draw perpendicular bisectors of two homologous points



5. Four Bar Mechanism

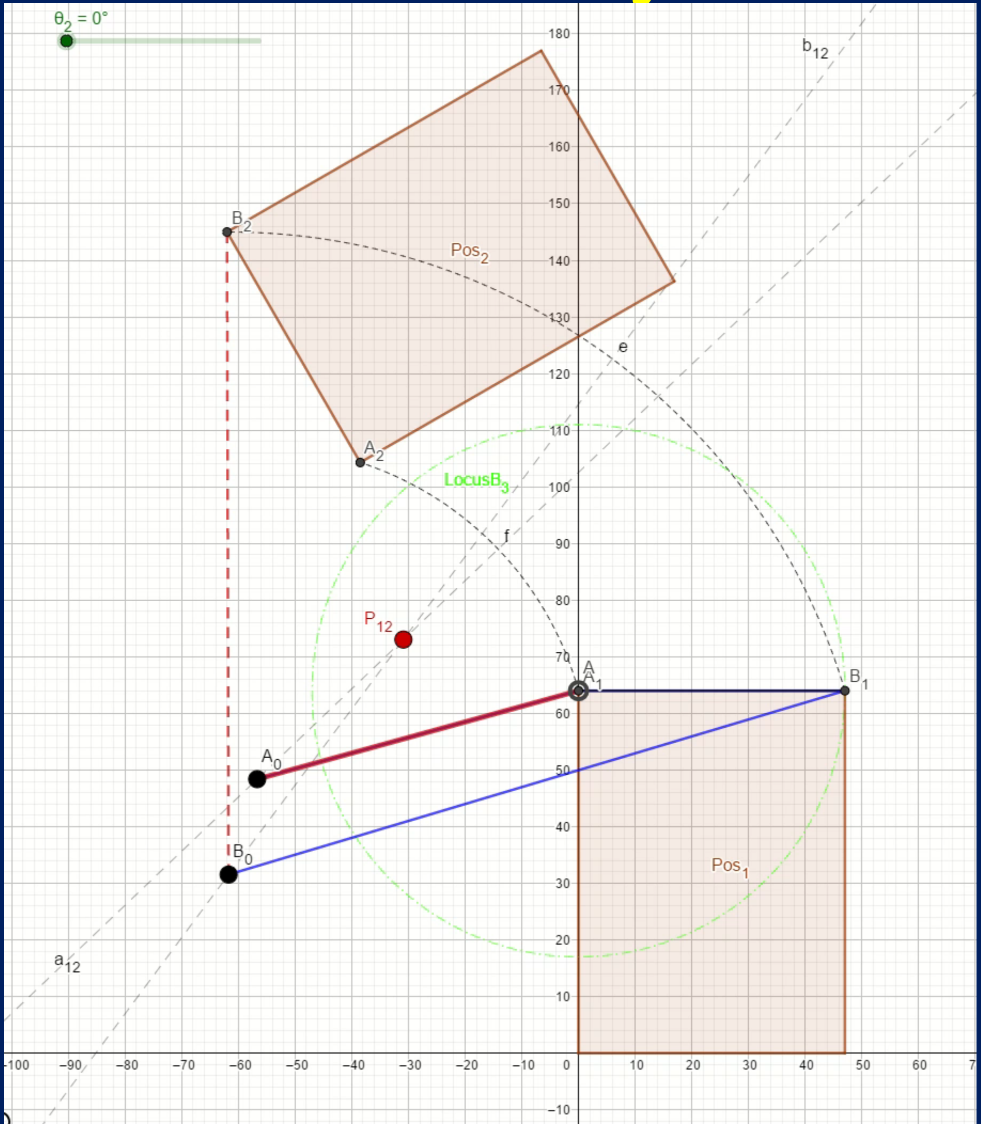
Body Guidance-Two Position Synthesis:

Select A_0 anywhere on a_{12} and B_0 anywhere on b_{12}.



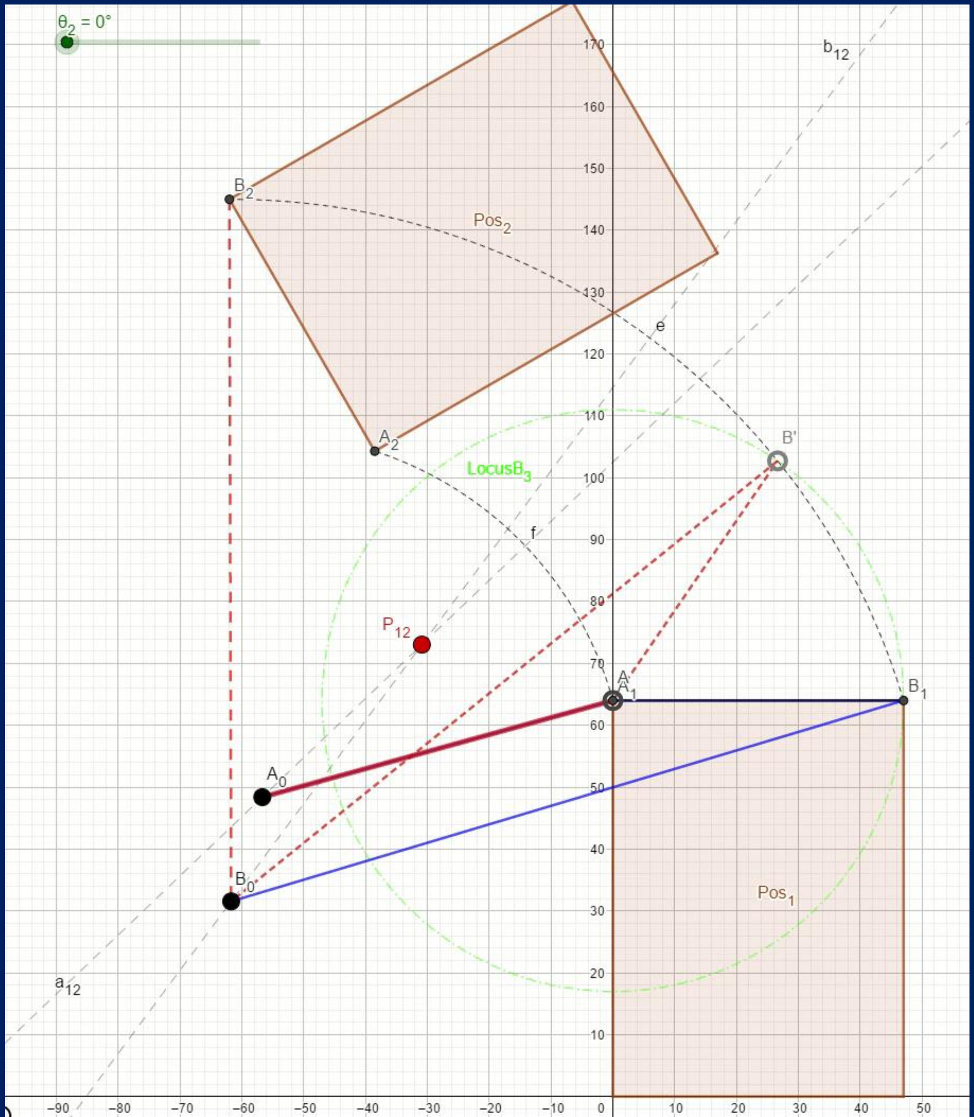
5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:



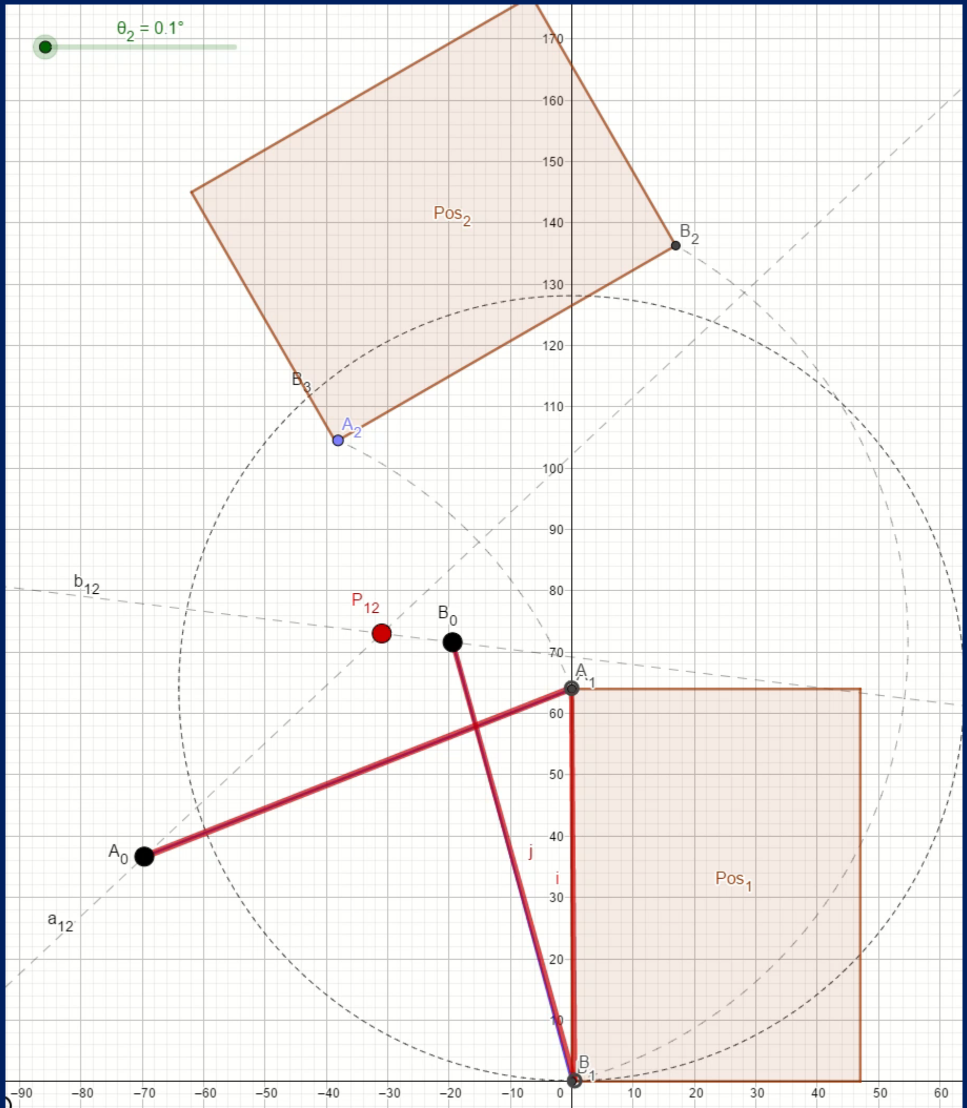
5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:



5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:



Switch to:

GeoGebra

5. Four Bar Mechanism

Body Guidance-Two Position Synthesis, *Problems*:

Although the mechanism exists in the two design positions,

- It may not move from one position to the other because the two positions may be at two different branches (assembly positions) of the mechanism.
- The transmission angle may be poor at some positions.
- The link lengths may be inappropriate.
- The body may interfere with an obstacle during motion.
- …

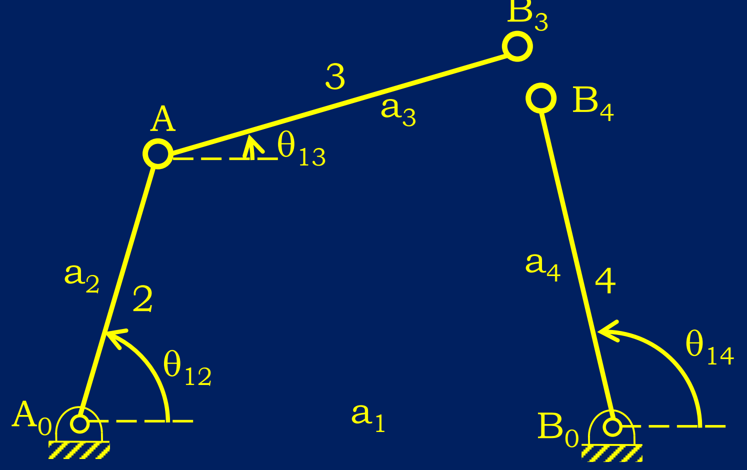
5. Four Bar Mechanism

Function Generation-Three Precision Points:

$$K_1 cos\theta_{14} - K_2 cos\theta_{12} + K_3 = cos(\theta_{14} - \theta_{12})$$

$$K_1 = \frac{a_1}{a_2}, K_2 = \frac{a_1}{a_4}, K_3 = \frac{{a_1}^2 + {a_2}^2 - {a_3}^2 + {a_4}^2}{2a_2a_4}$$

Freudenstein’s equation can be solved for three pairs of angles $(\theta_{12}, \theta_{14})_1$, $(\theta_{12}, \theta_{14})_2$ and $(\theta_{12}, \theta_{14})_3$ for the three link length parameters, K_1, K_2 and K_3.



5. Four Bar Mechanism

Function Generation-Three Precision Points:

$K_1 cos\theta_{14} - K_2 cos\theta_{12} + K_3 = cos(\theta_{14} - \theta_{12})$

$(\theta_{12}, \theta_{14})_1$, $(\theta_{12}, \theta_{14})_2$ and $(\theta_{12}, \theta_{14})_3$.

Example 5.6 (*Söylemez, Mechanisms, Second Ed., 1985*) Synthesize a four bar mechanism that will realize the function $y = lnx$ in the interval $1 \leq x \leq 2$.

Choose $\Delta\theta_{12} = 90°$ and $\Delta\theta_{14} = 60°$

Choose $\theta_{12_{initial}} = 30°$ and $\theta_{14_{initial}} = 30°$

Chebyshev spacing theory states that the error can be minimized (in *Chebyshev* or *mini-max* sense) if precision points are selected as:

$$x_i = \frac{x_{initial} + x_{final}}{2} + \frac{x_{initial} - x_{final}}{2} cos[30°(2i - 1)], i = 1, 2, 3.$$

5. Four Bar Mechanism

Function Generation-Three Precision Points:

Choose $\Delta\theta_{12} = 90°$ and $\Delta\theta_{14} = 60°$
Choose $\theta_{12_{initial}} = 30°$ and $\theta_{14_{initial}} = 30°$

$$x_i = \frac{x_{initial} + x_{final}}{2} + \frac{x_{initial} - x_{final}}{2} cos[30°(2i - 1)], i = 1, 2, 3.$$

$$r_x = \frac{\Delta\theta_{12}}{\Delta x} = 90°/x, r_y = \frac{\Delta\theta_{14}}{\Delta y} = \frac{60°}{ln2 - ln1} = 86.6°/y$$

$$\theta_{12_i} = \theta_{12_{initial}} + r_x(x_i - x_{initial})$$

$$\theta_{14_i} = \theta_{14_{initial}} + r_y(y_i - y_{initial})$$

i	x_i	y_i	$\theta_{12,i}$	$\theta_{14,i}$
1	1.07	0.06	36.03	35.61
2	1.50	0.41	75.00	65.10
3	1.93	0.66	113.97	87.05

5. Four Bar Mechanism

Function Generation-Three Precision Points:

i	x_i	y_i	$\theta_{12,i}$	$\theta_{14,i}$
1	1.07	0.06	36.03	35.61
2	1.50	0.41	75.00	65.10
3	1.93	0.66	113.97	87.05

$$K_1 cos(35.61°) - K_2 cos(36.03°) \; + K_3 = cos(35.61° - 36.03°)$$

$$K_1 cos(65.10°) - K_2 cos(75.00°) \; + K_3 = cos(65.10° - 75.00°)$$

$$K_1 cos(87.05°) - K_2 cos(113.97°) + K_3 = cos(87.05° - 113.97°)$$

$K_1 = 0.7232$

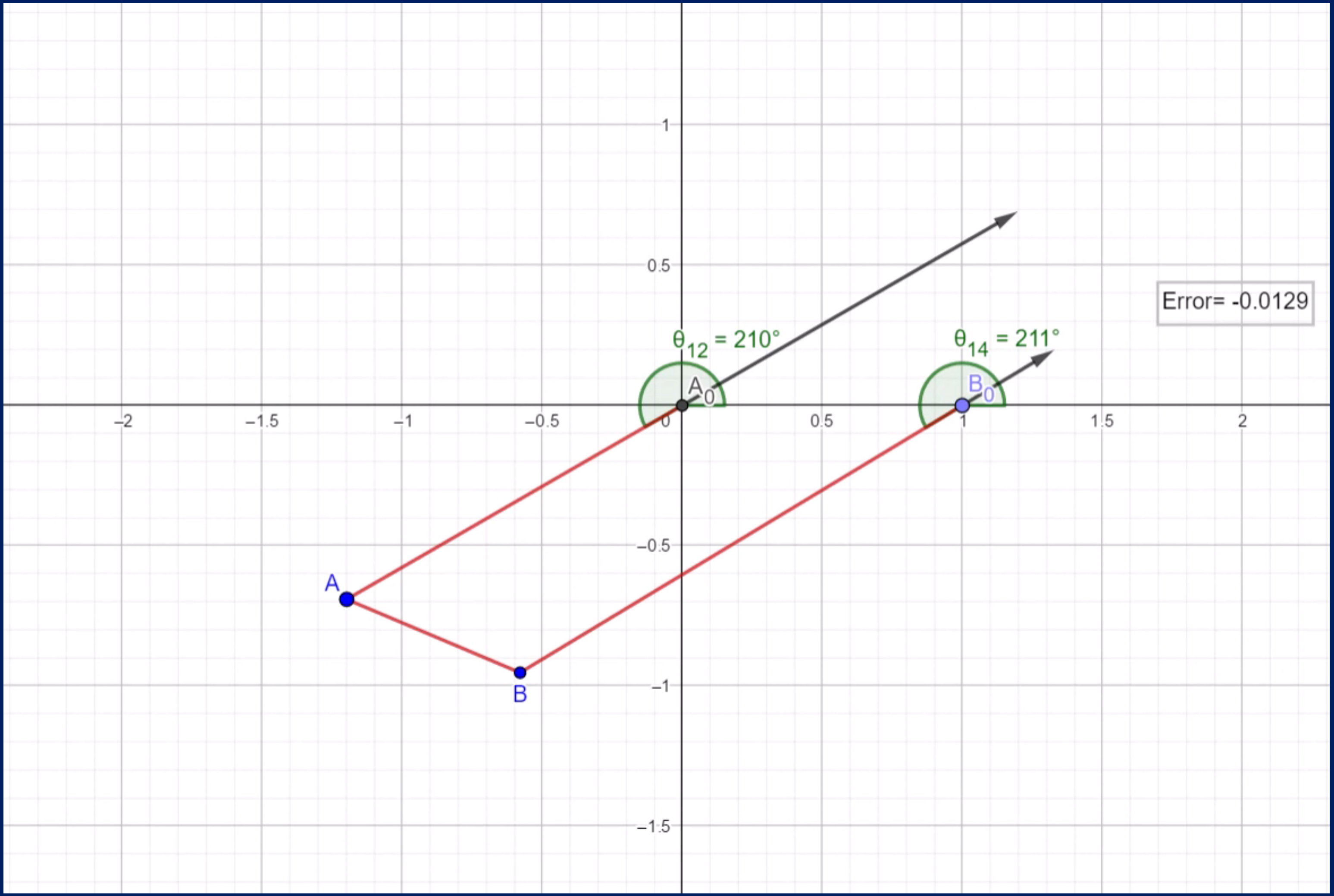
$K_2 = -0.5423$

$K_3 = 1.1293$

Let $a_1 = 1$, then $a_2 = -1.383$, $a_3 = 0.672$, $a_4 = -1.844$

5. Four Bar Mechanism

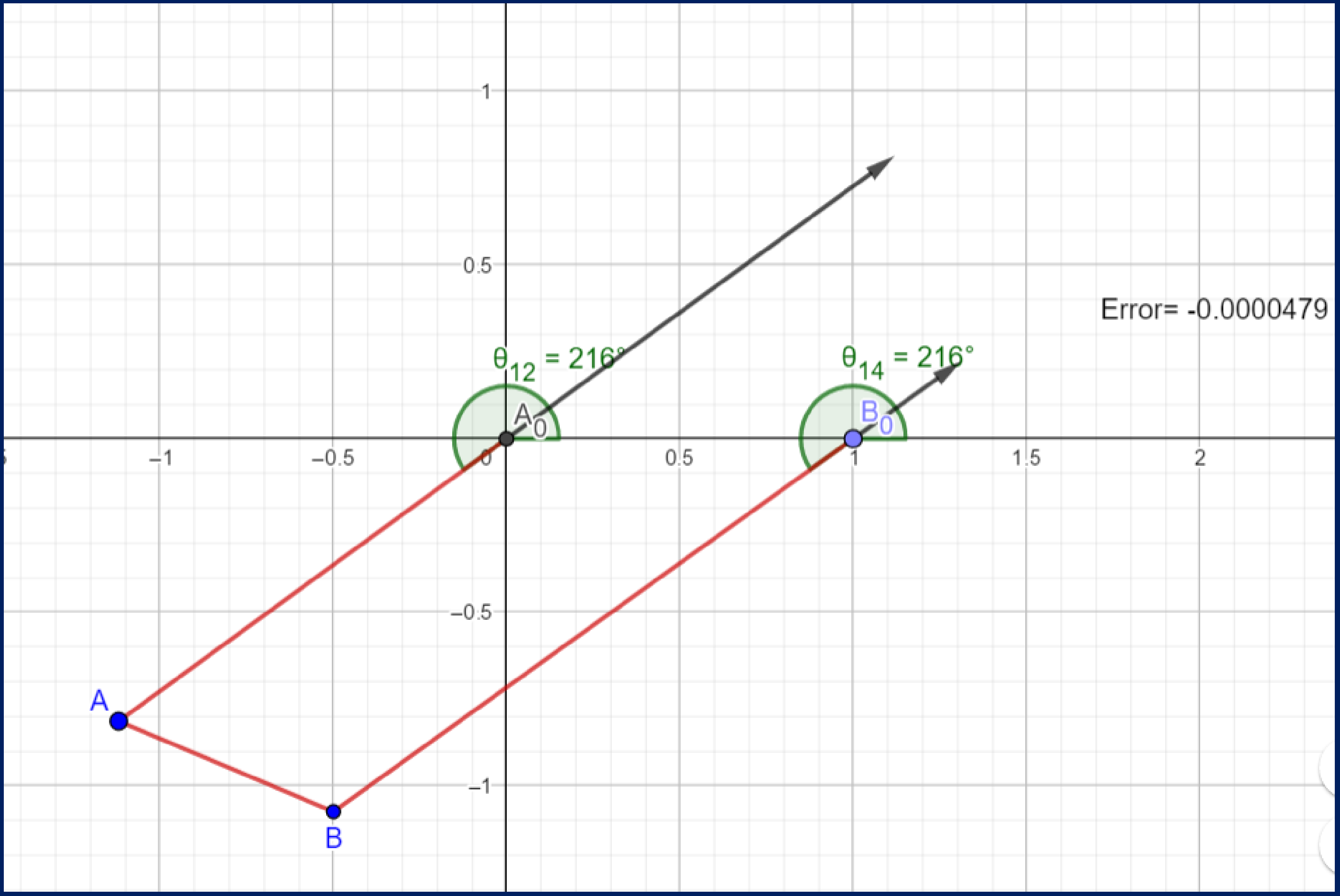
Function Generation-Three Precision Points:



5. Four Bar Mechanism

Function Generation-Three Precision Points:

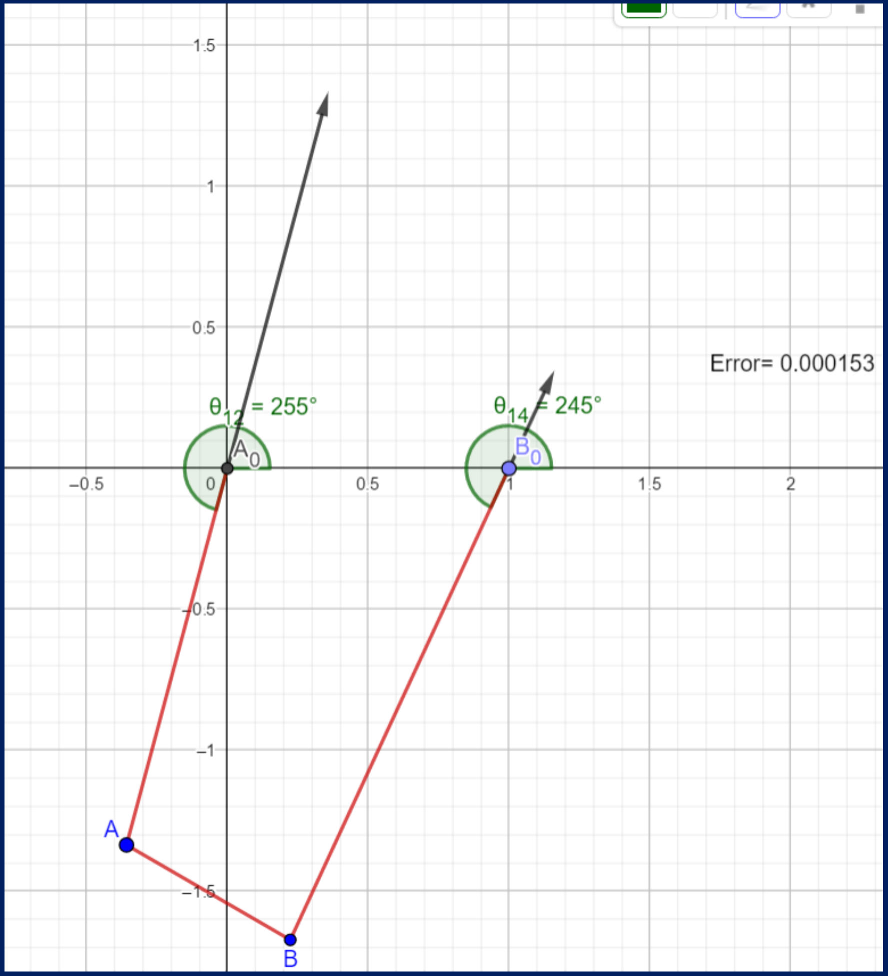
Precision point 1



5. Four Bar Mechanism

Function Generation-Three Precision Points:

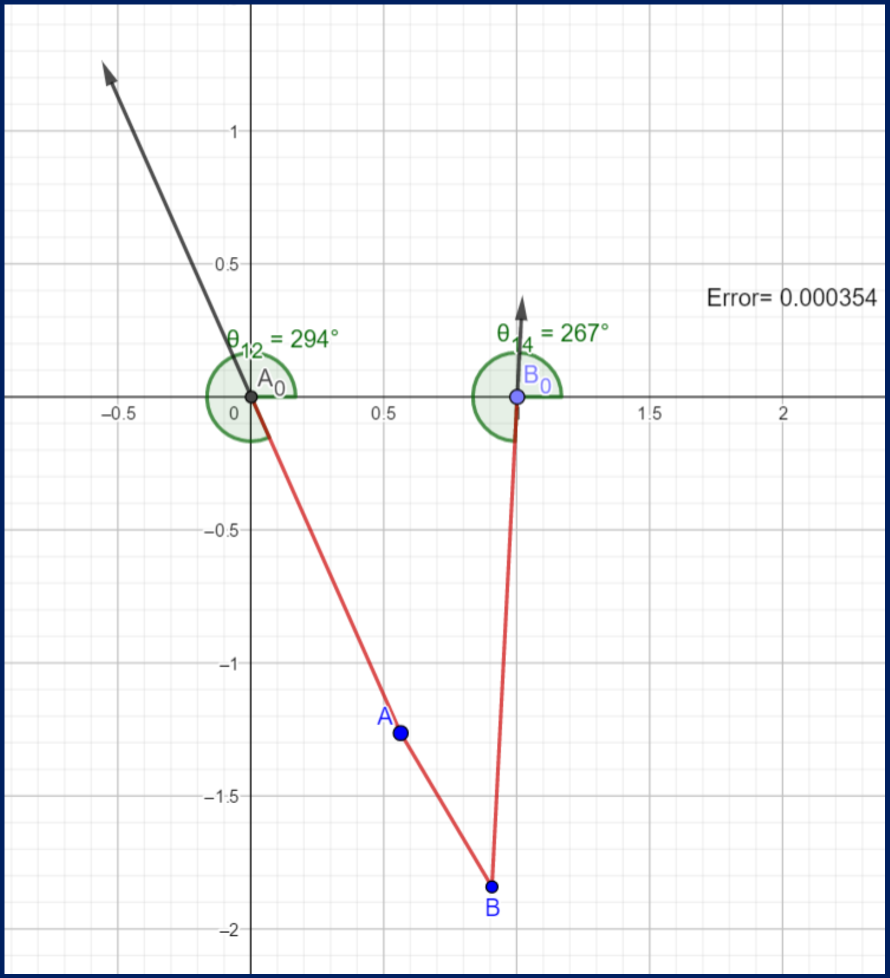
Precision point 2



5. Four Bar Mechanism

Function Generation-Three Precision Points:

Precision point 3



5. Four Bar Mechanism

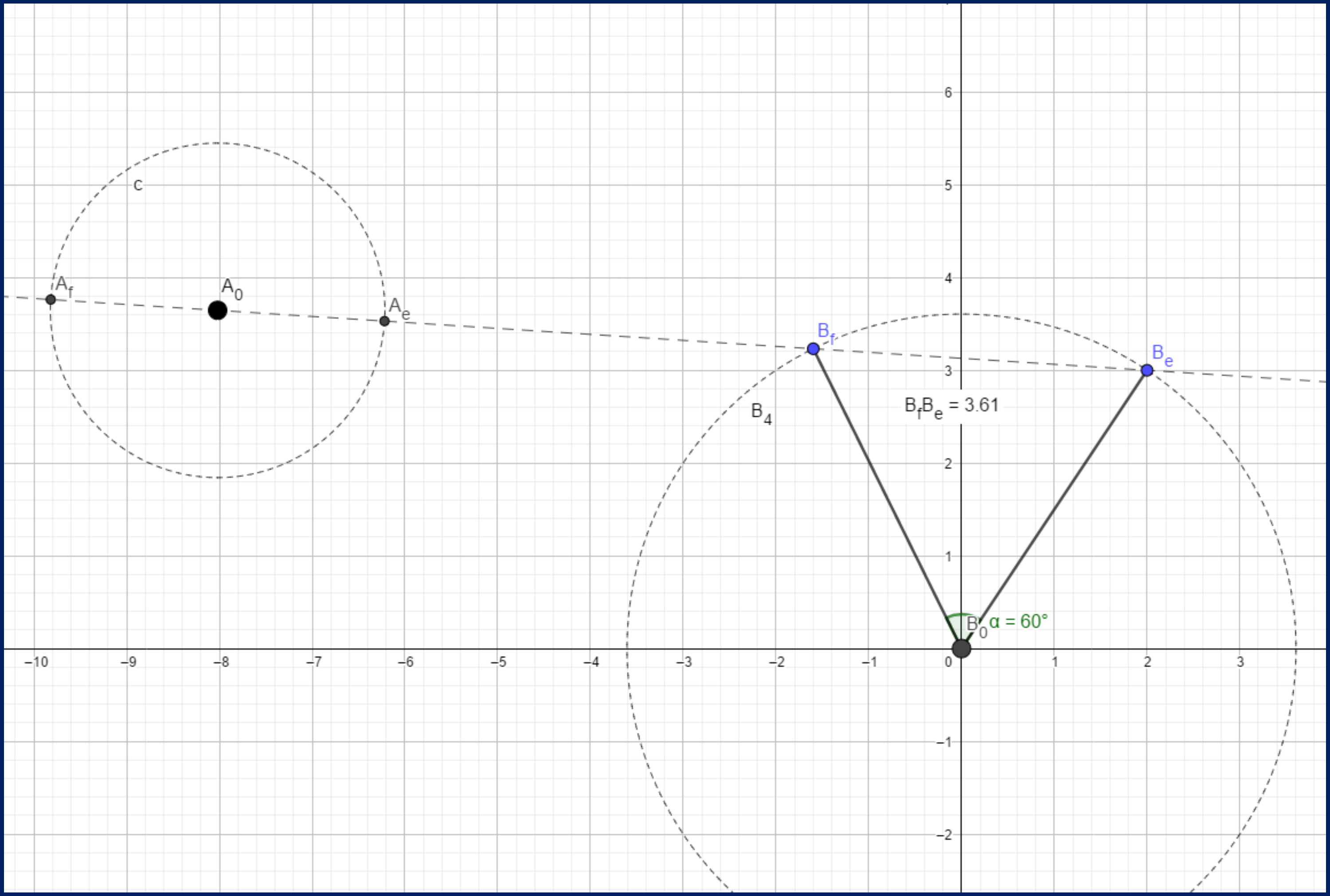
Design for Dead-Centers, *Centric* Four Bar:

1. Select the two dead-centers of the follower and a_4.
2. Draw two positions of point B, B_e extended dead center and B_f folded dead center.
3. Draw line B_eB_f and select A_0 at a suitable location on this line.
4. Draw a circle with diameter $|B_eB_f|$ centered at A_0.
5. $A_0A_eB_eB_0$ is the mechanism in extended dead center and $A_0A_fB_fB_0$ is the mechanism in folded dead center.

This is called a *centric* four-bar because crank rotates 180° between extended dead center and folded dead center.

5. Four Bar Mechanism

Design for Dead-Centers, *Centric* Four Bar:



5. Four Bar Mechanism

Design for Dead-Centers, *Centric* Four Bar: