ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 301
Theory of Machines I
Fall 2026

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Various Phases of (*Mechanical*) Machine Design

- Motion transmission
- Calculation of required driving force(s) and accompanying reactions (internal forces)

ME 301

- Dynamic requirements (smooth operation, control of vibration & noise)

ME 302

- Strength and rigidity

ME 307-8

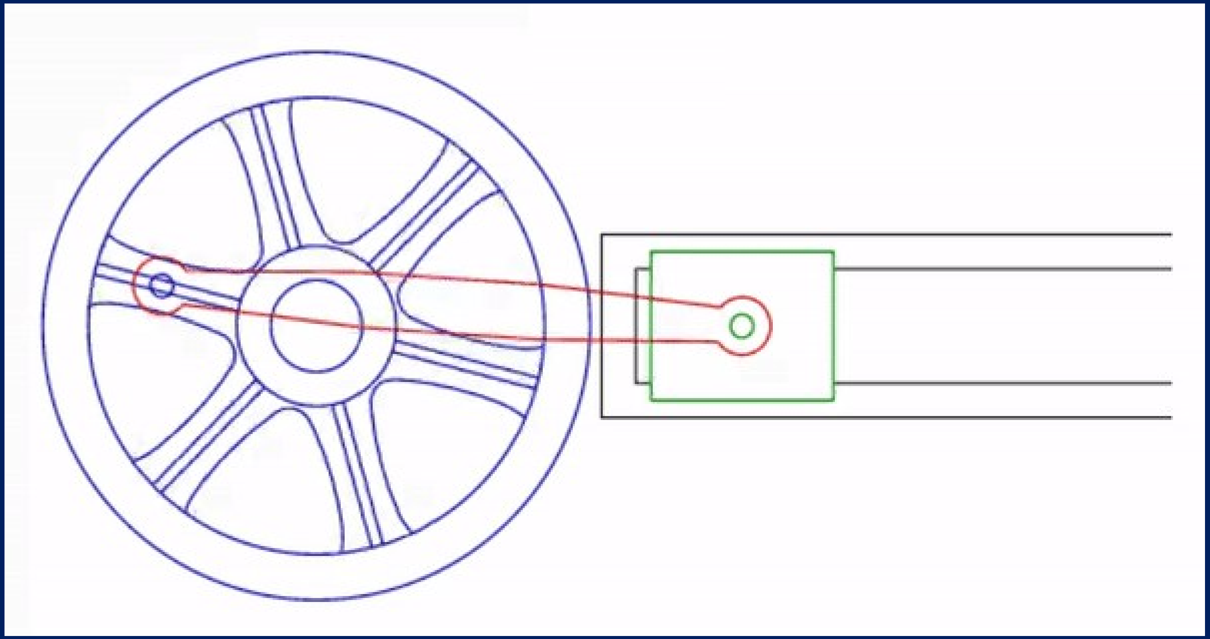
- Economic, ergonomic, esthetic, safety etc.

ME 407

- Production of designed parts, environmental impact, recycling…

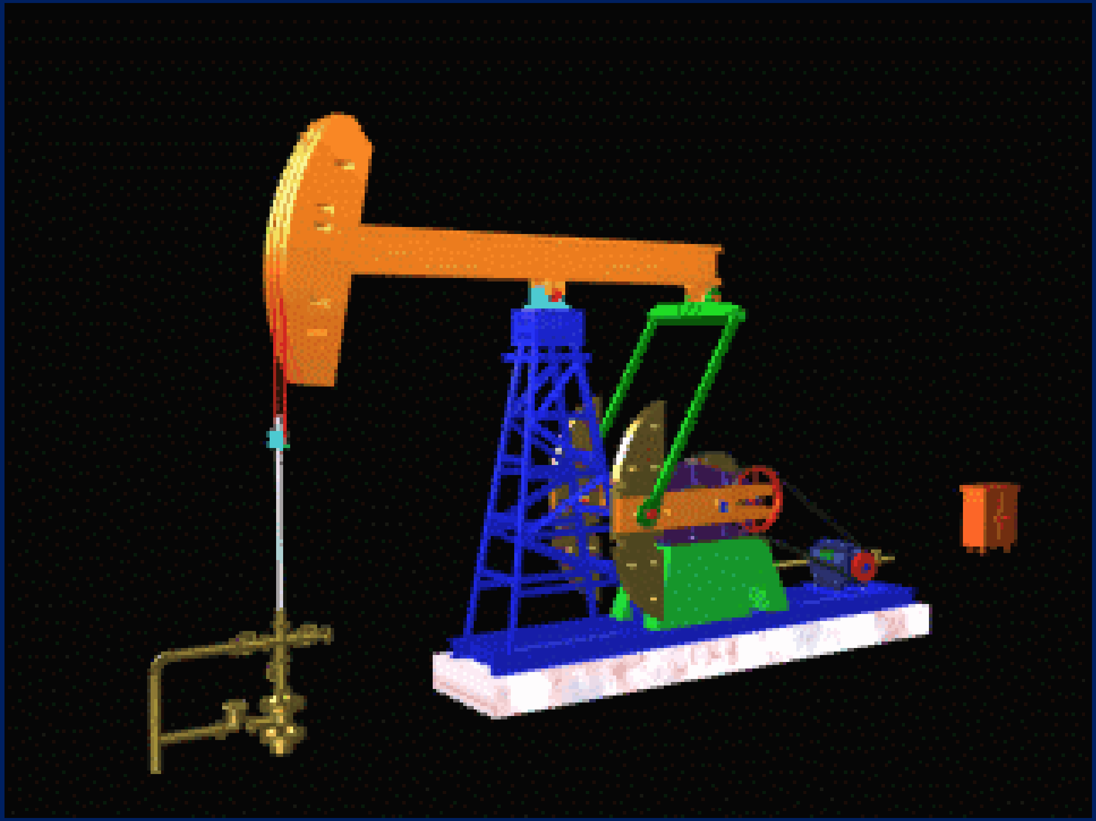
Question:

How do we convert reciprocating linear motion into continuous rotation (or vice versa)?



Question:

How do we convert continuous rotation into oscillatory rotation?



ME 301 Theory of Machines I

Course Content

Mechanisms

- Introduction and Basic Concepts
- Kinematic Analysis of Planar Linkage Mechanisms
  - Position Analysis
  - Velocity Analysis
  - Acceleration Analysis
- Gear Trains and Other Linear Mechanical Systems

Dynamics of Machinery

- Quasi-Static Force Analysis without Friction
- Dynamic Force Analysis without Friction
- Dry Friction Analysis
- Dynamic Motion Analysis and Derivation of Equation of Motion

Four Link Mechanisms

- Four-Bar Mechanisms
- Slider-Crank Mechanism and Its Inversions
- RRPP and RPRP Mechanisms

Tentative Course Outline

1. Introduction and Basic Concepts

Degree of Freedom, Kutzbach Formula, General Degree of Freedom Equation, Exceptions, Grübler's Equation, Classification of Mechanisms, Kinematic Inversion, Enumeration

2. Kinematic Analysis of Mechanisms

Complex Number Representation of Planar Vectors

Position Analysis of Mechanisms and Loop Closure Equation

Solution Techniques for Loop Closure Equation

- Graphical Solution Methods
- Analytical Solution Methods

Velocity Analysis and Singular Positions, Velocity Influence Coefficients

Acceleration Analysis

3. Gear Trains and Other Linear Systems

Belts, Rollers, Chain Drives, Simple and Compound Gear Trains, Planetary Gear Trains, Gear Trains with Bevel Gears

4. Vectorial Force Analysis in Machinery

Static Force Analysis without Friction

Dynamic Force Analysis without Friction

5. Four Link Mechanisms

Four Bar Mechanism, *Grashof's Rule, Transmission Angle, Dead-Center Positions of Four-Bar and Slider-Crank Mechanisms, Body Guidance: Four Bar Mechanism Design for Two Positions, Function Generation: Use of Freudenstein's Equation with Chebyshev Spacing*

Course Policy

This course is offered in **full coordination**.

INSTRUCTORS

Section 1 : Dr. Kıvanç Azgın (G-207/azgin@metu.edu.tr/210 25 35)

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Sections 4&5 : Dr. Ergin Tönük (B-319/tonuk@metu.edu.tr/210 5293)

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Semih Durmaz (G-210/ semihd@metu.edu.tr /)

Course Policy

COURSE WEB SITE

http://odtuclass.metu.edu.tr

GRADING

2 Midterms (25% each), Final (30%), Homework Assignments (10%), Computer Implementation Project (5%), Attendance and/or Quiz (5%).

TEXT BOOK

E. Söylemez, Mechanisms, METU, 5th/6th Edition, 2018/2025.

REFERENCES

Prof. Dr. Eres Söylemez Online Material, ME301 Theory of Machines I

METU Open Courseware, ME 301 Theory of Machines (by Dr. Ergin Tönük)

A. G. Erdman, G. N. Sandor, Mechanism Design: Analysis and Synthesis, Prentice-Hall TJ230 E67 1991.

B. Paul, Kinematics and Dynamics of Planar Machinery , Prentice-Hall TJ175 P38.

J.E. Shigley, J.J. Uicker, Theory of Machines and Mechanisms, Mc-Graw Hill Book Co. Second Edition, 1995.

Course Policy

REGULAR EXAMINATIONS

Examination dates, hours and places will be announced by the Mechanical Engineering Department. All examinations will be closed book and notes. The front page of the exam paper will contain some formulae which are considered to be difficult to remember. Please bring a scientific calculator to all exams.

MAKE-UP EXAMINATIONS

One make-up examination, covering the whole course material, will be given after the final exam date for the students who missed a midterm and/or final examination due to a valid excuse approved by the Department.

HOMEWORK ASSIGNMENTS

There will be about 7 homework assignments where you need to solve problems related to the topics covered in the class. Homework assignments should reflect your own effort and should be returned on due date. No late submissions are accepted.

COMPUTER IMPLEMENTATION PROJECT

Computer implementations of the solution of the equations/set of equations derived in homework assignments will be assigned as computer implementation project where full cycle kinematic and dynamic analysis will be realized. You are free to use any mathematical tool of your preference. If you prefer, guiding material for matlab, which is one of the mathematical tools you may use, will be available.

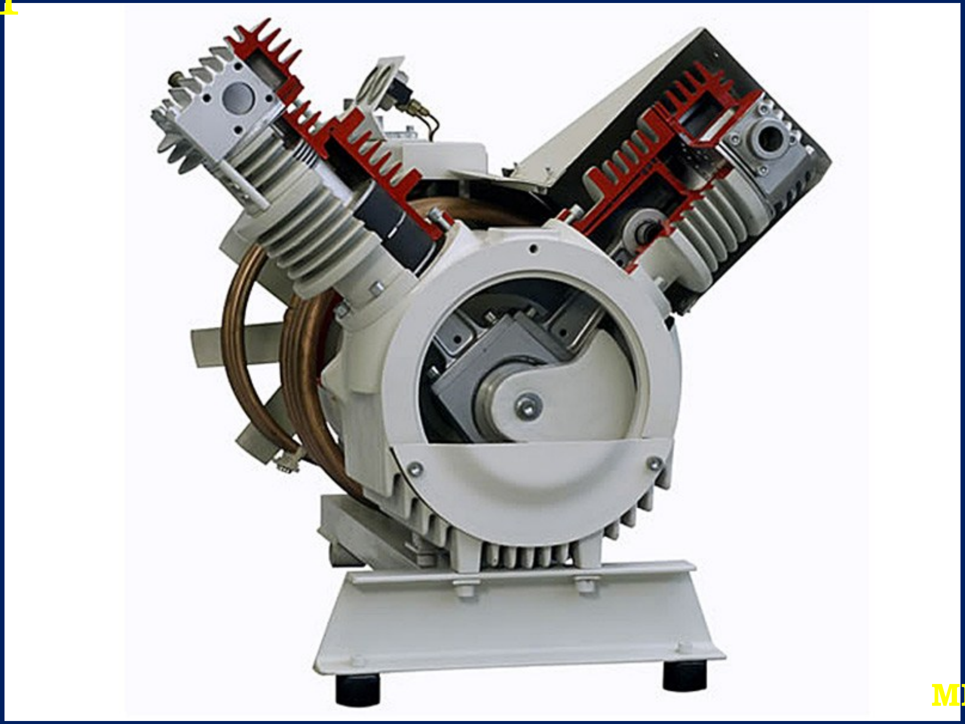
1. Introduction and Basic Concepts

- Introduction and Application Examples
- Definitions
- Degree of Freedom
  - Kutzbach Formula
  - General Degree of Freedom Equation
    - Exceptions
- Grübler’s Equation
- Kinematic Inversion
- Enumeration

Basic Definitions

Mechanical Machine: A self contained assembly of components that transmits / modifies energy to perform or assist a specific task.

- It has an energy source (input / prime mover / motor / actuator)
- It accomplishes a well defined task (output / work)



Basic Definitions

(Mechanical) Machine: Mechanical forces of nature, accompanied by motion, are compelled to do the work (task).

Basic Definitions

Theory of Machines: An applied *exact* science used to understand / generate motions of parts of the machine and the forces that produce / accompany these motions.

We will need kinematics and kinetics of rigid bodies from ME 208 Dynamics.

Mechanics

is an *exact* science concerned with the behavior of bodies/matter under the action of forces

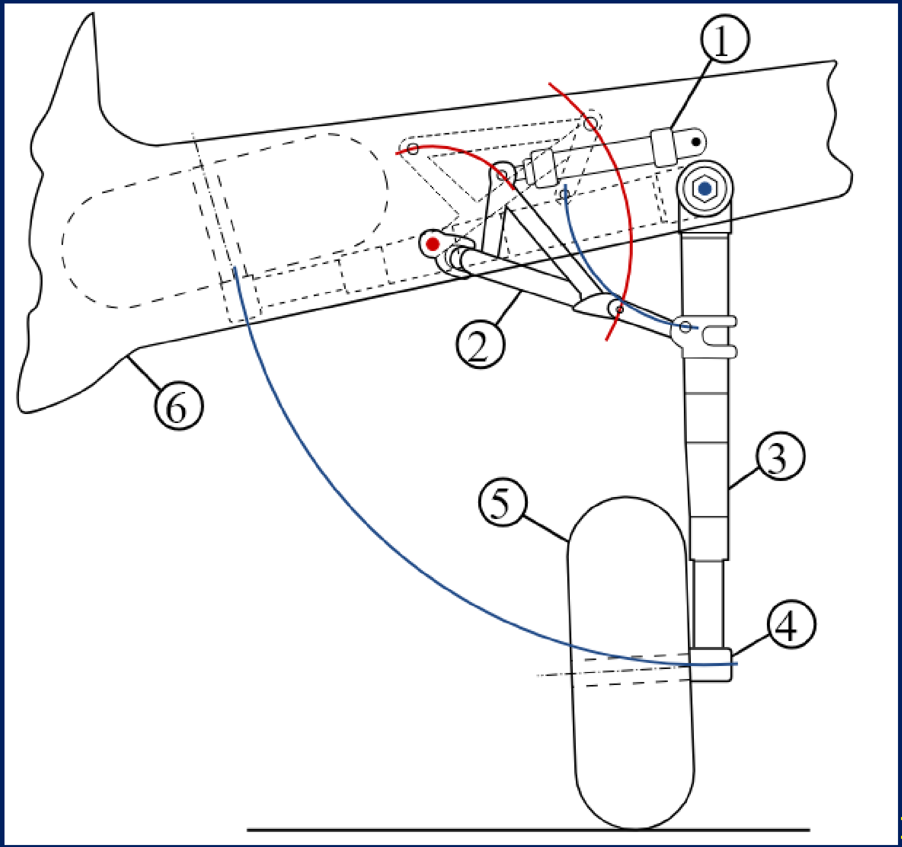
- Solid Mechanics
  - Mechanics of Deformable Bodies (Strength of Materials)
  - Mechanics of Rigid Bodies
    - Statics
    - Dynamics
      - Kinematics
      - Kinetics
- Fluid Mechanics

ME 301 Theory of Machines

Matter is assumed to be a continuous distribution of infinitely many infinitesimal particles (**continuum** *approximation*)!

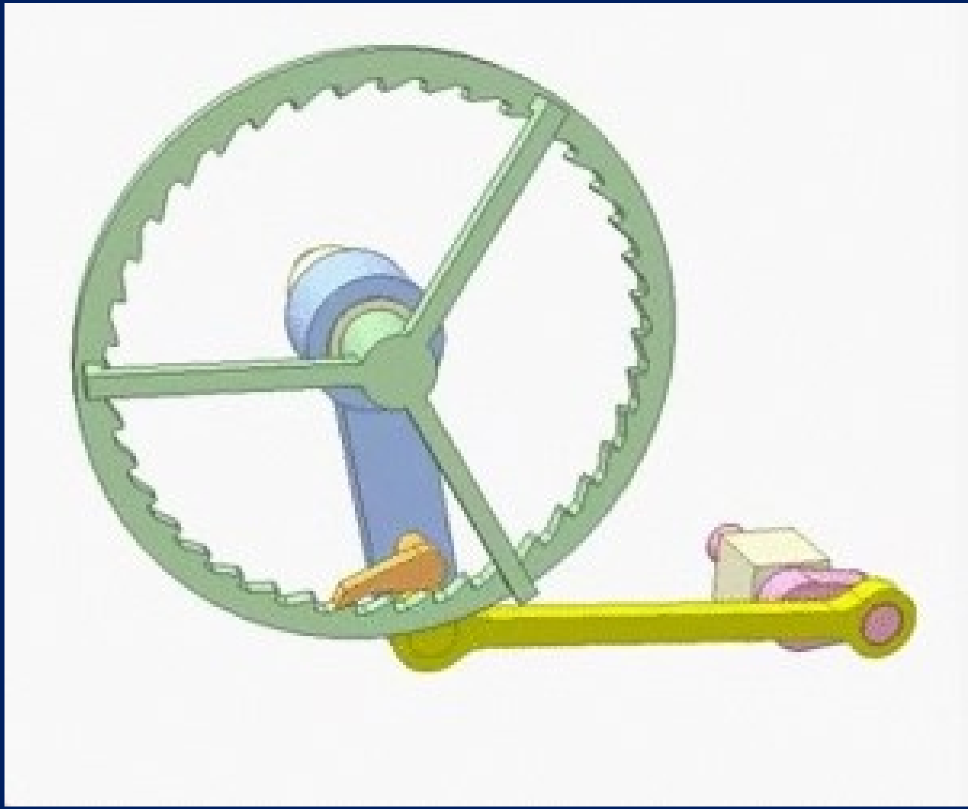
Definitions

Kinematics of machinery deals with the motion transmission characteristics in isolation from forces. This branch of science is also named as *Mechanisms. The motion of interconnected rigid bodies.*



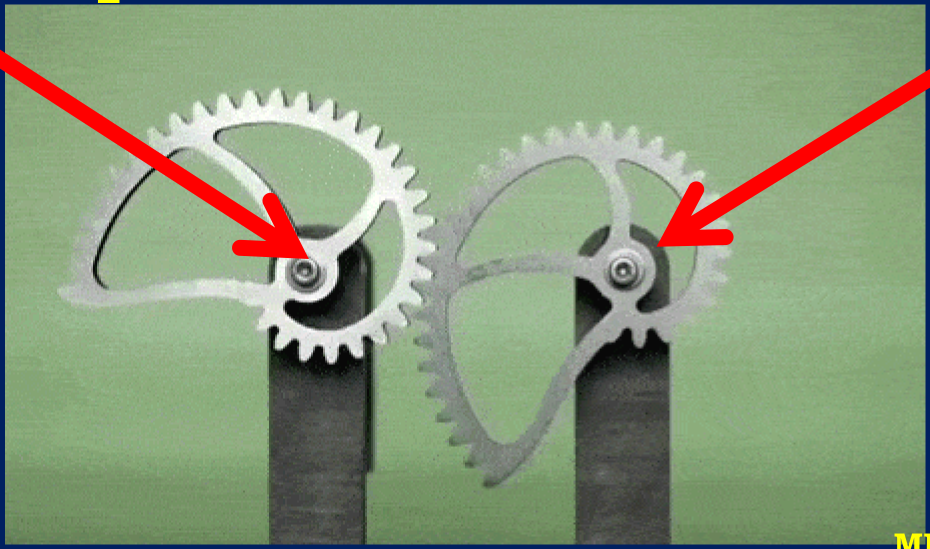
Definitions

Mechanism is a group of rigid bodies (**links**) connected to each other by joints (**rigid kinematic pairs**) having the purpose of motion transmission.



Definitions

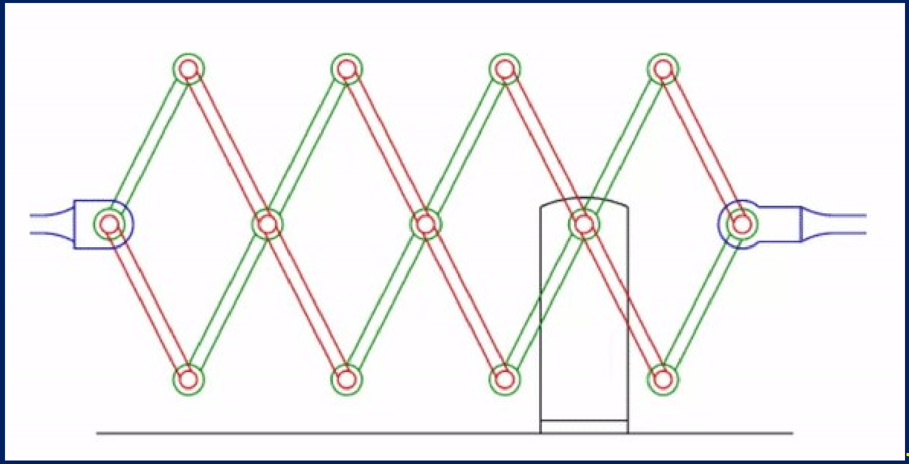
Kinematic Pair / Joint: Two or more kinematic elements joining two or more links and **permitting only certain relative motion** between the links, (Unlike many joints in ME 307 Machine Elements, which may be permanent or detachable and may not permit relative motion).



Mechanism

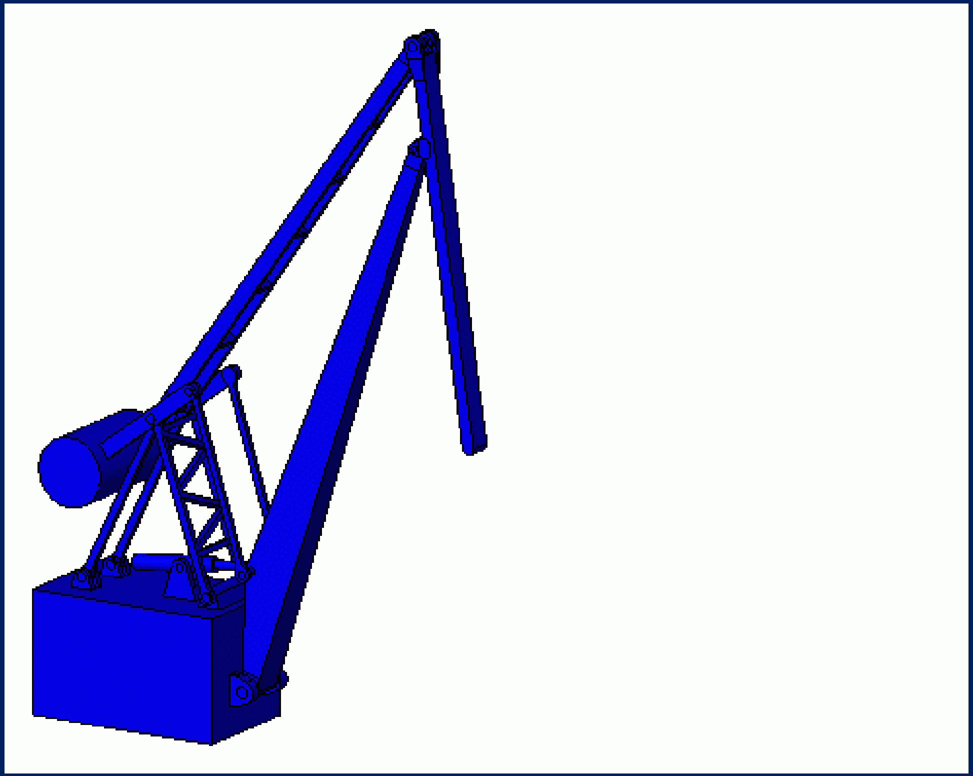
- It is a group of rigid bodies (*links*) connected to each other by rigid kinematic pairs (*joints*) to transmit force and motion.
- It is a *kinematic chain* where **one of the links is fixed**.
- Kinematic chain is an assembly of links connected by kinematic pairs **before any link is chosen as the frame**.

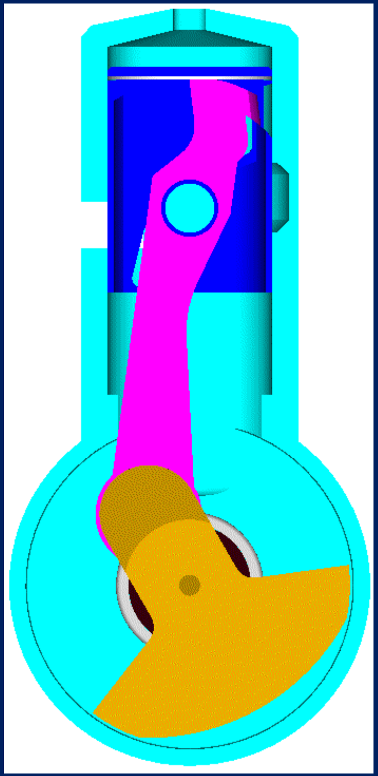
Mechanisms are <u>the basic building blocks</u> of mechanical machines. A machine is created by combining appropriate mechanisms to perform a specific task.



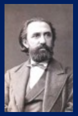
Mechanism

A ***mechanical machine*** is defined as a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motion[1].



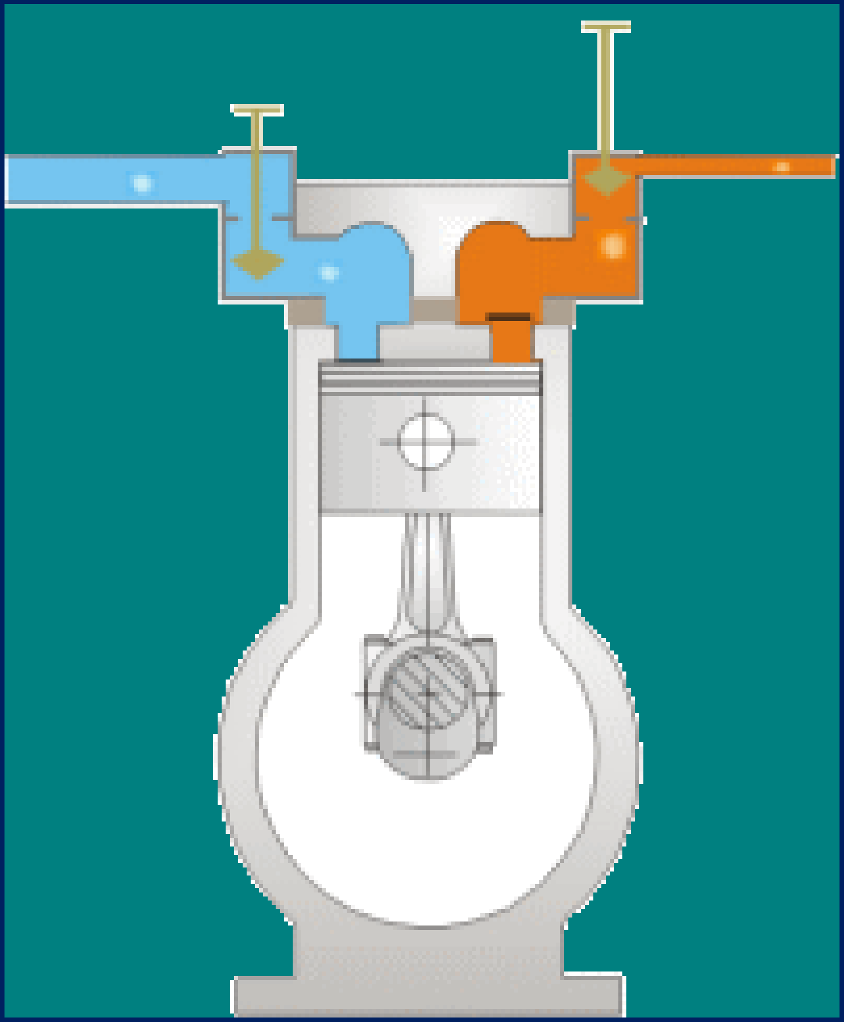


[1] Franz Reuleaux (1829-1905, German), Kinematics of Machinery, 1876

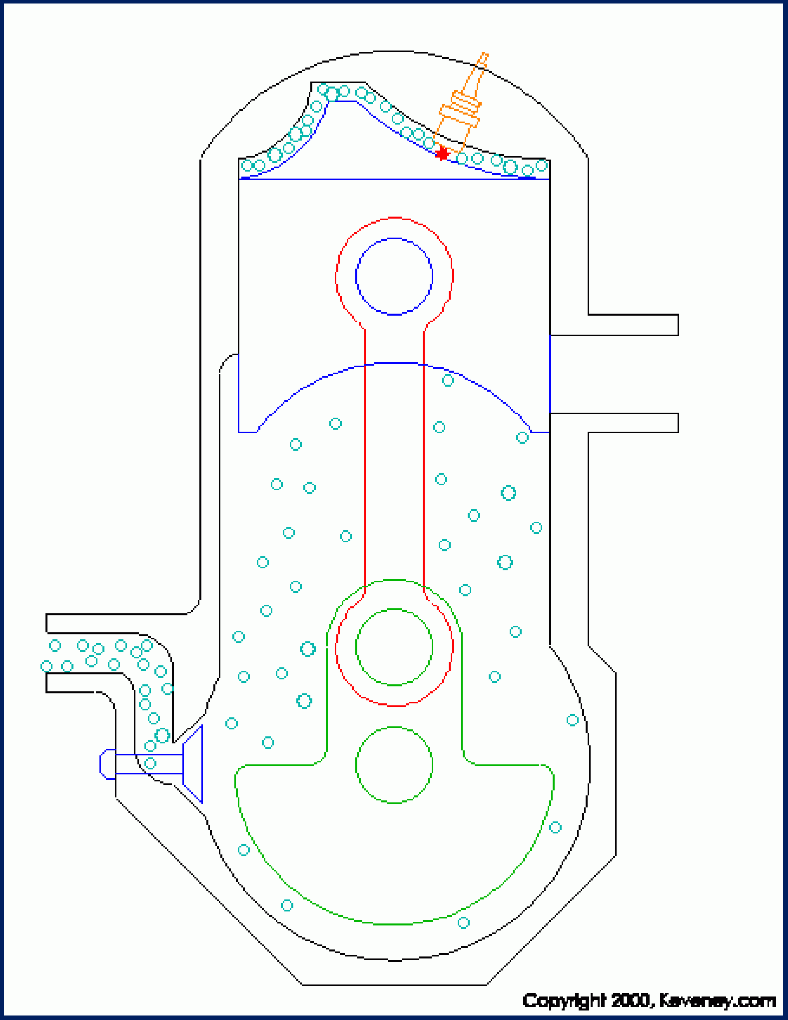


Examples

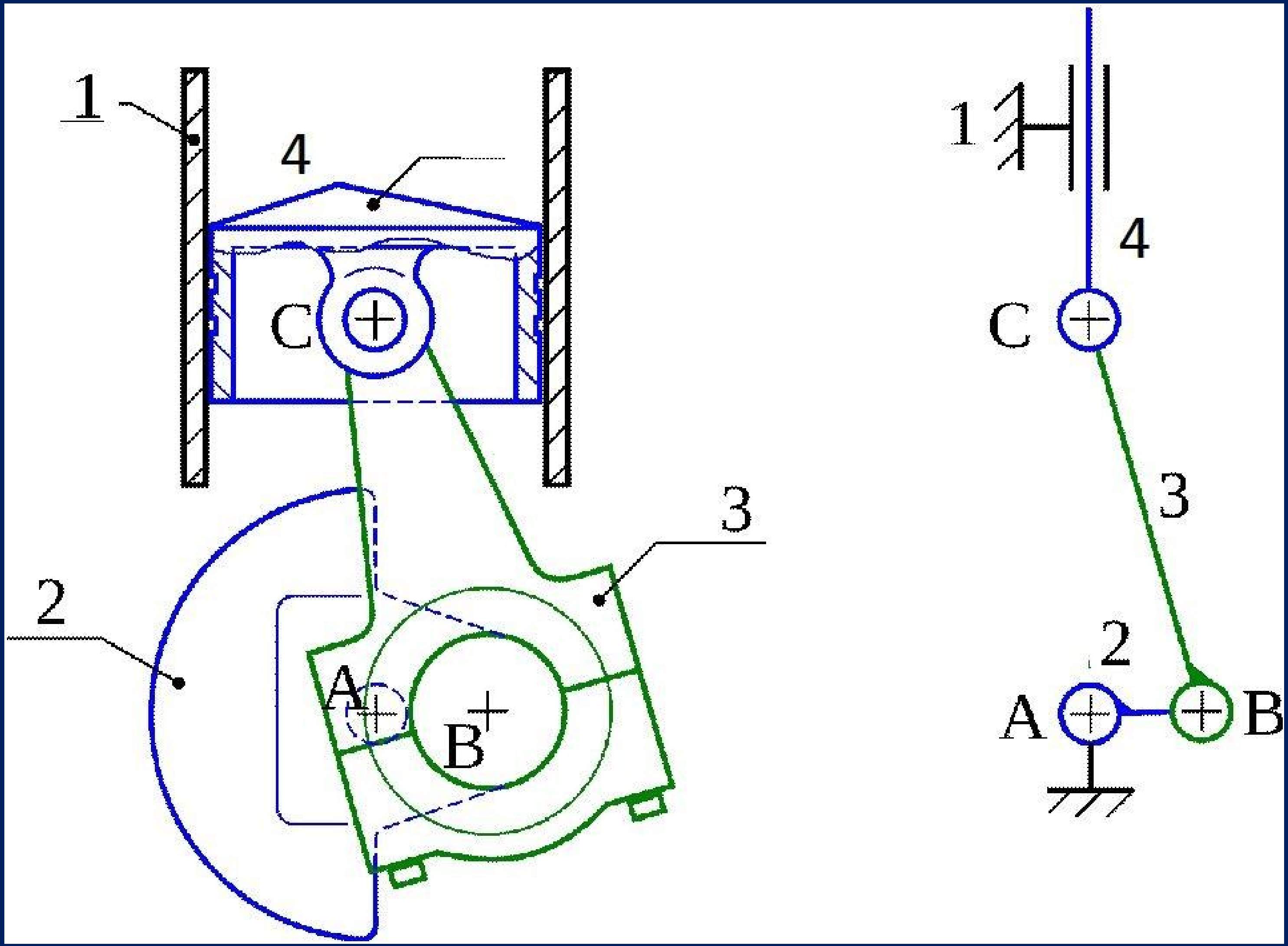
Reciprocating Air Compressor



Two-Stroke Internal Combustion Engine



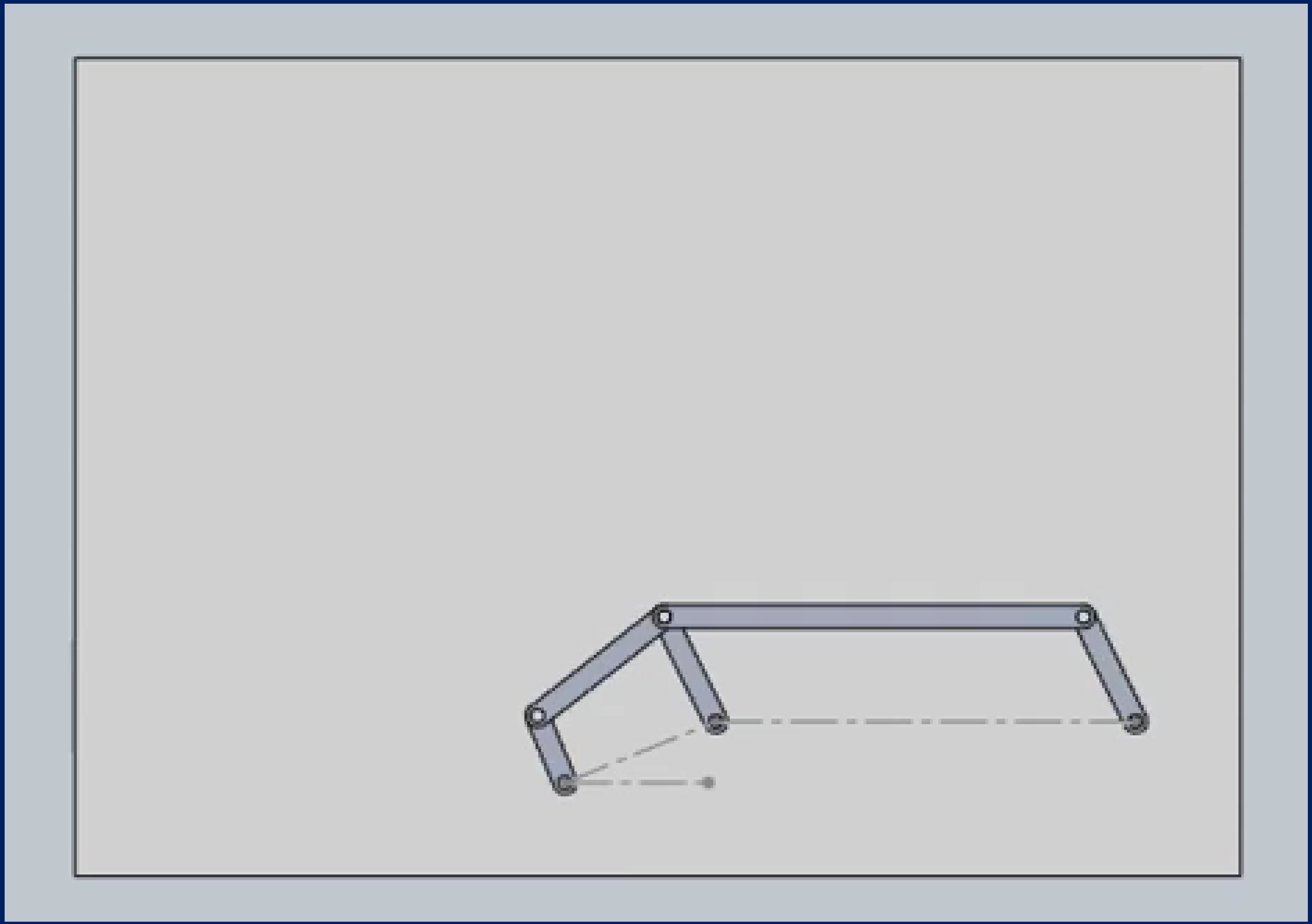
Examples



Slider-crank mechanism　　　Kinematic Diagram

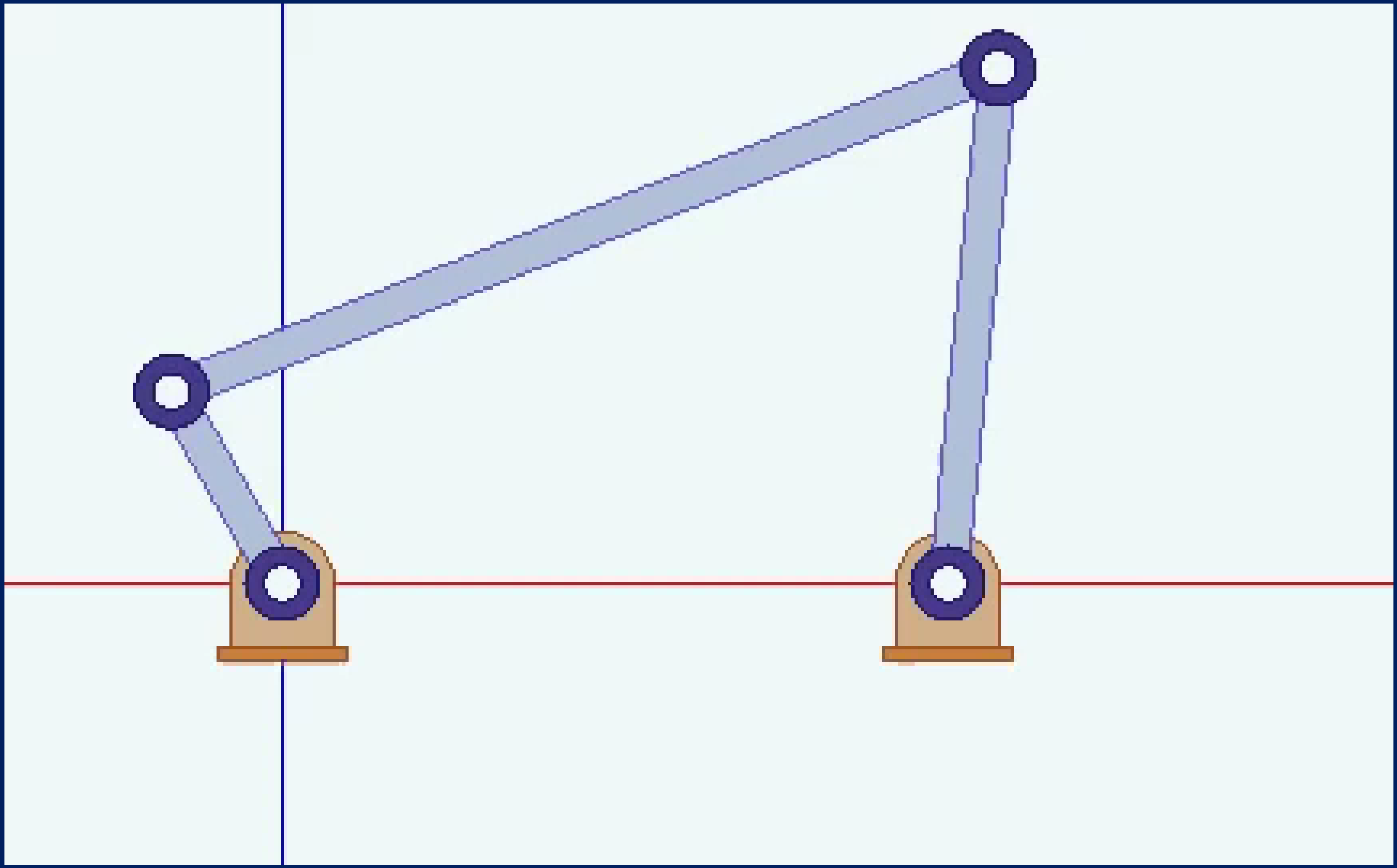
Examples

Windshield Wiper



Examples

Crank-Rocker Four Bar Mechanism

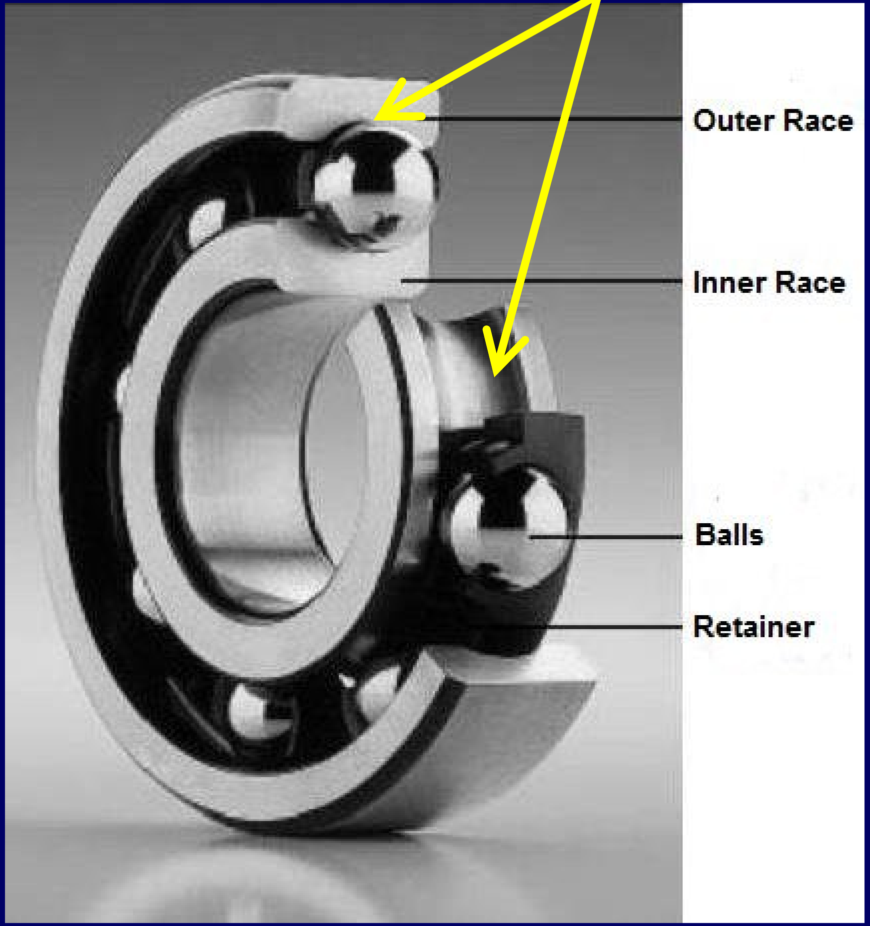


Examples

See Examples in the textbook pp. 5-9

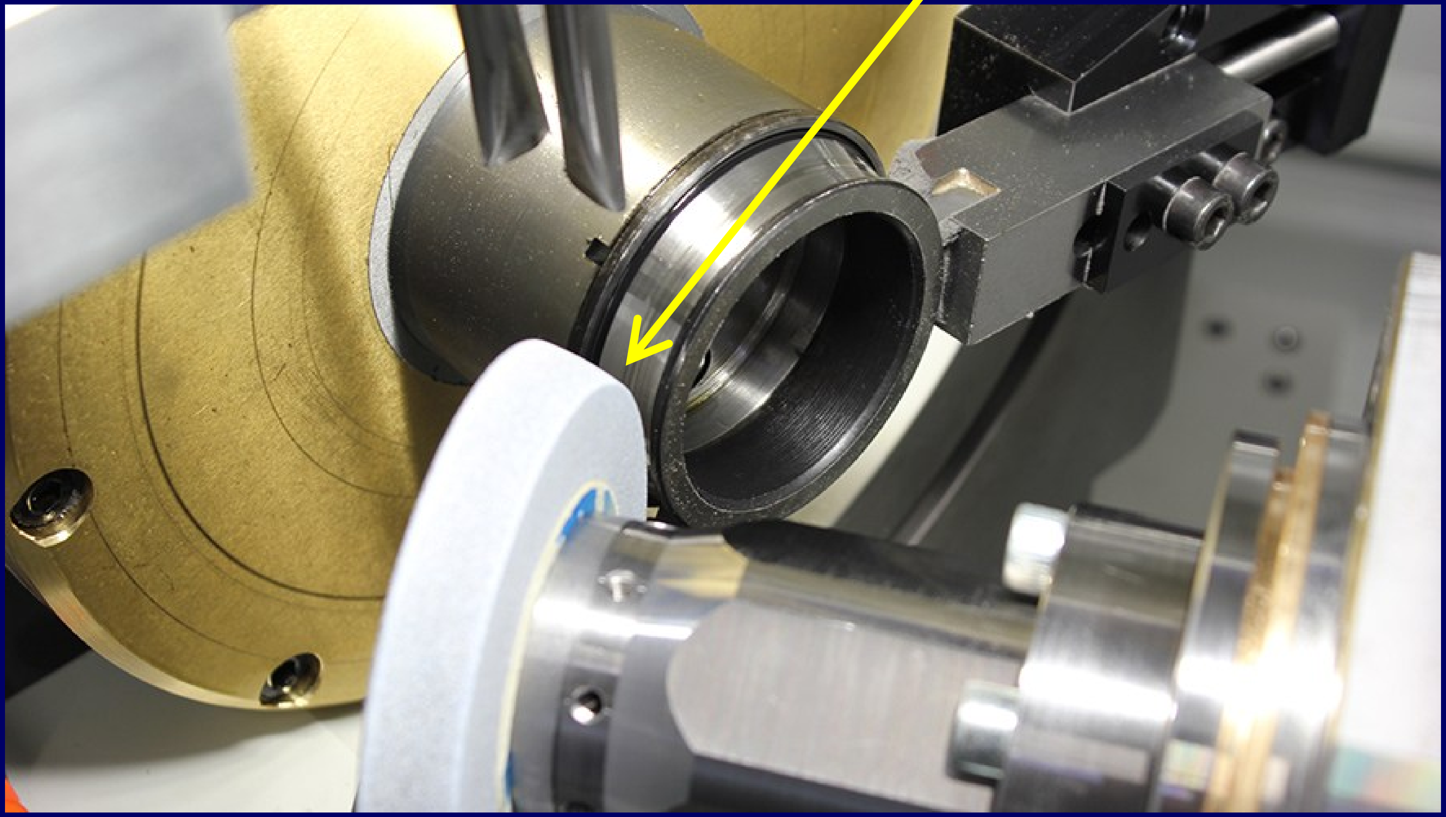
Physical Examples of Instructor

Roller Bearing Runway Grinding Wheel Reshaping

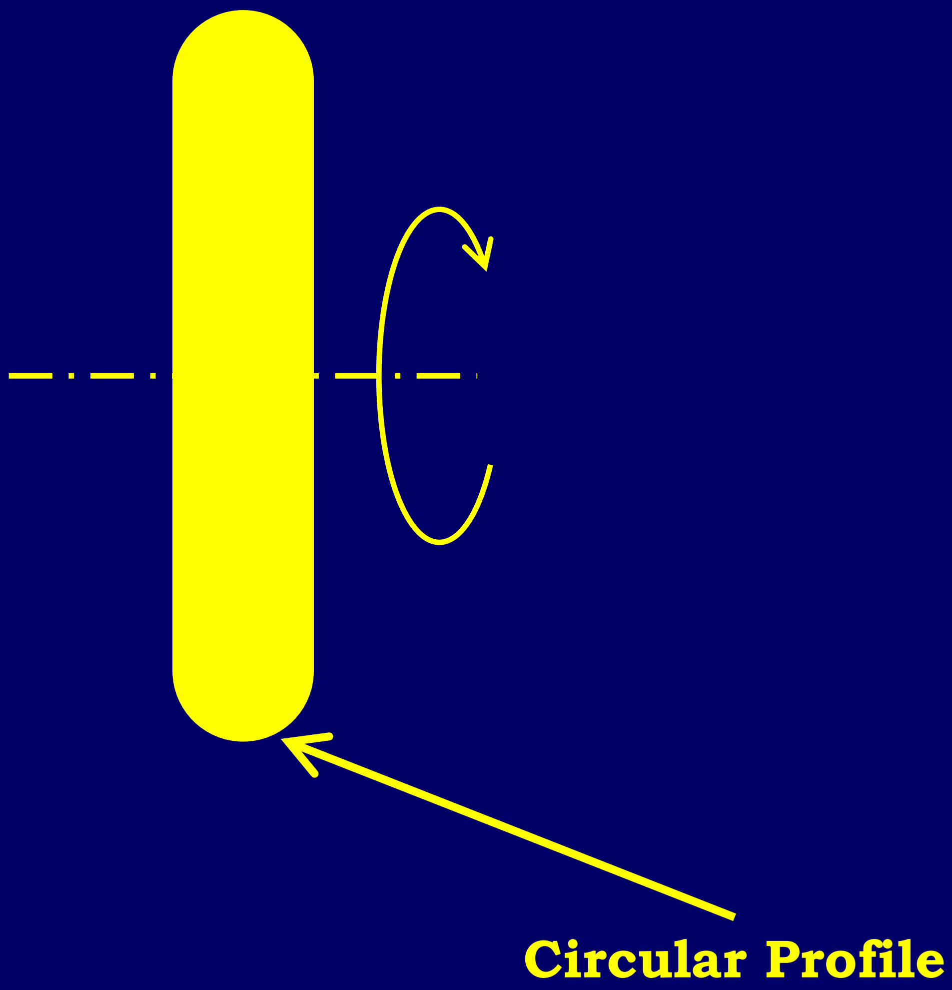


Roller Bearing Runway Grinding Wheel Reshaping

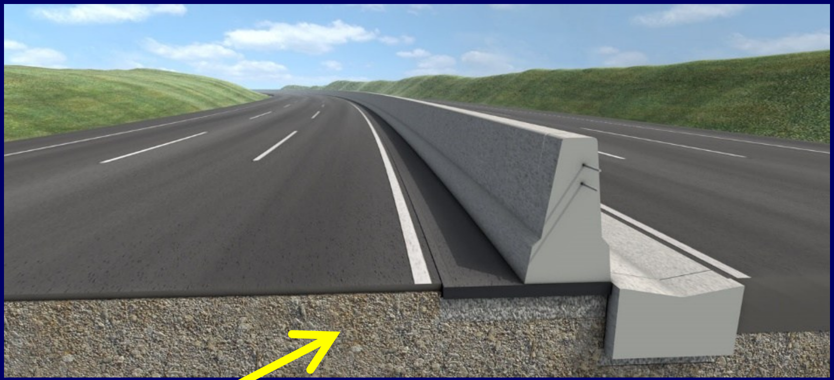
Grinding the Runway

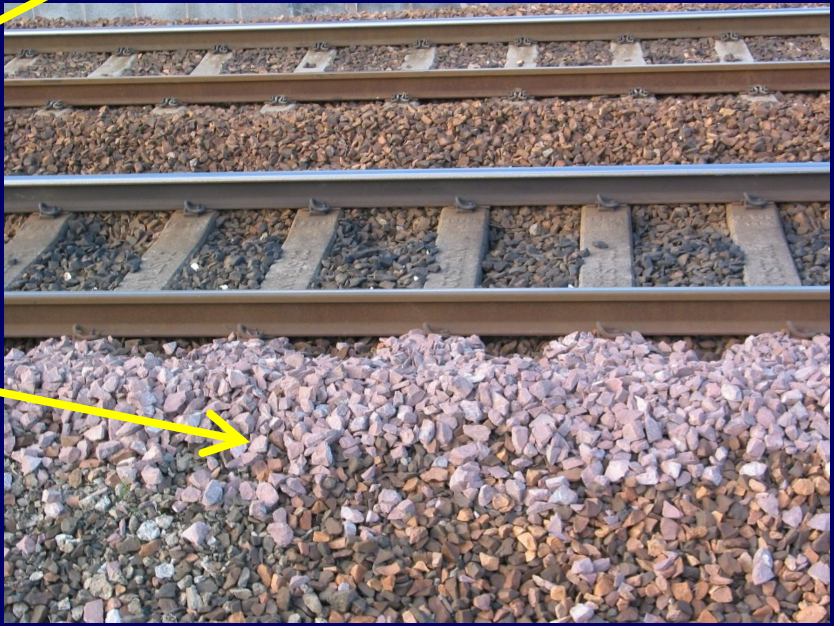


Roller Bearing Runway Grinding Wheel Reshaping



Rock Crusher





Crushed Rock

Parallelogram Windshield Wiper

Windshield Wipers

Parallelogram Windshield Wiper

Windshield Wipers

Compliant (Elastic) Mechanisms

Examples

Examples of Open Sources

https://blog.metu.edu.tr/eresmech/mechanisms/ch1/1-2-1/

Similar to above but in Turkish

http://makted.org.tr/wp-content/uploads/kaynaklar/ders_sunumlari/1.IntroductionToMechanisms.pdf

Slides 15-21, 24-28, 43

http://makted.org.tr/wp-content/uploads/kaynaklar/mekanizma_teknigi_konu_anlatimi/ch1/1-2_1.htm
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http://makted.org.tr/wp-content/uploads/kaynaklar/mekanizma_teknigi_konu_anlatimi/ch1/1-2_5.htm

Animations may not run…

Classification of Mechanism Problems

a. Functional Synthesis: Determination of candidate mechanisms that can realize a set of given (or implied) functional requirements.

b. Type Determination: Investigation of known mechanisms for their topological characteristics.

c. Kinematic Analysis: Prediction of kinematic quantities (position, velocity and acceleration) of a known mechanism.

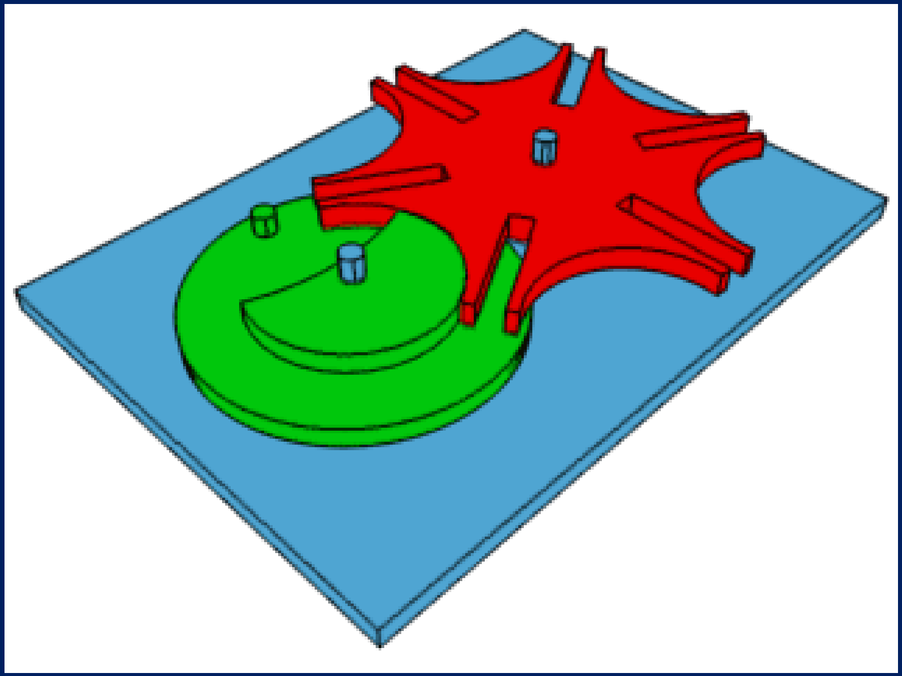
d. Kinematic Synthesis: Determination of mechanism parameters (mostly link dimensions) to realize a given motion (position, velocity and/or acceleration) for a mechanism whose topological characteristics are known.

Mechanisms

Kinematic element is the part of a body used to connect it to another body *permitting relative motion of two bodies*.

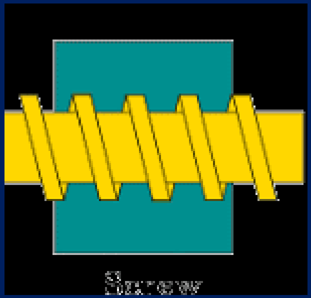
Kinematic pair is the combination of two kinematic elements and mostly termed as **joint**.

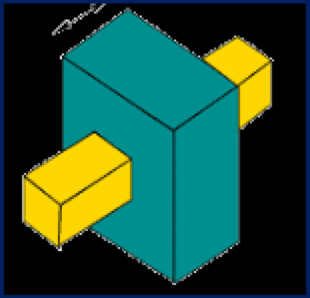
- **Open kinematic pair** permits separation of the two elements during operation.

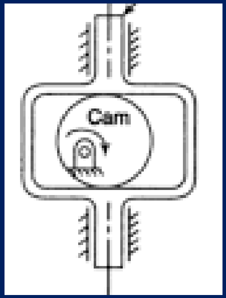


Mechanisms

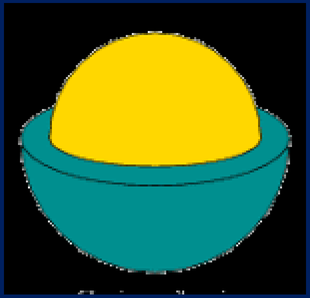
- **Closed kinematic pair** maintains contact for all possible positions of the mechanism
  - In **form closed kinematic pairs** one element envelopes the other

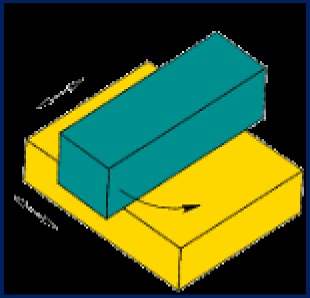


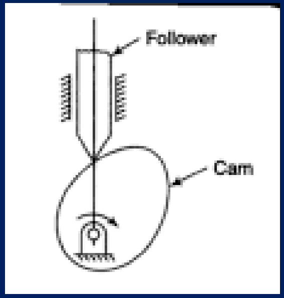




  - In **force closed kinematic pairs** an external force maintains contact of two kinematic elements







Kinematic Pair (*Joint*)

Open Kinematic Pair

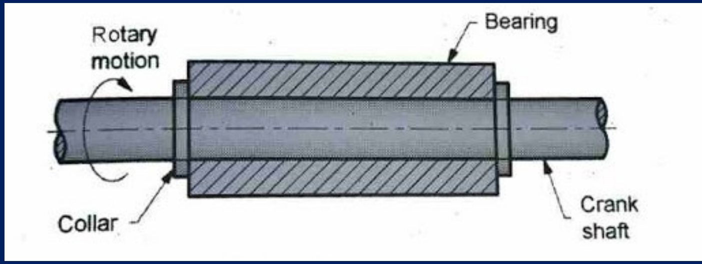
Closed Kinematic Pair

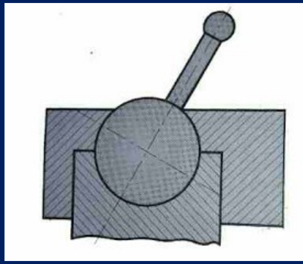
Force Closed Kinematic Pair

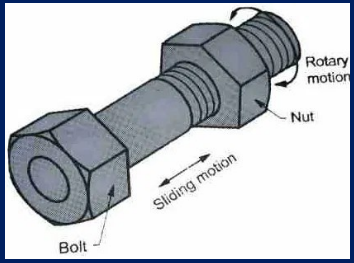
Form Closed Kinematic Pair

Mechanisms

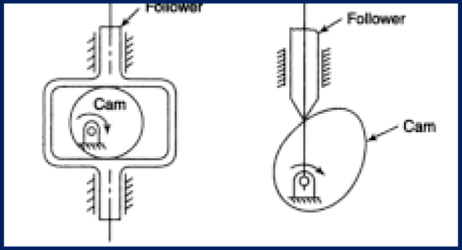
- **Lower kinematic pair** has contact of two kinematic elements along a surface therefore contact stresses are ***lower***

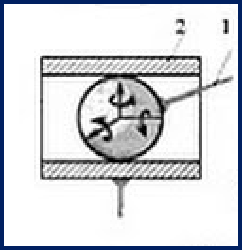


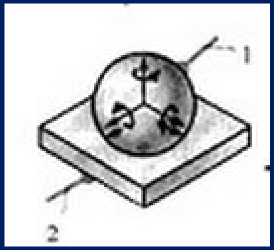


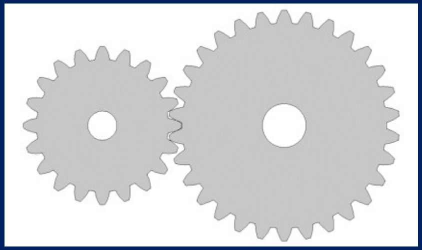


- **Higher kinematic pair** has contact of two kinematic elements along a line or at a point therefore contact stresses are ***higher***









Kinematic Pair

Open Kinematic Pair

Closed Kinematic Pair

Force Closed Kinematic Pair

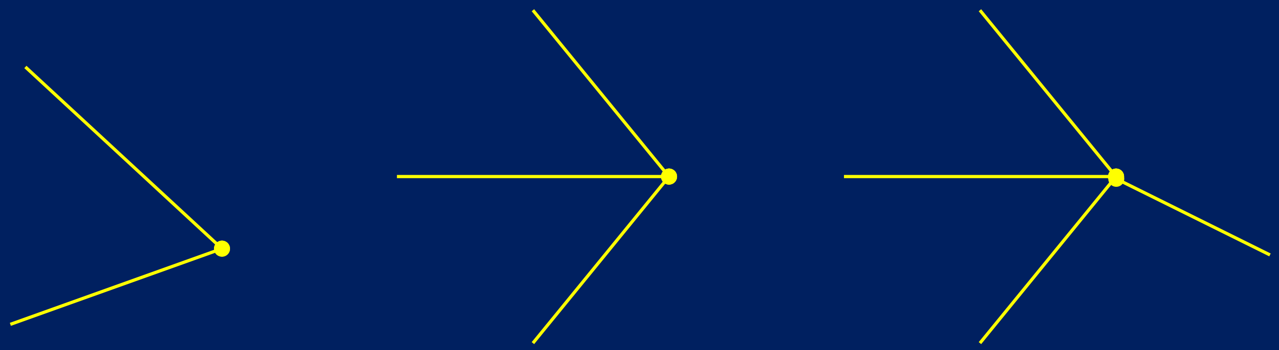
- Force closed lower kinematic pair
- Force closed higher kinematic pair

Form Closed Kinematic Pair

- Form closed lower kinematic pair
- Form closed higher kinematic pair

Mechanisms

Degree of a kinematic pair (*number of equivalent binary joints*) is number of links that come together at the kinematic pair *minus one*.



Binary Joint (Degree: 1) | *Ternary Joint* (Degree: 2) | *Quaternary Joint* (Degree: 3) | ...

Degree of Freedom

Degree of Freedom of a *Free* Rigid Body in Space: It is the number of *independent* parameters to define the position of a rigid body in that space.

3-D Space:

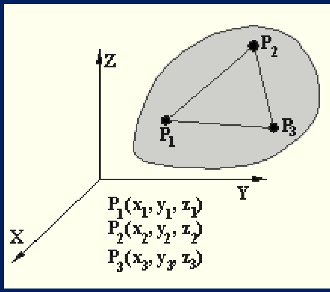
The rigid body may translate along any three orthonormal coordinate axes (3 DOF)

The rigid body may rotate about above coordinate axes (3 more DOF)

A free rigid body in 3-D space has 6 (3 translational and 3 rotational) degrees of freedom.

Degree of Freedom

Degree of Freedom of a *Free* Rigid Body in Space: It is the number of *independent* parameters to define the position of a rigid body in that space.



3-D Space: Three *non-collinear* points

9 parameters: x_1, y_1, z_1; x_2, y_2, z_2; x_3, y_3, z_3

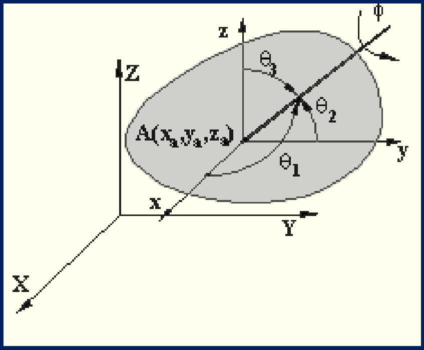
3 equations:
$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$|P_2P_3| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$$
$$|P_3P_1| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$$

We may select six of the nine parameters as we like to locate the body as we like, however the distances $|P_1P_2|$, $|P_2P_3|$ and $|P_3P_1|$ are fixed once the points P_1, P_2 and P_3 are selected on the rigid body.

Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Degree of Freedom

Degree of Freedom of a *Free* Rigid Body in Space: It is the number of *independent* parameters to define the position of a rigid body in that space.



3-D Space: One point, four angles

7 parameters: x_A, y_A, z_A; θ_1, θ_2, θ_3; ϕ

1 equation: $\sqrt{cos^2\theta_1 + cos^2\theta_2 + cos^2\theta_3{}^2} = 1$

We may select six of the seven parameters as we like to locate the body as we like, however the angles θ_1, θ_2 and θ_3 cannot be *all* arbitrary.

Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Degree of Freedom

Degree of Freedom of a *Free* Rigid Body in Space: It is the number of *independent* parameters to define the position of a rigid body in that space.

2-D Space:

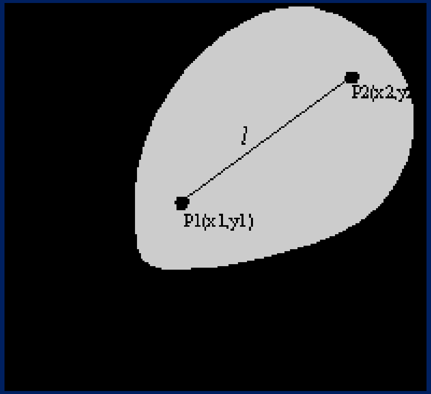
The rigid body may translate along any two orthonormal coordinate axes (2 DOF)

The rigid body may rotate about an axis perpendicular to the plane of motion (1 more DOF)

A free rigid body in 2-D space has 3 (2 translational and 1 rotational) degrees of freedom.

Degree of Freedom

Degree of Freedom of a *Free* Rigid Body in Space: It is the number of *independent* parameters to define the position of a rigid body in that space.



2-D Space: Two points

4 parameters: x_1, y_1; x_2, y_2

1 equation: $|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

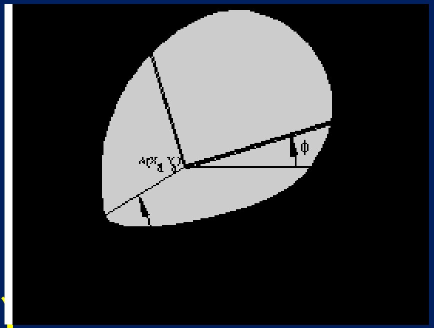
We may select three of the four parameters as we like to locate the body as we like, however the distance $|P_1P_2|$ is fixed once the points P_1 and P_2 are selected on the rigid body.

Therefore degree of freedom of a rigid body in plane motion is three.

Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Degree of Freedom

Degree of Freedom of a *Free* Rigid Body in Space: It is the number of *independent* parameters to define the position of a rigid body in that space.



2-D Space: One point, one angle

3 parameters:

- x_A, y_A; ϕ (Cartesian representation of point A)
- r, θ; ϕ (polar representation of point A)

In any case we may select three parameters as we like to locate the body in plane.

Therefore degree of freedom of a rigid body in plane motion is three.

Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Degree of Freedom

Degree of Freedom of a Kinematic Pair (Joint): It is the number of *independent* parameters to define the position of one link (rigid body) *relative to the other link* connected by the kinematic pair.

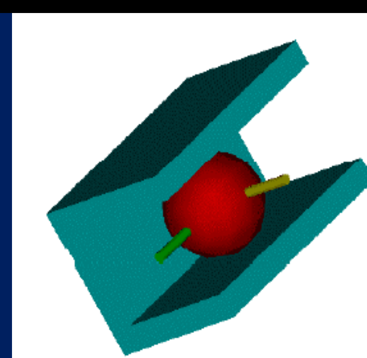
Since degree of freedom of a free rigid body in 3-D space is 6, the body is fully free without any constraint (i.e. joint) for degree of freedom 6.

$f_i < 6$

Similarly the body is fully free without any constraint (i.e. joint) for degree of freedom 3 for planar motion.

$f_i < 3$

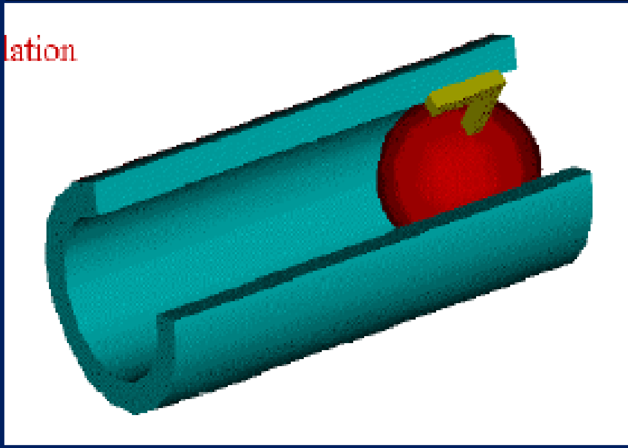
Degree of Freedom of Joints (1/6)
Independent Joint Motions

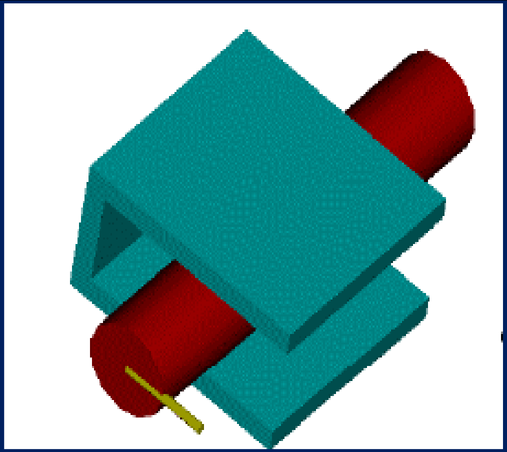


In Table I, rotational and translational motions are independent: specifying **one does not determine the other**.

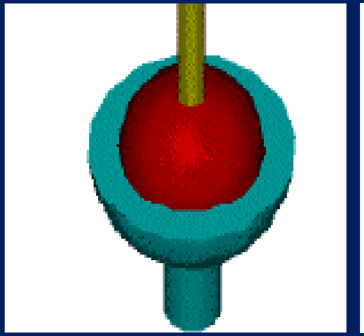
Textbook pg. 16 (5^{th}), 19 (6^{th})

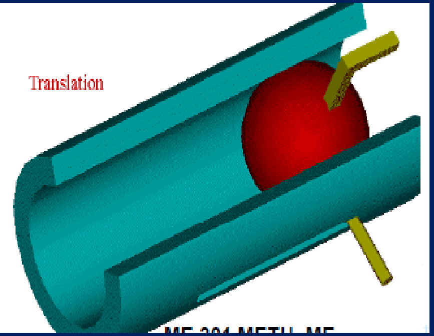
Degree of Freedom of Joints (2/6)

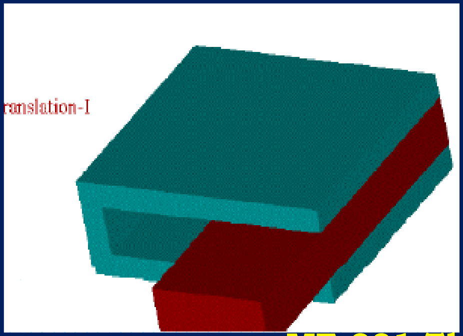




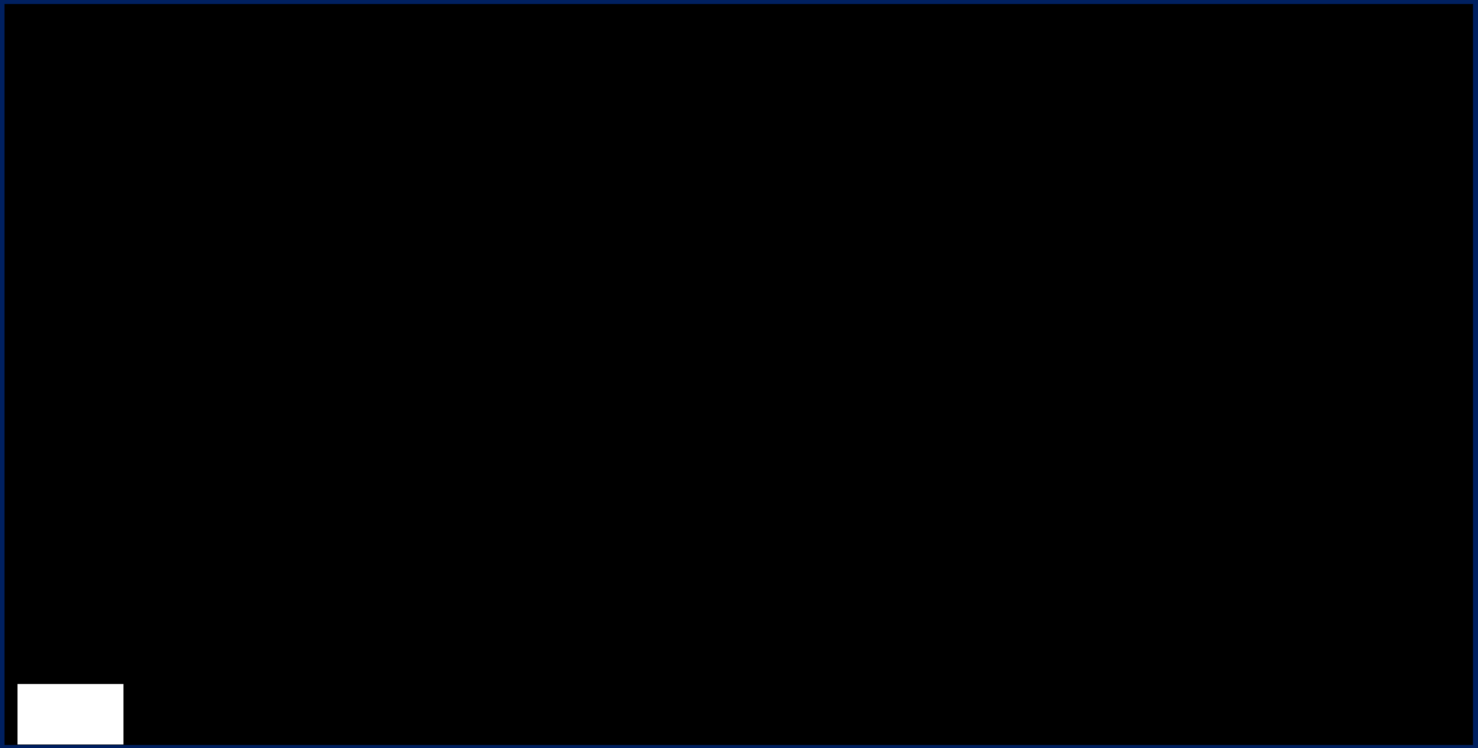
Degree of Freedom of Joints (3/6)



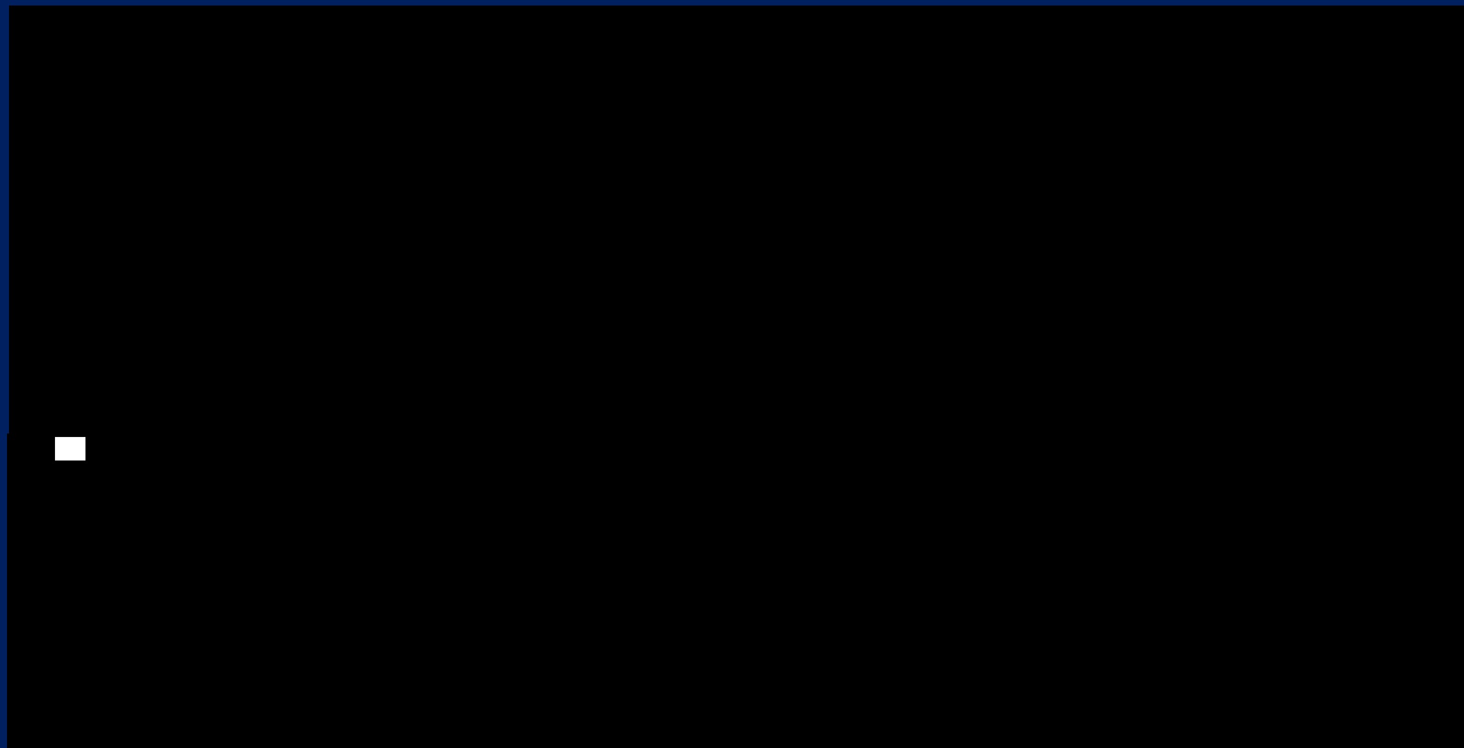


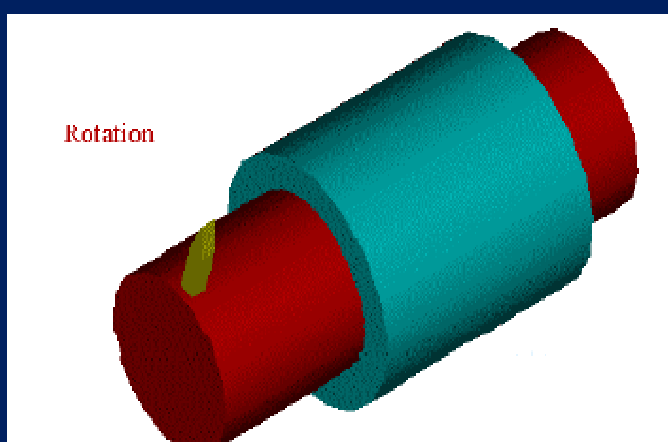


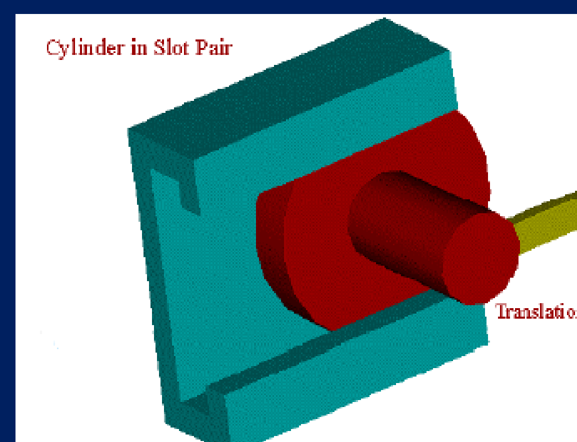
Degree of Freedom of Joints (4/6)



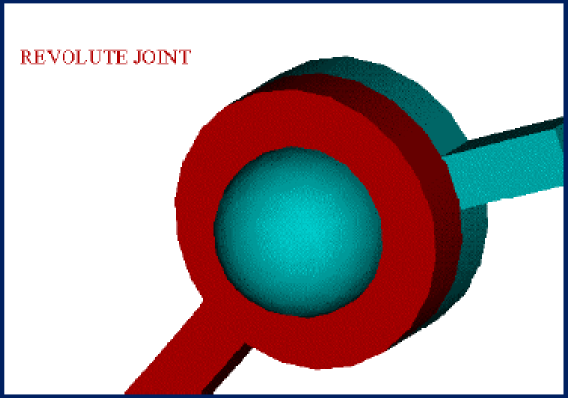
Degree of Freedom of Joints (5/6)

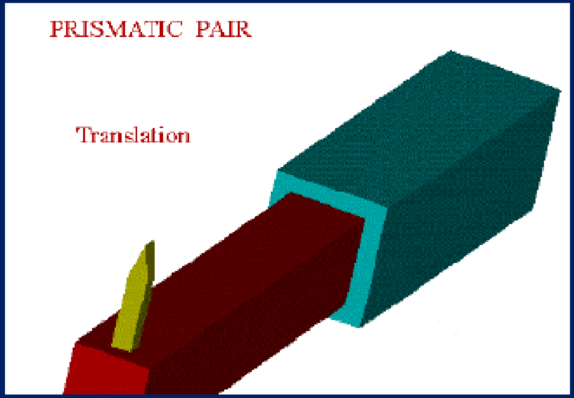






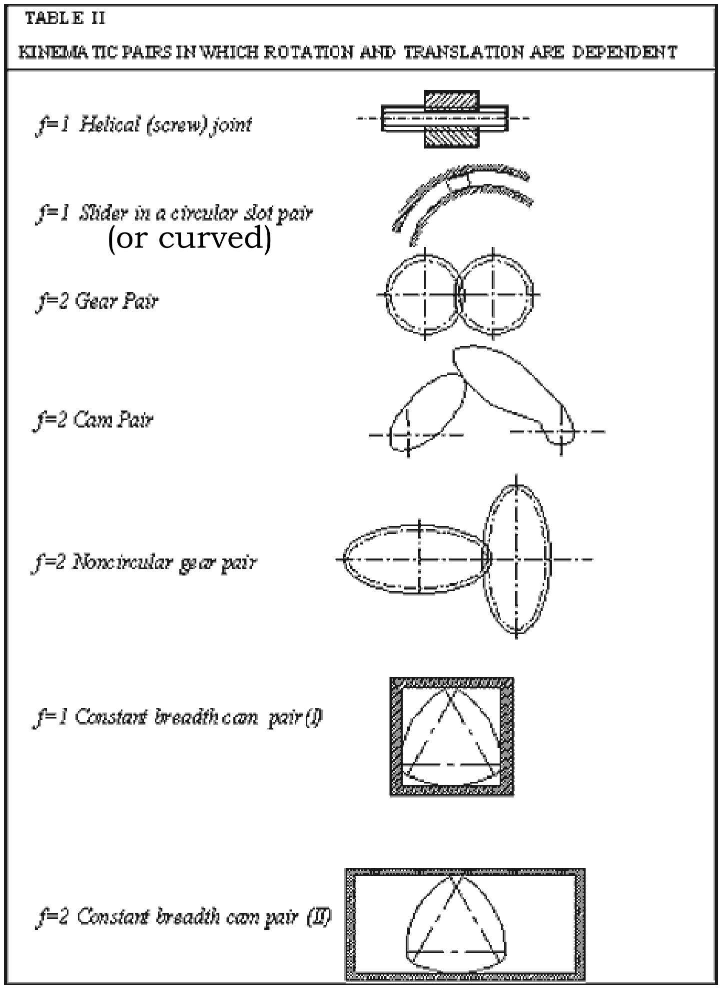
Degree of Freedom of Joints (6/6)

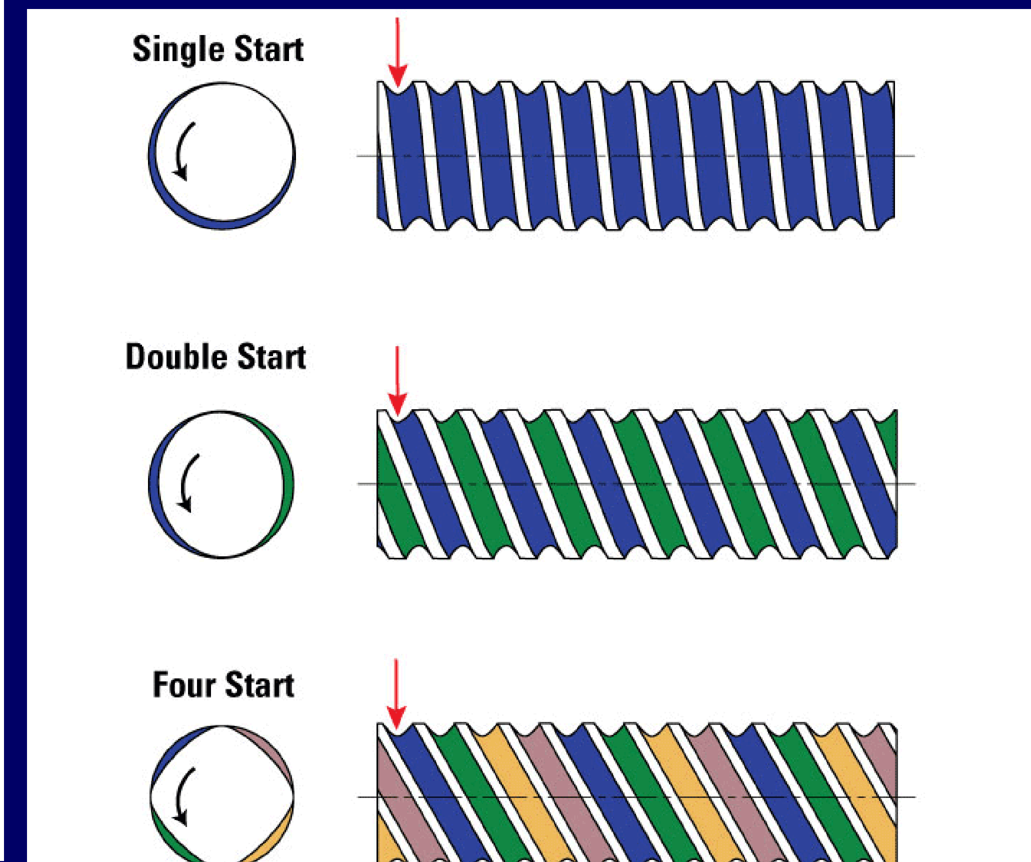




Degree of Freedom of Joints
Dependent Joint Motions

In Table II, some motions are dependent: specifying **one determines the other**.





Textbook pg. 17 (5th), 20 (6th)

Degree of Freedom of Joints
Dependent Joint Motions

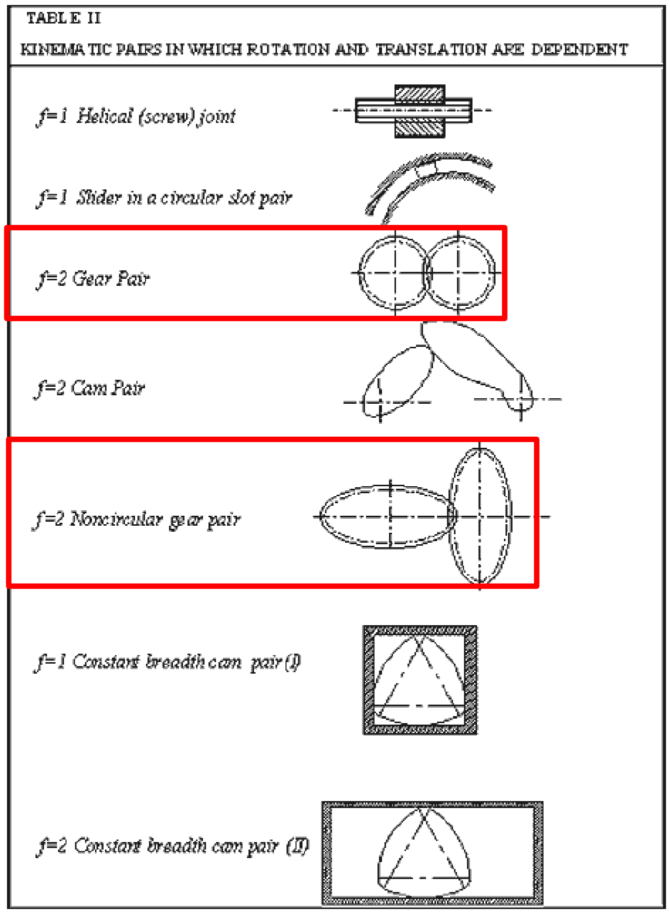


TABLE II

KINEMATIC PAIRS IN WHICH ROTATION AND TRANSLATION ARE DEPENDENT

$f=1$ Helical (screw) joint

$f=1$ Slider in a circular slot pair

$f=2$ Gear Pair

$f=2$ Cam Pair

$f=2$ Noncircular gear pair

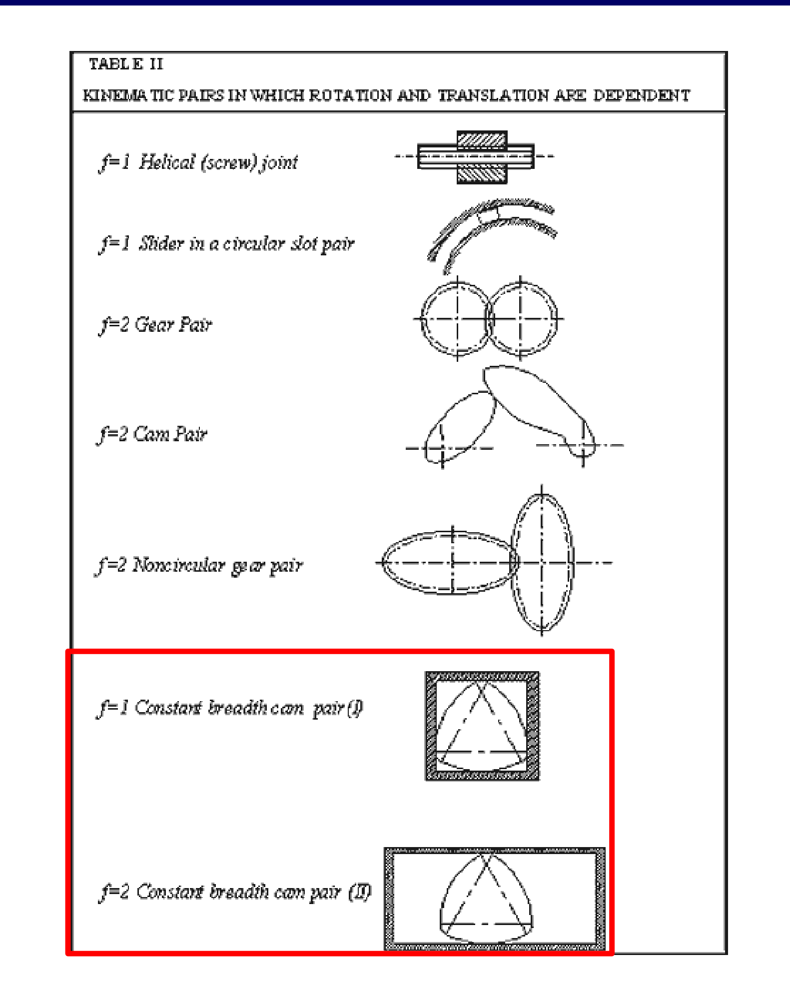
$f=1$ Constant breadth cam pair (I)

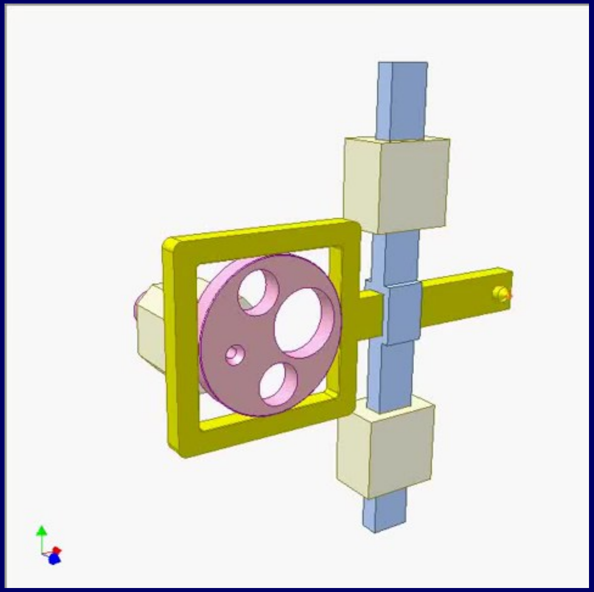
$f=2$ Constant breadth cam pair (II)

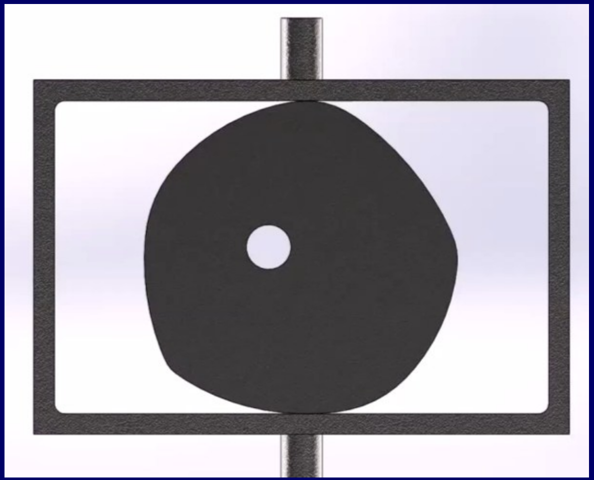
Gear Pair
- Force Closed → f = 1
- Form Closed
  - Irregular → f = 1
  - Regular → f = 2

Textbook pg. 17 (5th), 20 (6th)

Degree of Freedom of Joints
Dependent Joint Motions



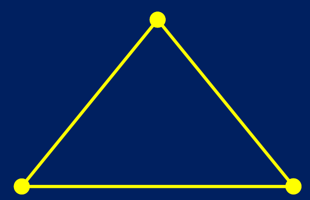




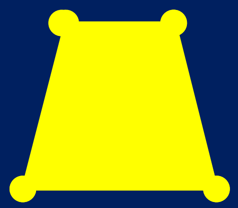
Definitions

Link: A rigid body with *at least two* (*in robotics there are open kinematic chains, therefore one is also allowed*) kinematic elements

Binary Link



Ternary Link



Quaternary Link

Kinematic Chain: Links connected by kinematic elements *but none is fixed*.

Mechanism: When one fixes one of the links of a kinematic chain a mechanism is obtained.

Dimension of a Link: The linear and angular measurements of relative positions of kinematic elements on a link.

Degree of Freedom of Mechanisms

Degree of Freedom (DOF or F) is the total number of ***independent*** parameters that is required to define the configuration of the mechanism (hence position of every point on any link) in a mechanism.

For simple mechanisms, the DOF can often be determined by inspection. For more complicated mechanisms, a systematic approach is required. The **general degree-of-freedom equation** provides such an approach.

Degree of Freedom of Mechanisms

Inspection: Four-bar mechanism (*British call it three-bar mechanism because there are three moving bars but Germans and Americans call it four-bar because it is obtained from a four-bar kinematic chain by fixing one of the bars*. We will use German-American nomenclature).

The lengths of links are known, let's say:

$|A_0A| = a_2 = 3\ cm$

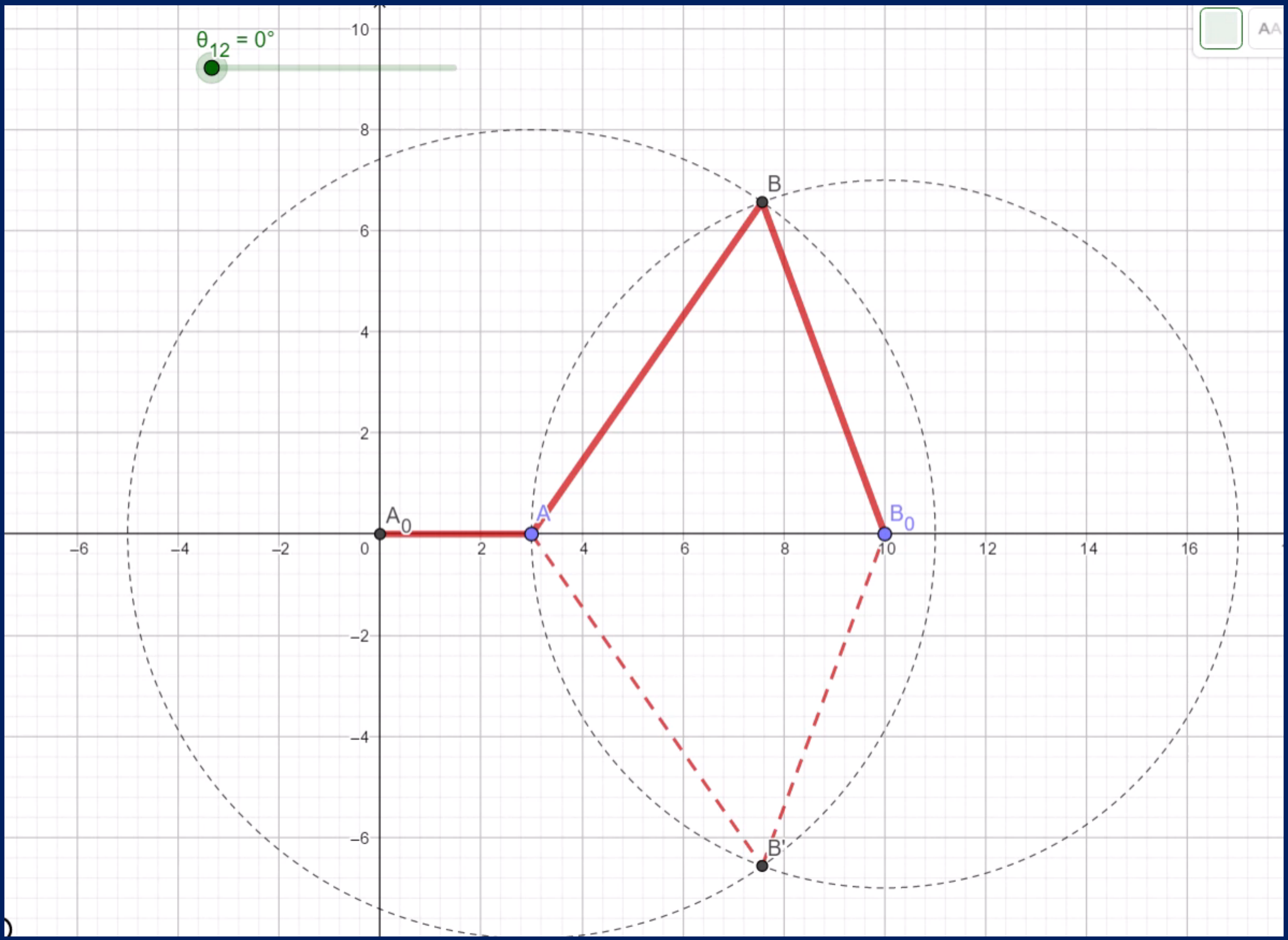
$|AB| = a_3 = 8\ cm$

$|B_0B| = a_4 = 7\ cm$

$|A_0B_0| = a_1 = 10\ cm$

Degree of Freedom of Mechanisms

Inspection: Four-bar mechanism, say $0° \leq \theta_{12} \leq 360°$



Software: Geogebra Classic 6, https://www.geogebra.org/

Degree of Freedom of Mechanisms

Inspection: Five-bar mechanism

The lengths of links are known, let's say :

$|A_0A| = a_2 = 4\ cm$

$|AB| = a_3 = 3\ cm$

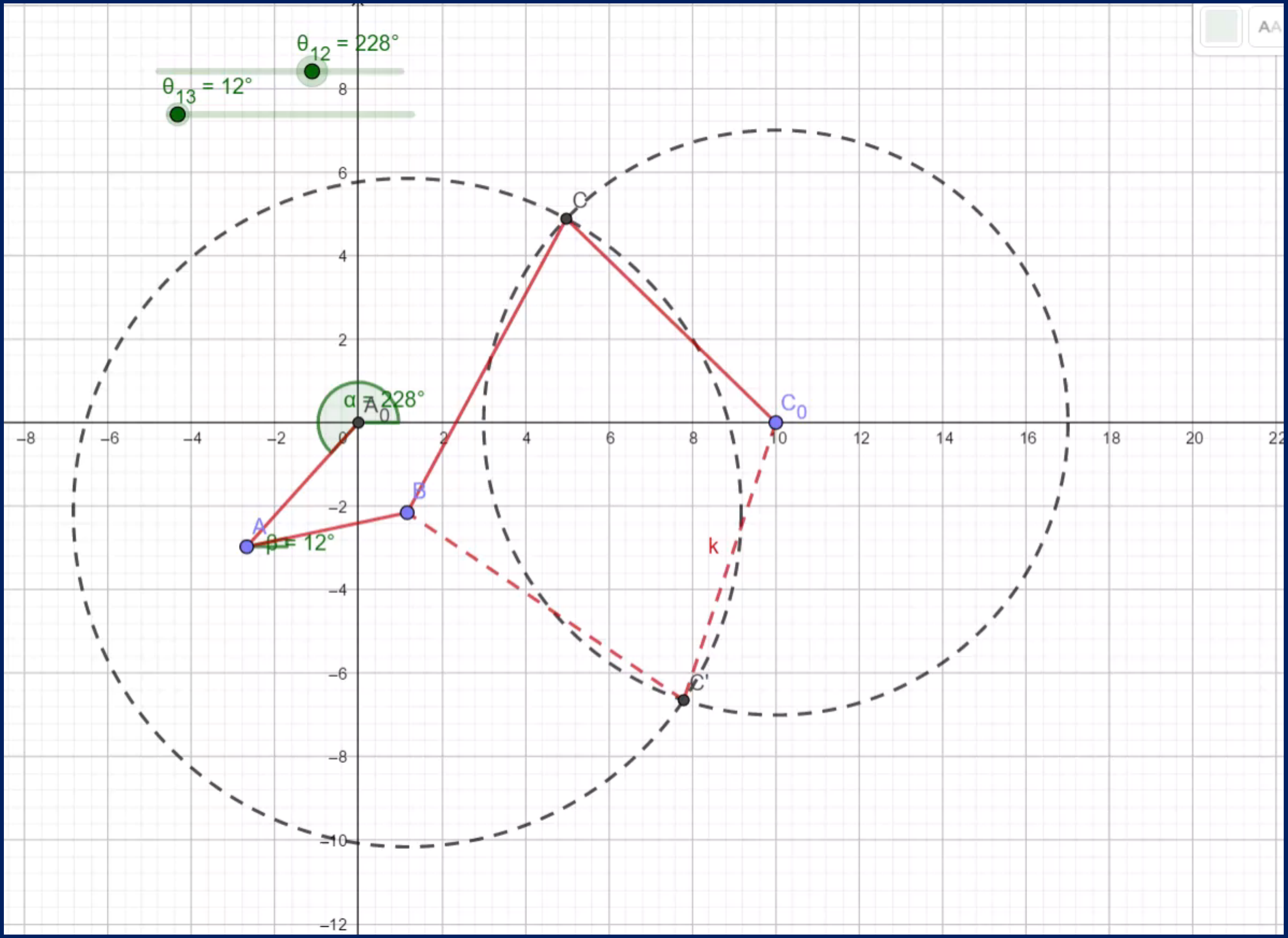
$|BC| = a_3 = 8\ cm$

$|C_0C| = a_4 = 7\ cm$

$|A_0C_0| = a_1 = 10\ cm$

Degree of Freedom of Mechanisms

Inspection: Five-bar mechanism



Degree of Freedom of Mechanisms

Systematic determination of degree of freedom (1/2):

- In planar motion $\ell - 1$ moving links with no joints has $F = 3(\ell - 1)$
- k_1 joints (revolute & prismatic) constrain 2 freedoms
- k_2 joints (cylinder in slot) constrain 1 freedom

$$F = 3(\ell - 1) - 2k_1 - k_2$$

Chebyshev–Grübler criterion

$$F = 6(\ell - 1) - 5k_1 - 4k_2 - 3k_3 - 2k_4 - k_5$$

Chebyshev (1869) – Grübler (1884) – Kutzbach (1929) formula!

Systematic determination of degree of freedom (2/2):

Replace 3 and 6 in the Chebyshev – Grübler – Kutzbach formula with λ, degree of freedom of a free rigid body in the space (or *degree of freedom of the space the mechanism is in*),

Constraints imposed by ith joint with joint degree of freedom f_i is $\lambda - f_i$

Constraints imposed by all *j* joints in the mechanism:

$$\sum_{i=1}^{j}(\lambda - f_i) = \lambda j - \sum_{i=1}^{j} f_i$$

$$F = \lambda(\ell - 1) - \left[\lambda j - \sum_{i=1}^{j} f_i\right]$$

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

Degree of Freedom of Mechanisms

λ: Degree of freedom of the unconstrained rigid bodies in the mechanism space

ℓ: Number of links of the mechanism (***including fixed link***)

j: Number of joints of the mechanism (*ternary joints!*)

f_i : Degree of freedom of i^th^ joint

F : Degree of freedom of the mechanism

$$\boxed{F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i}$$

General Degree of Freedom Equation

Degree of Freedom of Mechanisms

$$\boxed{F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i}$$

Remember there are exceptions!

$F > 0$ mechanism requires F actuations for kinematically deterministic motion

$F = 0$ structure (immobile) *unless has special dimensions*

$F < 0$ over-constraint (number of *redundant* supports is $|F|$) and immobile *unless has special dimensions* (*also forces cannot be determined unless equations of equilibrium/motion are complemented with $|F|$ number of equations relating deformations, **indeterminate***).

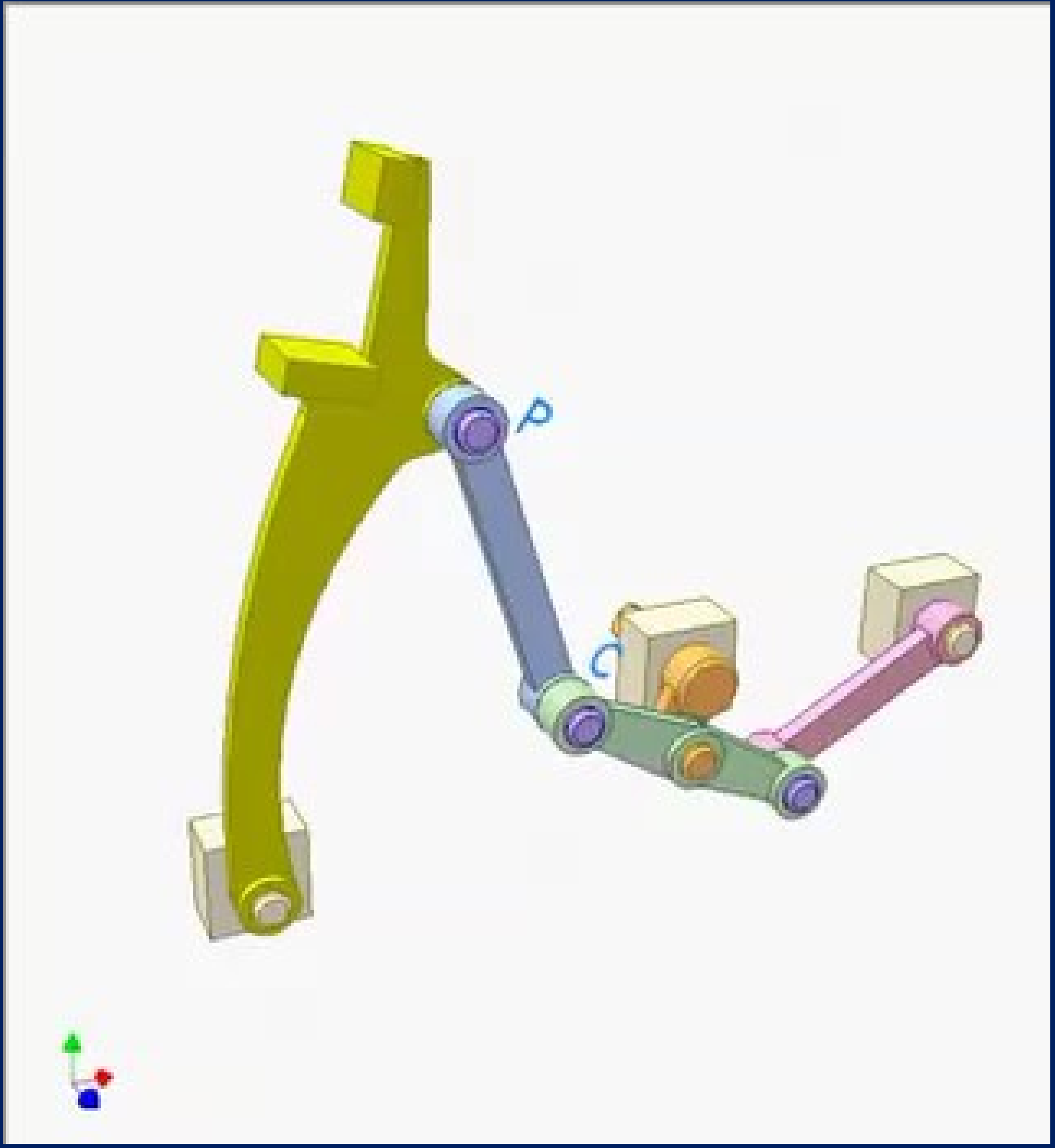
Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

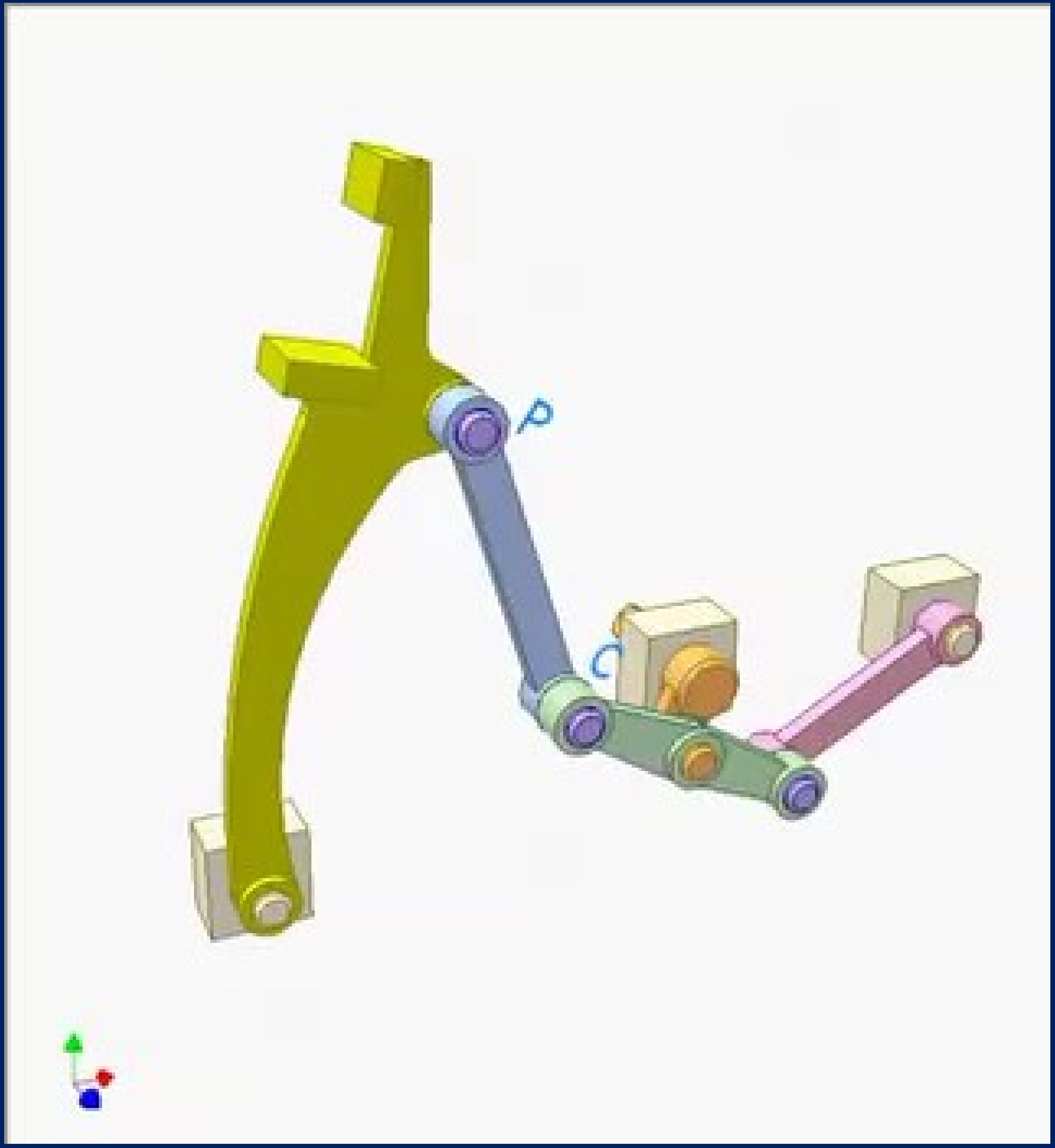
Restrictions of the General DOF Equation:

- All links must be in the same motion space with equal λ (i.e. $\lambda = const$ for all links!),
- The constraints imposed by the joints should be independent of each other.

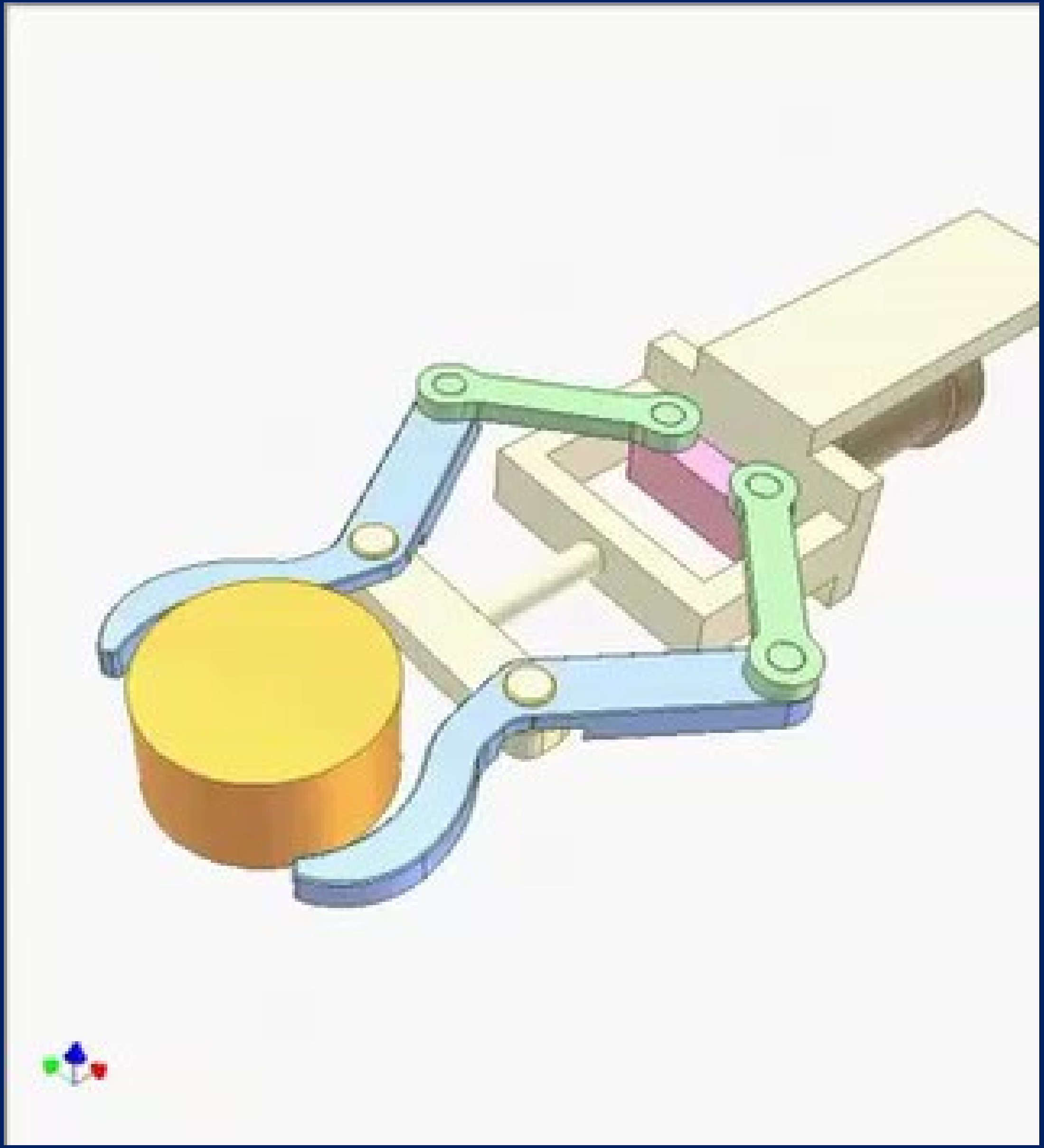
Degree of Freedom of Mechanisms



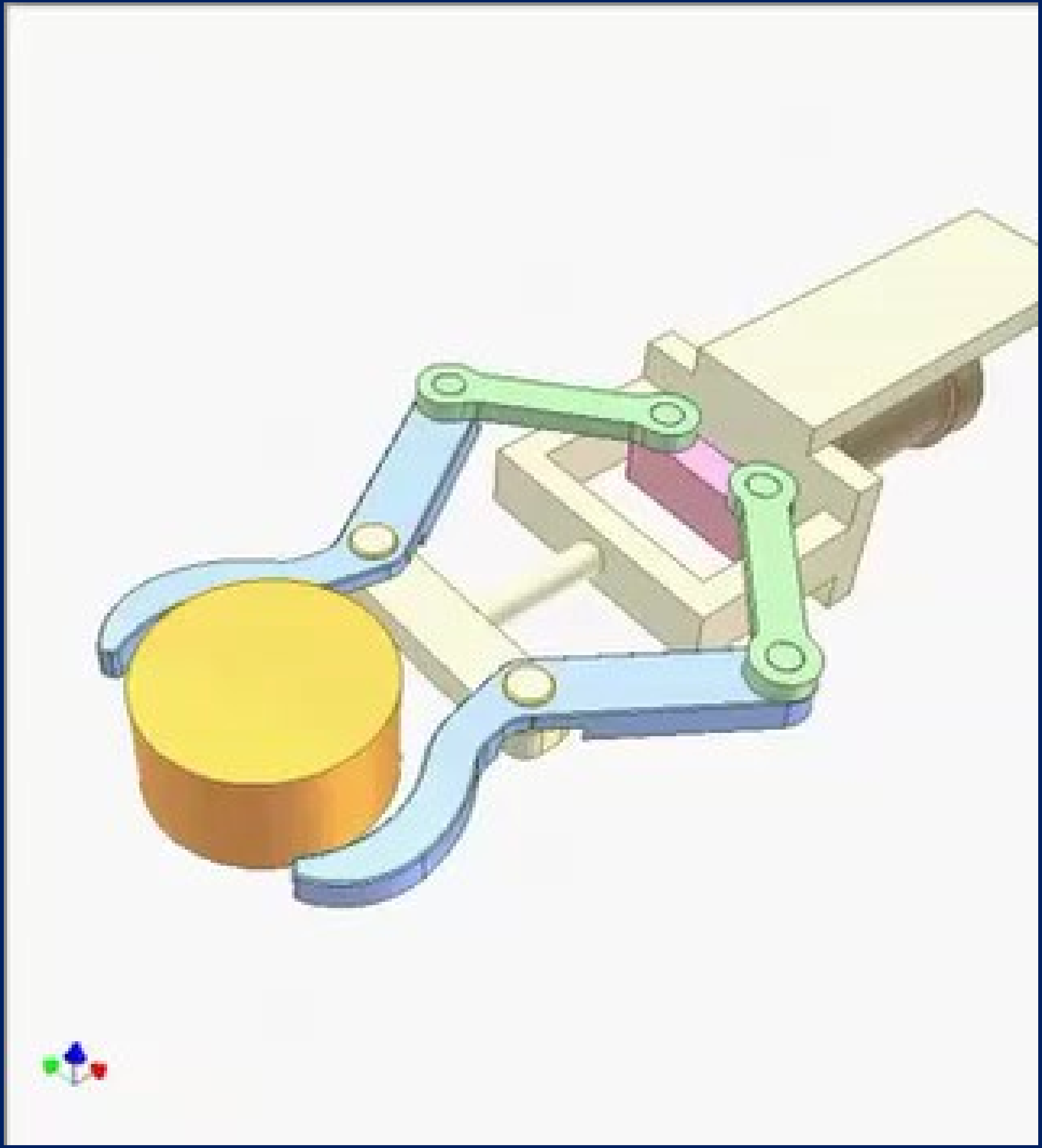
Degree of Freedom of Mechanisms



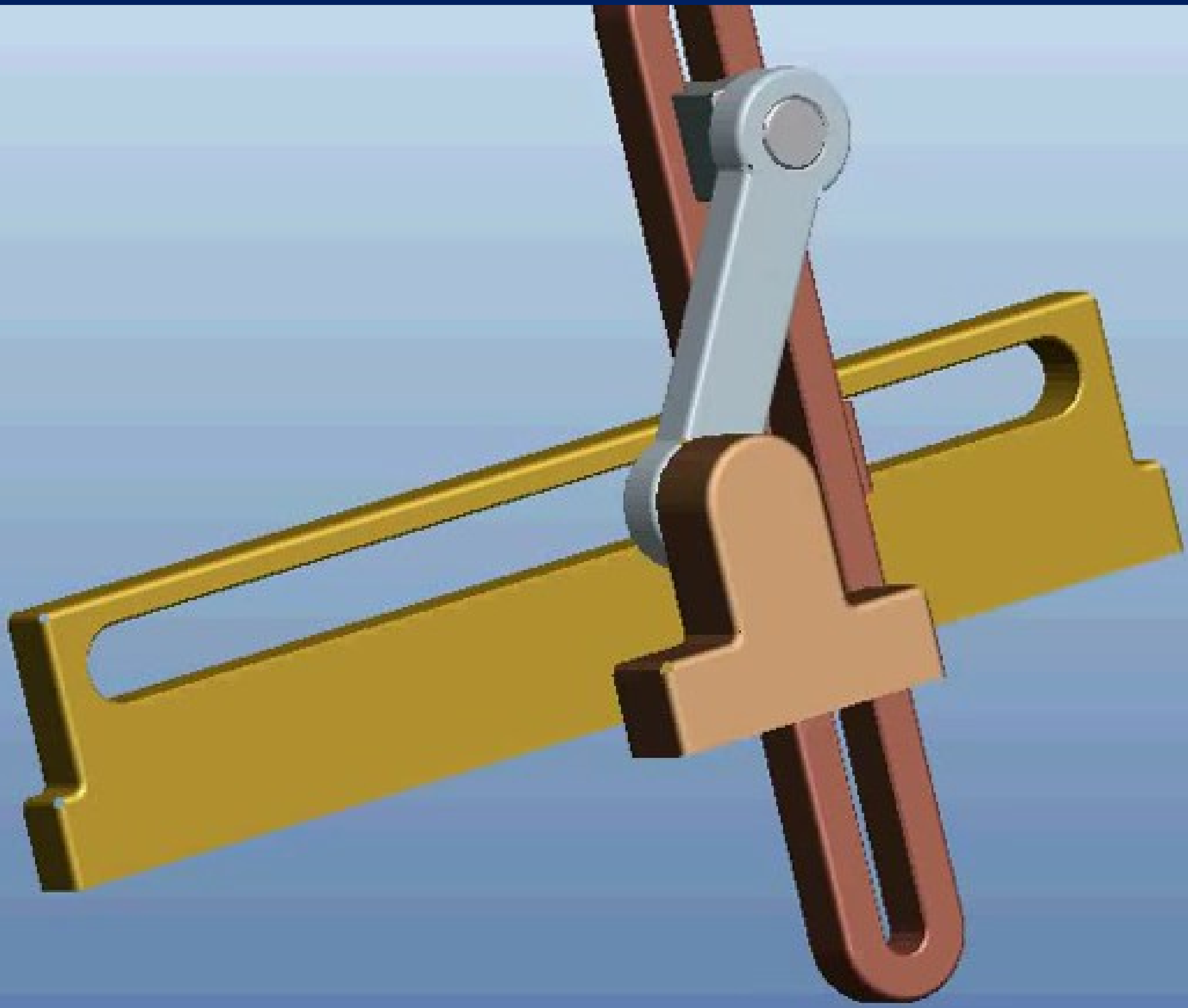
Degree of Freedom of Mechanisms



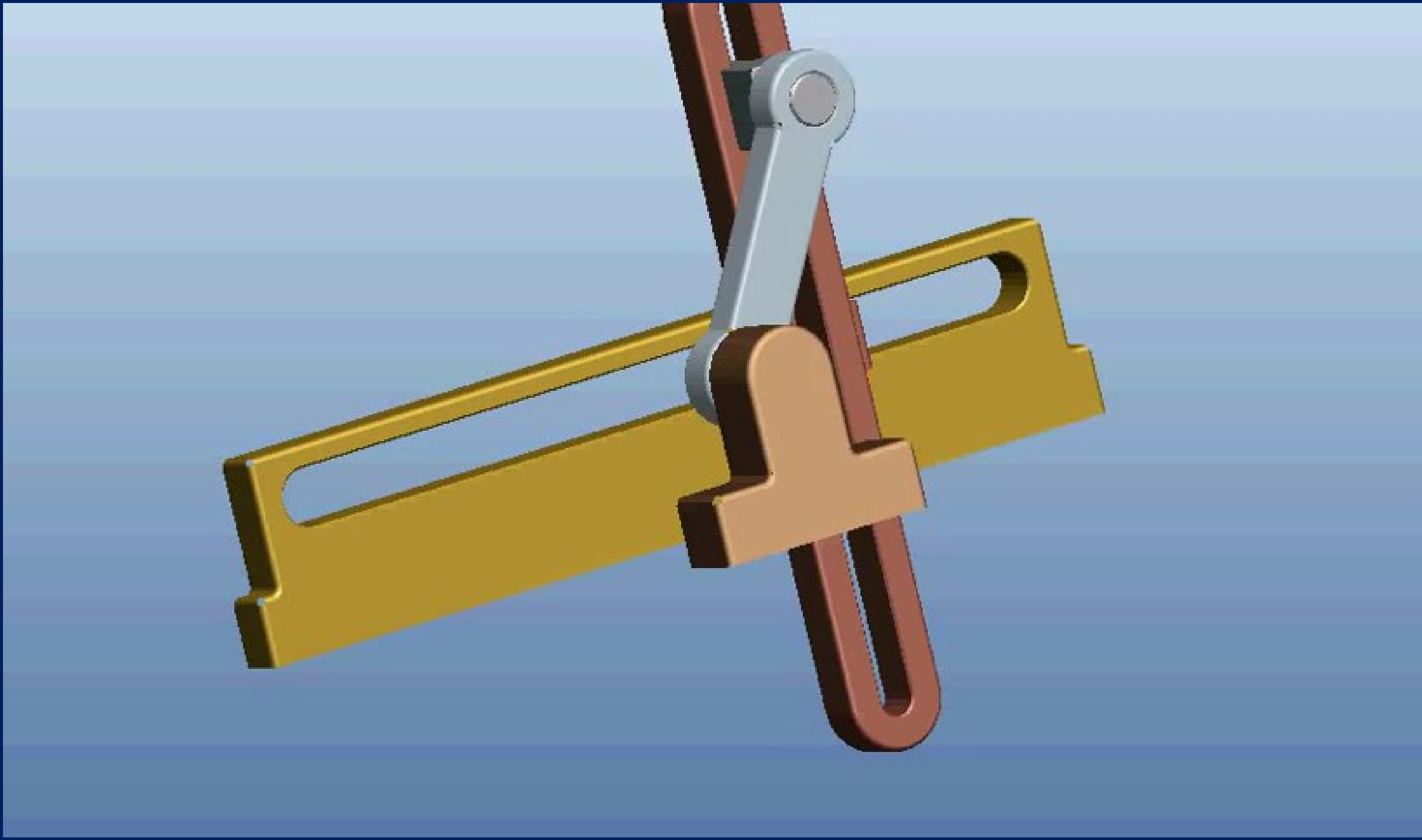
Degree of Freedom of Mechanisms



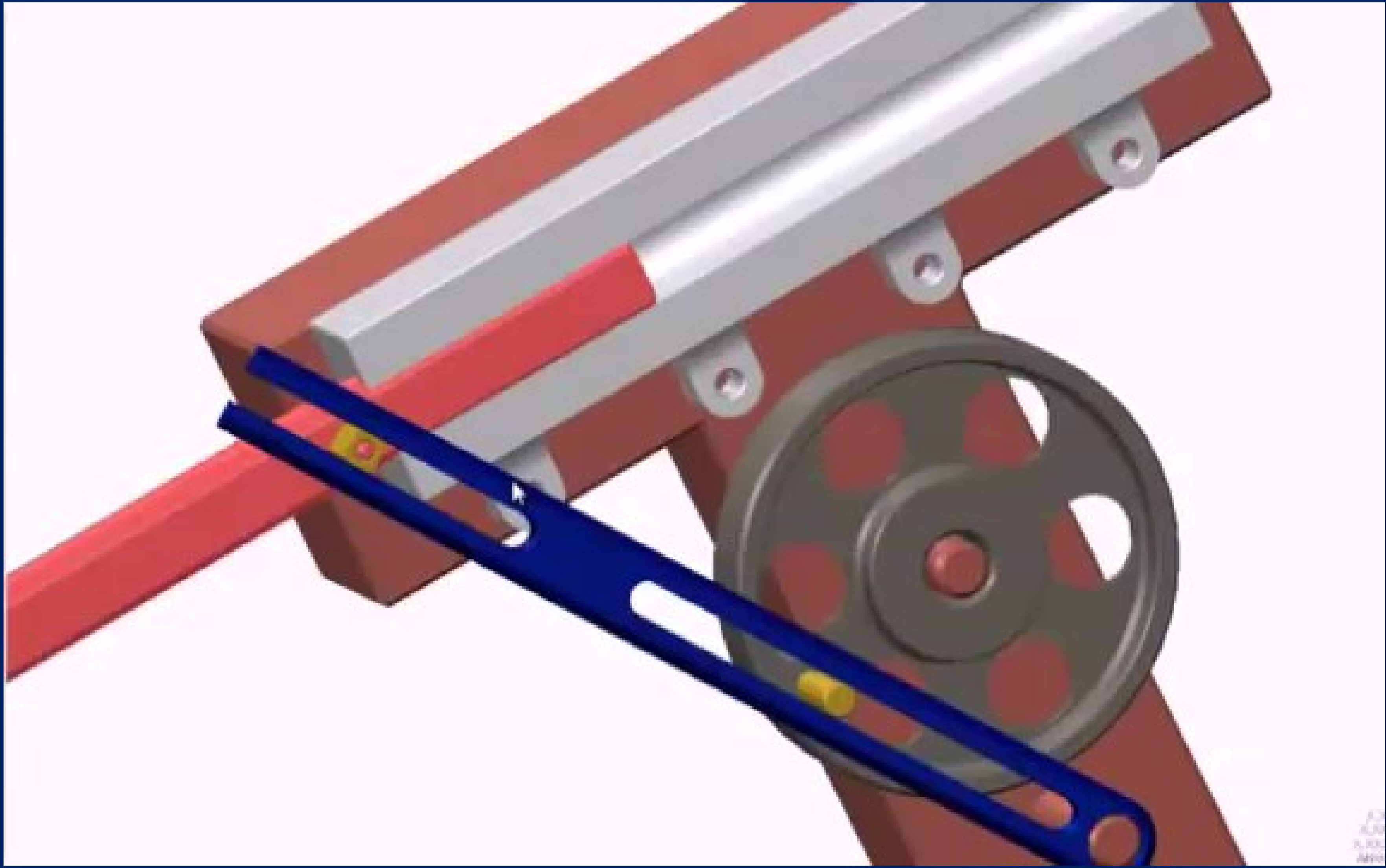
Degree of Freedom of Mechanisms



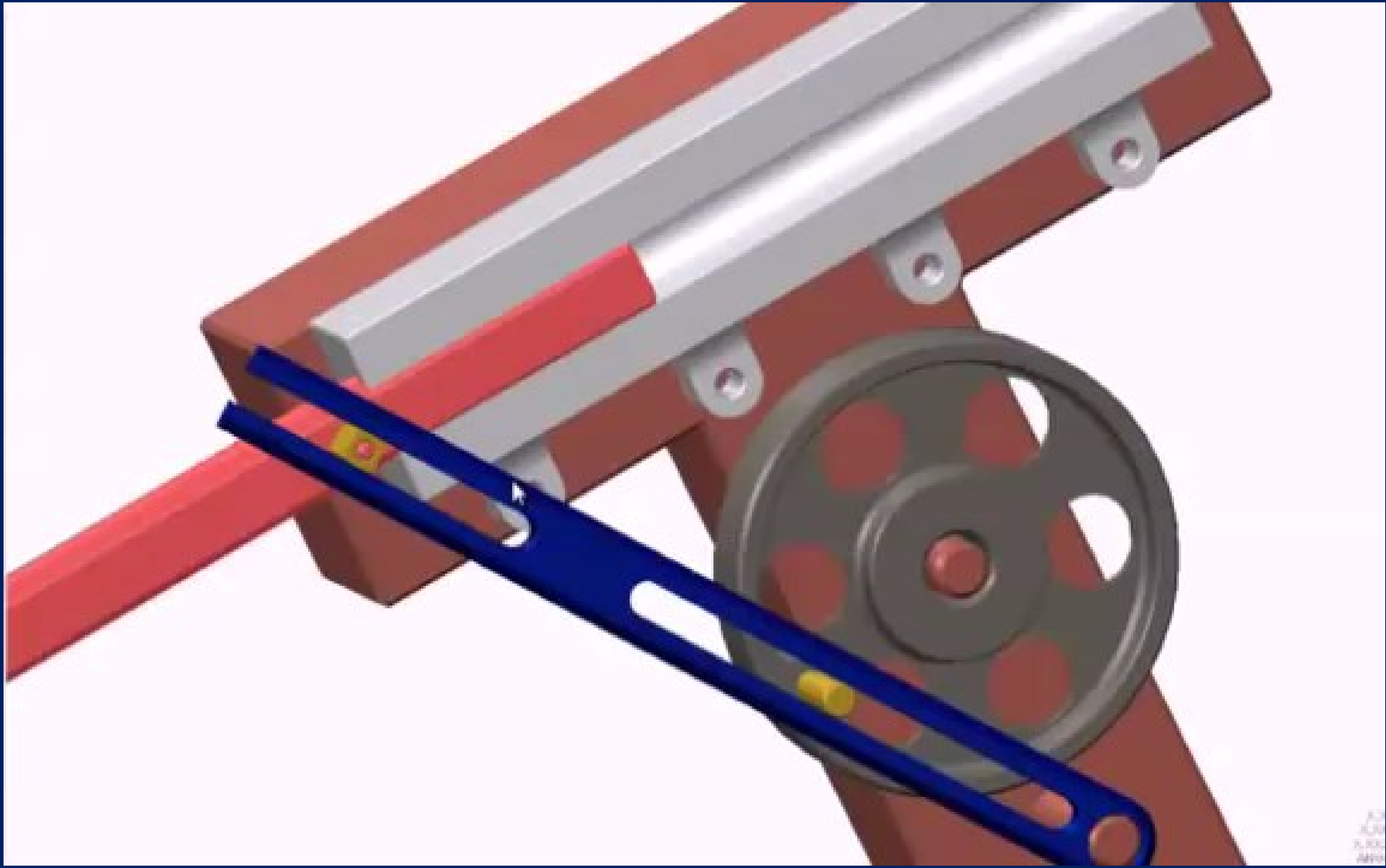
Degree of Freedom of Mechanisms



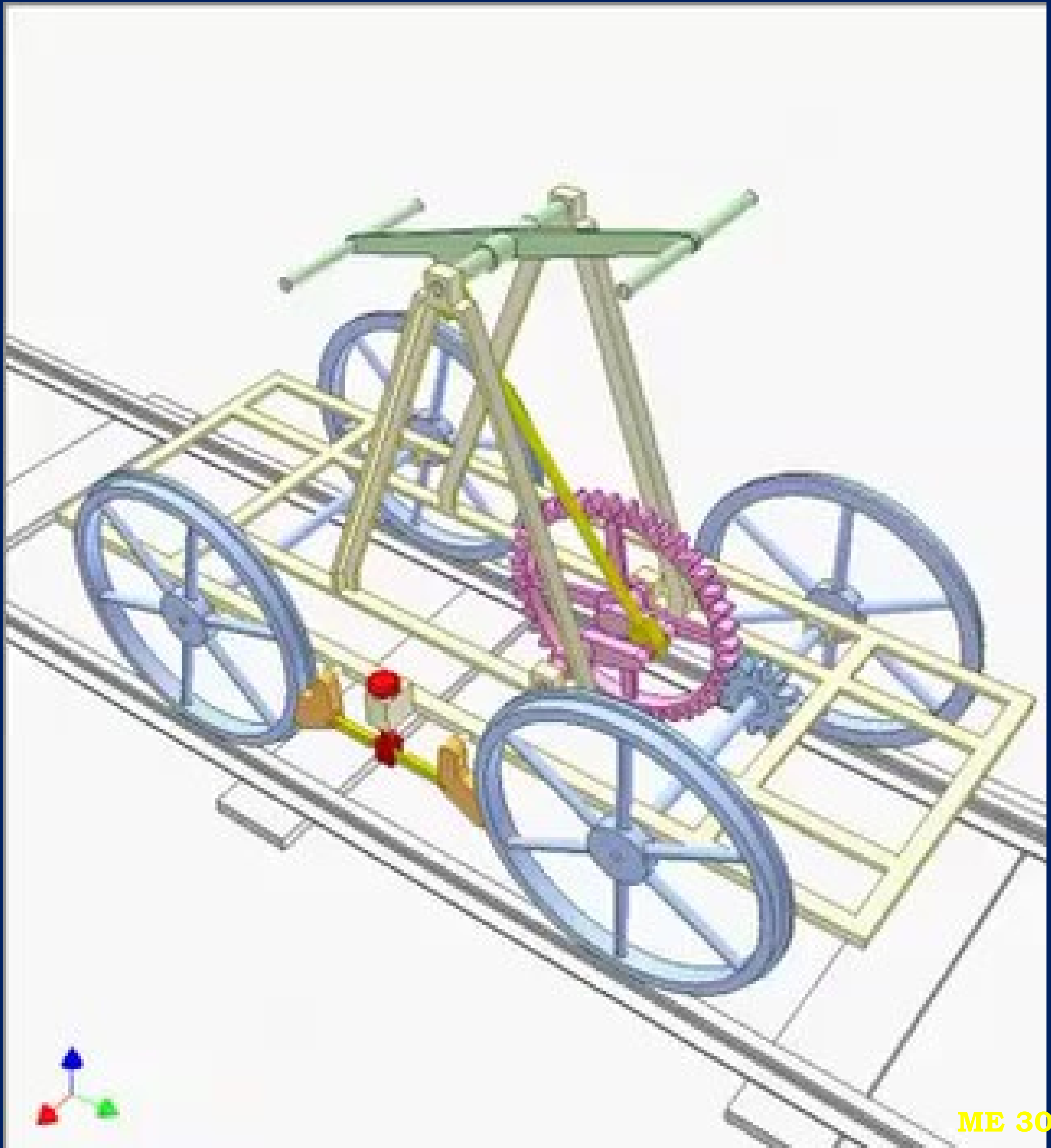
Degree of Freedom of Mechanisms



Degree of Freedom of Mechanisms

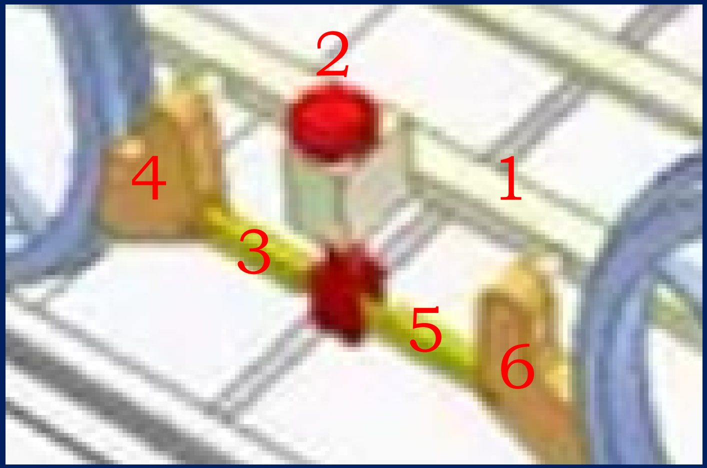


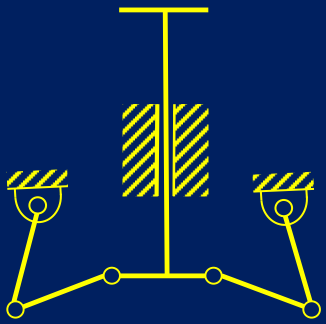
Degree of Freedom of Mechanisms



Degree of Freedom of Mechanisms

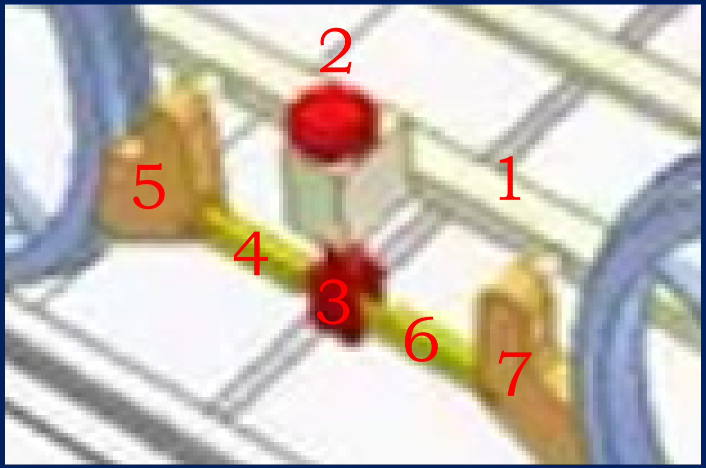
The Brakes (Approach 1):

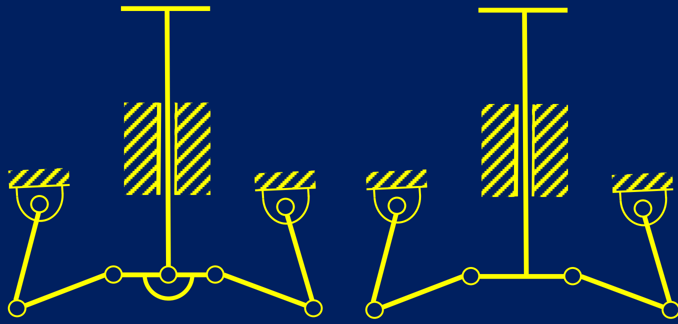




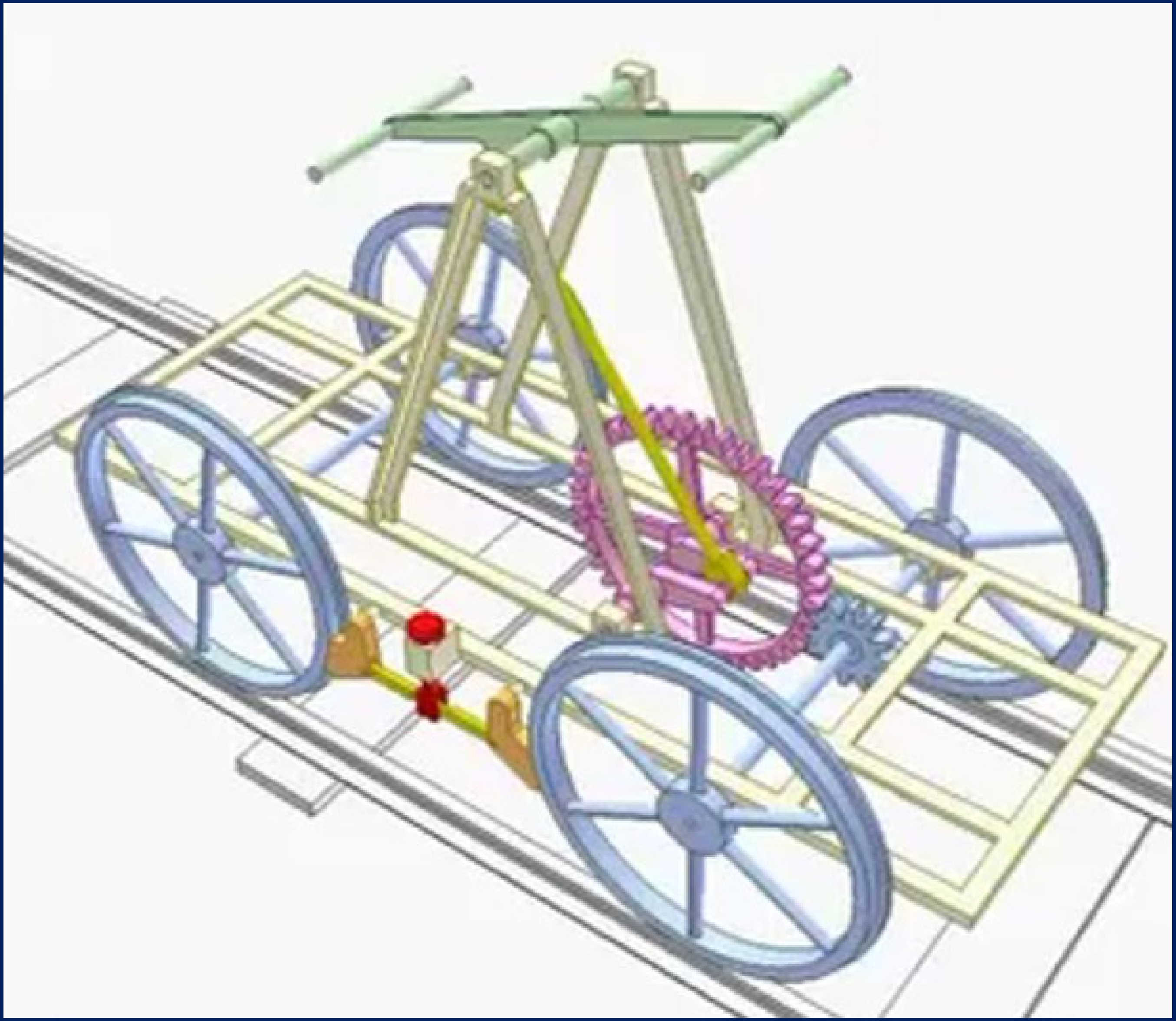
Degree of Freedom of Mechanisms

The Brakes (Approach 2):

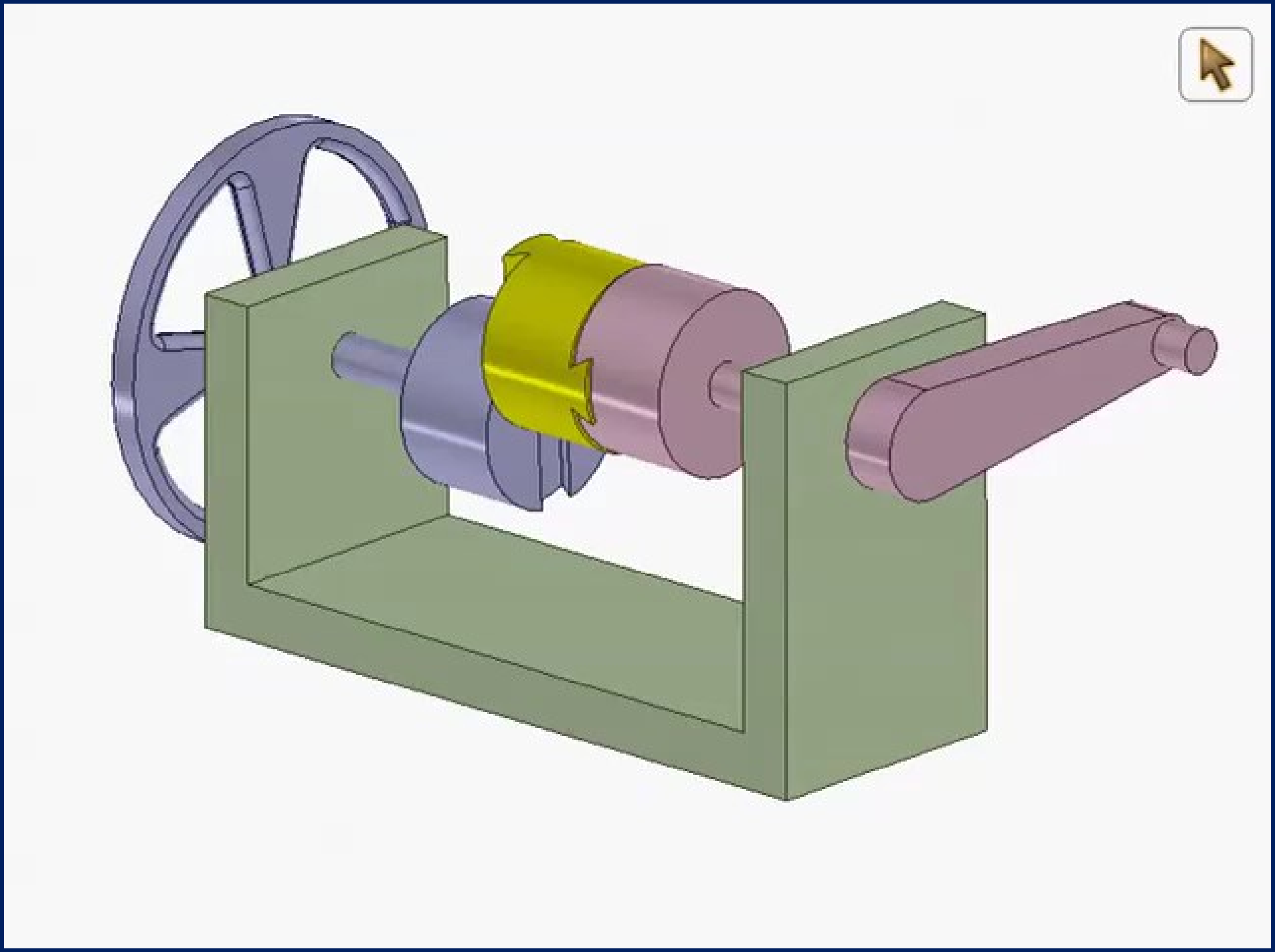




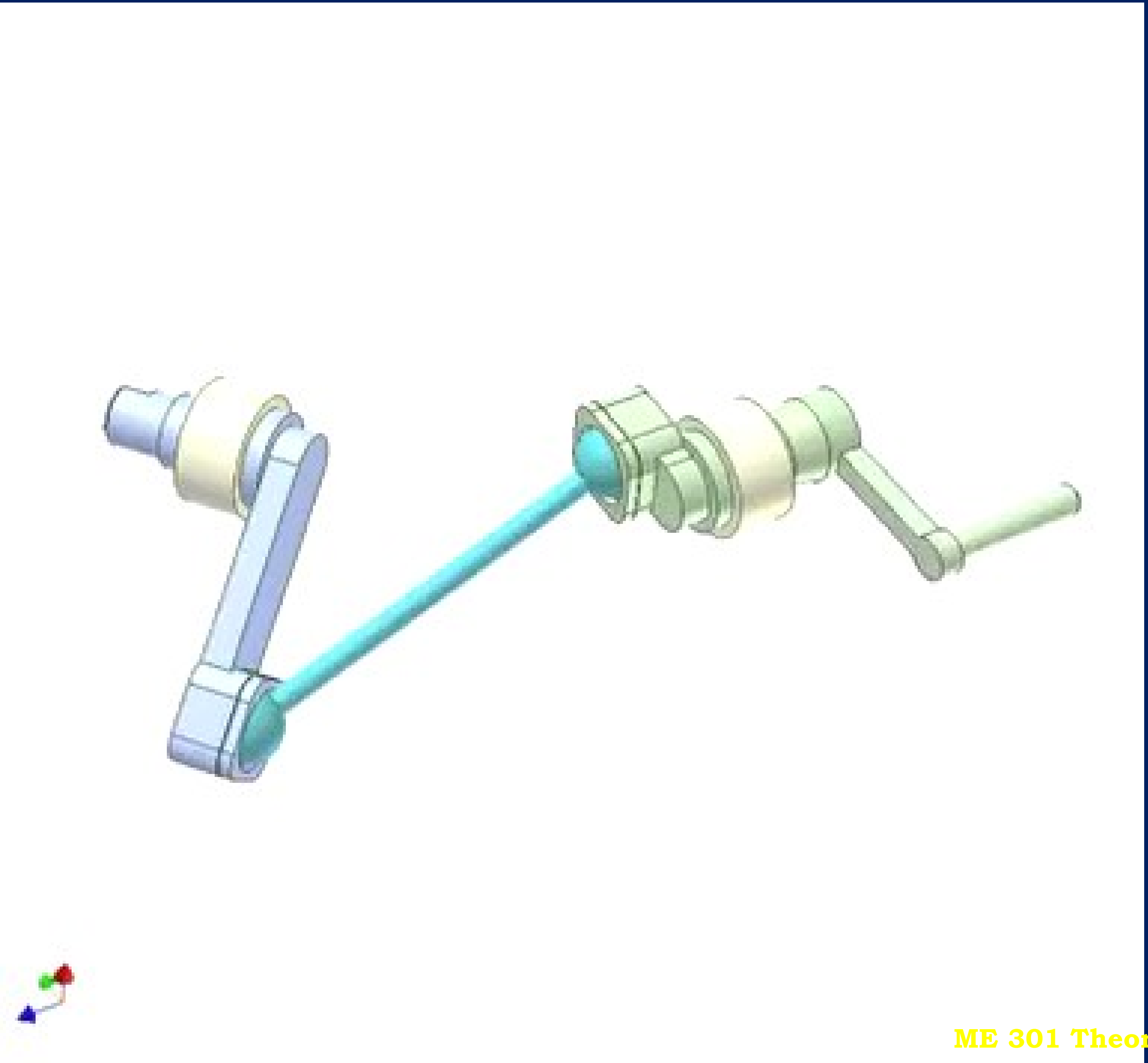
Degree of Freedom of Mechanisms



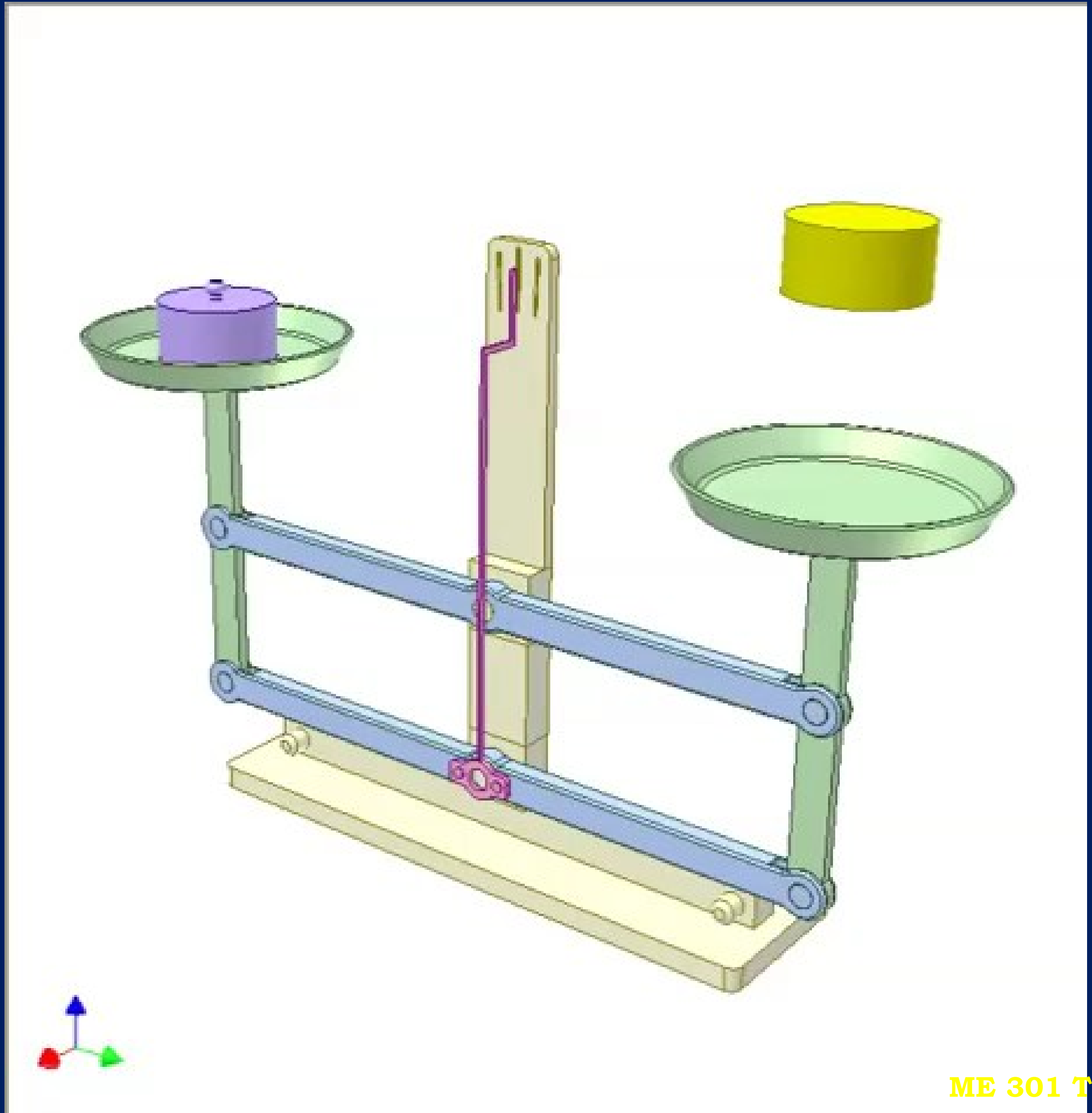
Degree of Freedom of Mechanisms



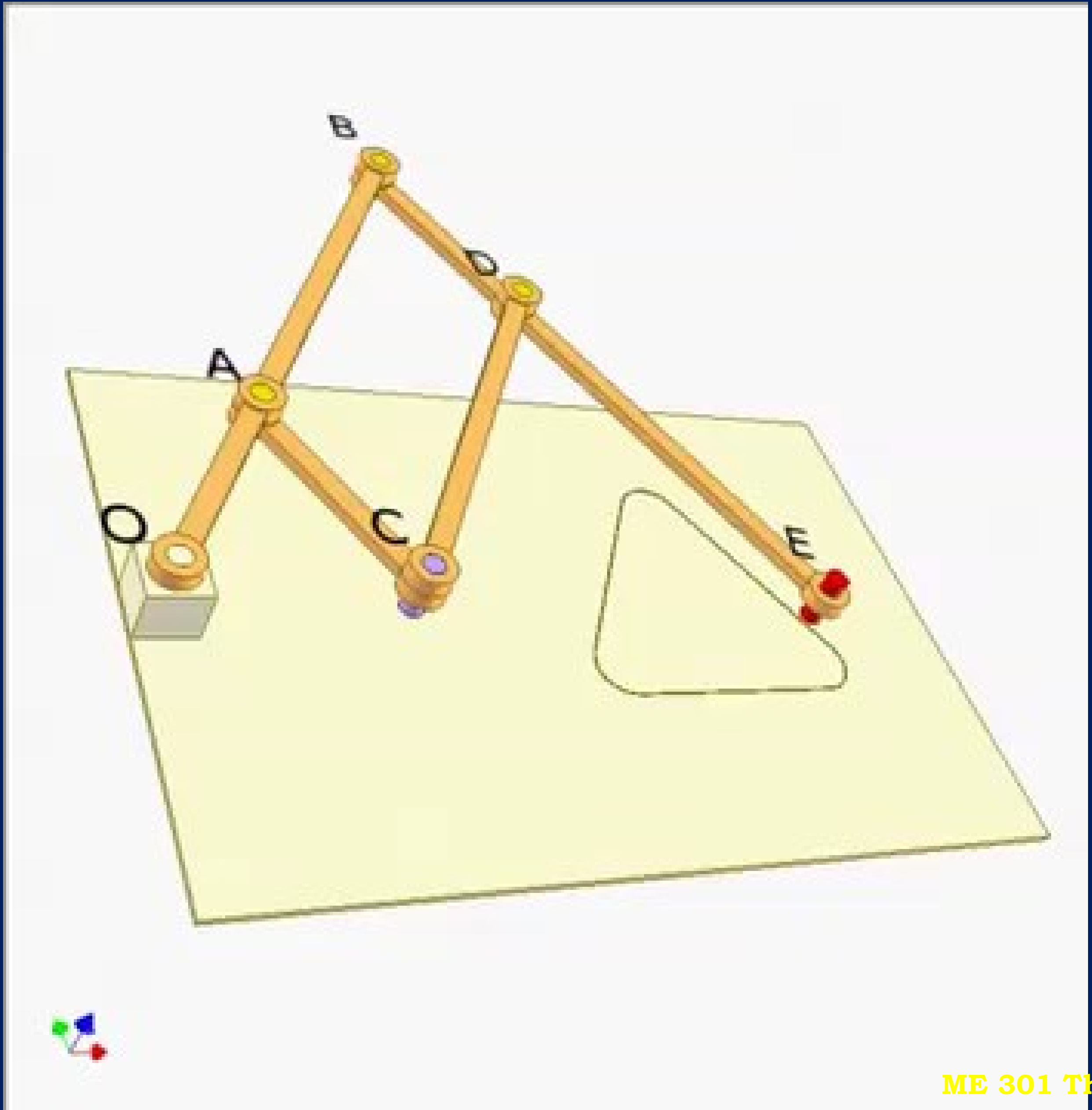
Degree of Freedom of Mechanisms



Degree of Freedom of Mechanisms

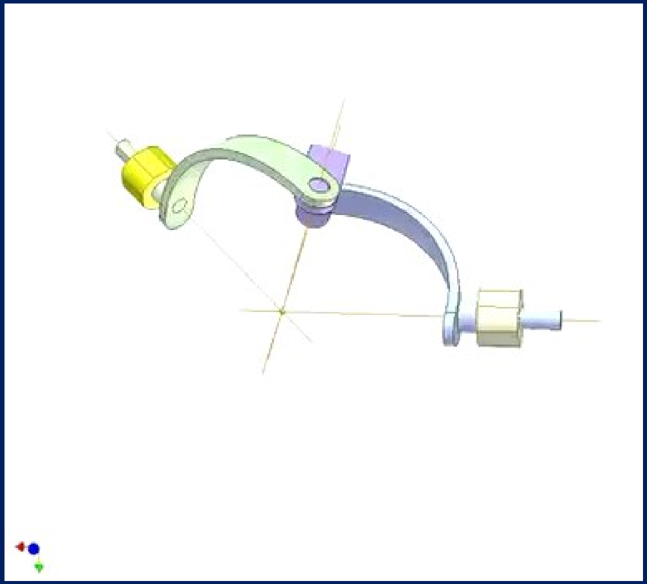


Degree of Freedom of Mechanisms



Special Spaces

Spherical Space $\lambda = 3$



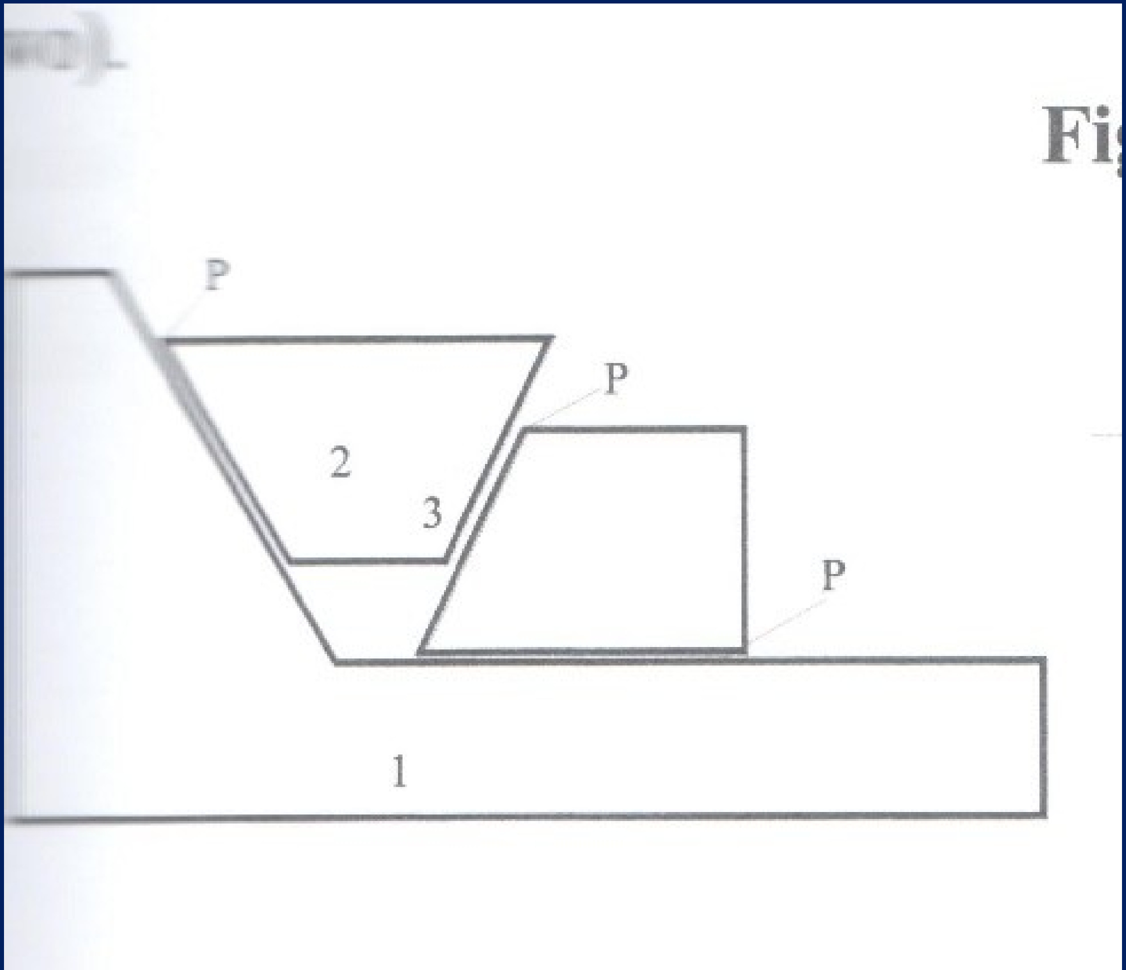
Spherical Four-Bar

Cardan (Universal) "*Joint*"s

Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Special Spaces

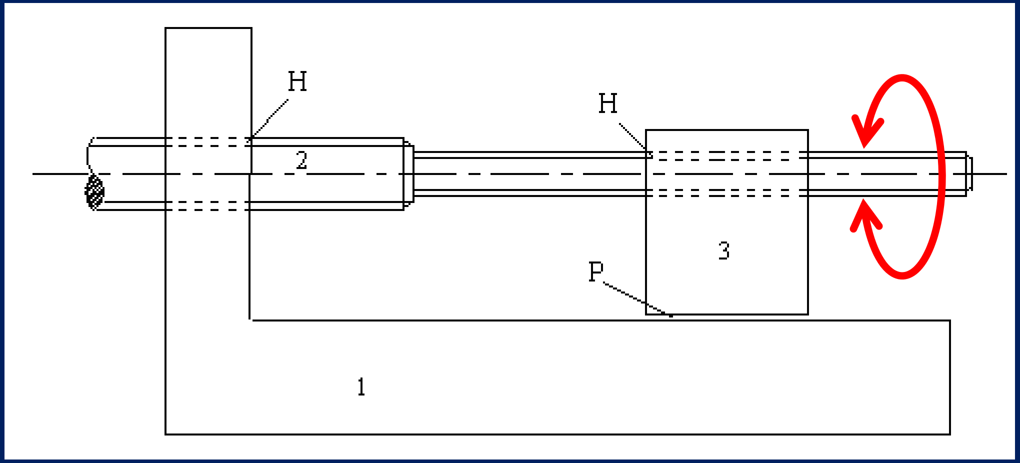
Non-rotational Planar (Wedge) Space $\lambda = 2$



Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132 and https://solomon.desertcart.com/products/50863575-multi-ware-car-scissor-lift-car-jack-1-ton

Special Spaces

Mechanisms with Screws



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Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

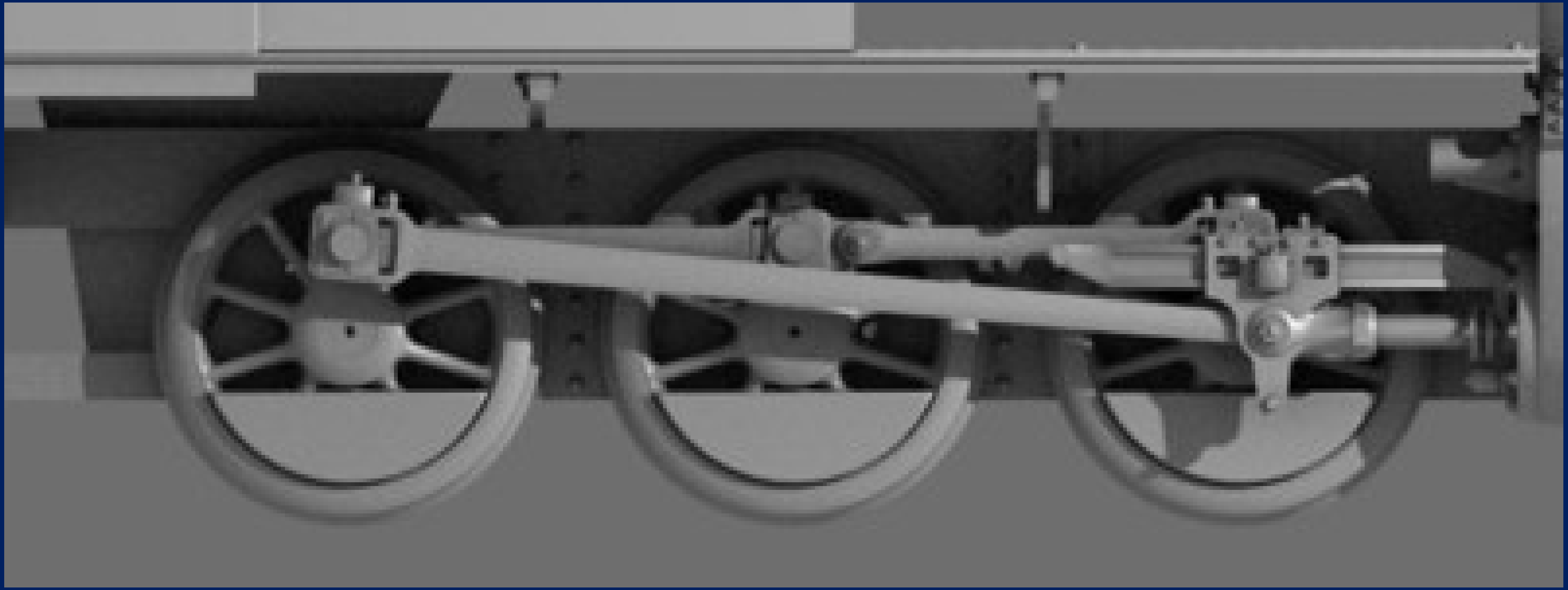
The general degree of freedom equation cares only the **topological characteristics** of the mechanism (e.g. number of links, number and type of joints *but* not the link dimensions) and the degree of freedom of the space the bodies are in.

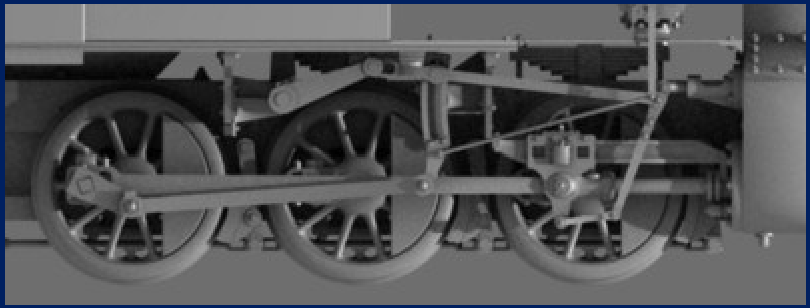
Therefore a mechanism in *instantaneous* or *permanent critical form* may move due to special dimensions although the general degree of freedom equation predicts it is immobile (which is true for any arbitrary dimensions).

Degree of Freedom of Mechanisms

Exceptions: Mechanism in Permanent Critical Form

Steam Locomotive power distribution among axles





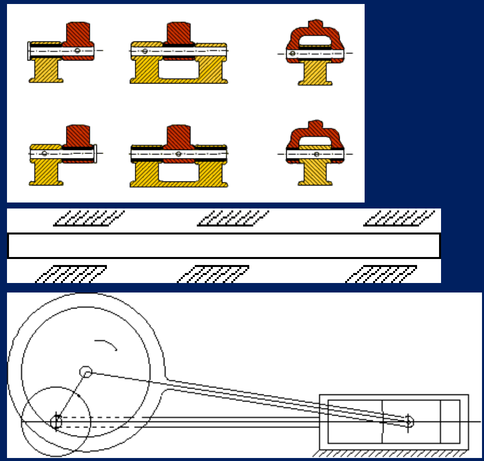
Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

- A joint should permit relative motion of a body with respect to other.

  If there is no relative motion permitted, it is *not* a joint **kinematically** and those parts of the body should be accounted as a single link.

- What identifies the joint type is the relative motion of one body relative to the other not the constructional details.



Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Degree of Freedom of Mechanisms

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

- The degree of freedom of the space of all the links in the mechanism (i.e. λ) should be the same. If not the mechanism should be separated.

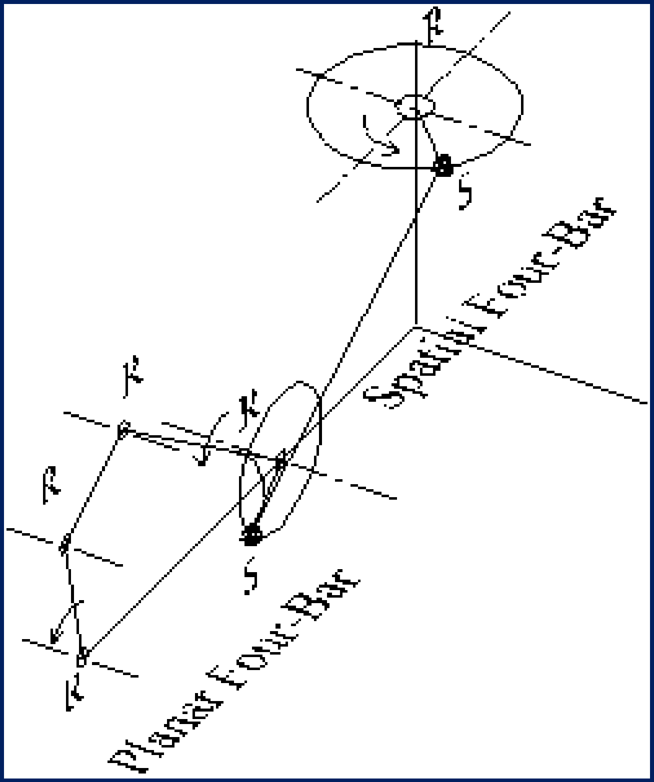


Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Degree of Freedom of Mechanisms with Gear Pairs

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

- The degree of freedom of *ordinary* gear pair, $f_i = 1$ because it is rolling without slipping.
- The *regular* gear pair (GP*) is in **permanent critical form** because of special dimensional requirements:
  - The gears are centered at certain special points (generally centers of circles),
  - The distances between these joints are the sum (or difference if internal mesh) of the radii of the gears.

  Therefore GP* is a *workaround* to obtain correct degree of freedom for the gears **in permanent critical form**.

Degree of Freedom of Mechanisms with Gear Pairs

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

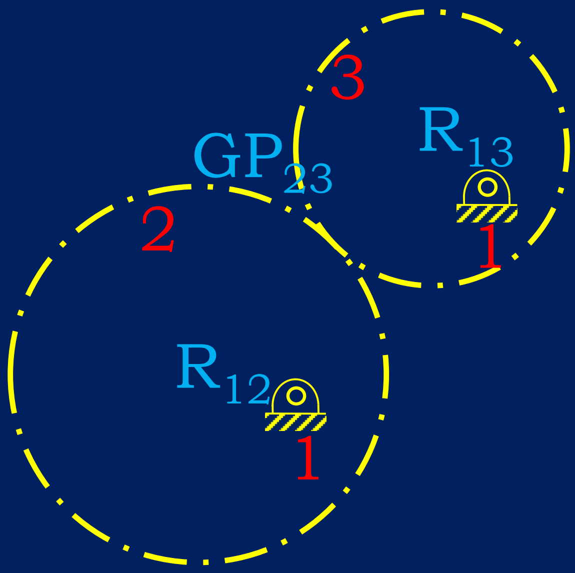
Therefore GP* is a workaround to obtain correct degree of freedom for the gears in permanent critical form.

λ = 3 (co-planar)

ℓ = 3

j = 3

$\sum_{i=1}^{3} f_i = 3$ (2R+GP)



$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$$F = 3(3 - 3 - 1) + 3$$

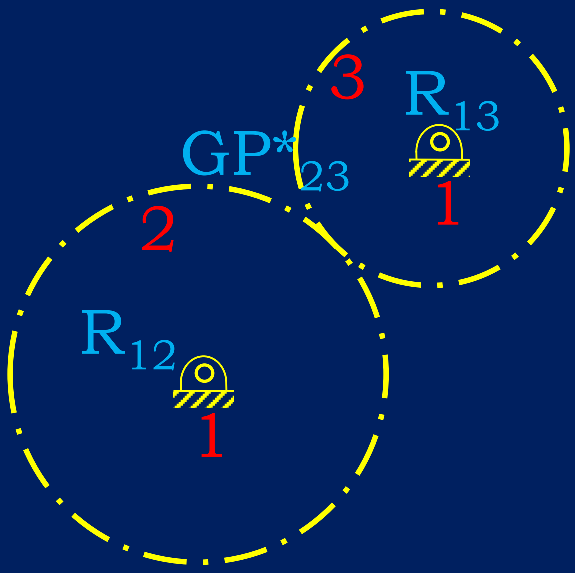
$$F = 0$$

Will not work unless R_{12} and R_{13} are centers of gears **AND** $|R_{12}R_{13}| = r_2 + r_3$

Degree of Freedom of Mechanisms with Gear Pairs

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

Therefore GP* is a workaround to obtain correct degree of freedom for the gears in permanent critical form.



$\lambda = 3$ (co-planar)

$\ell = 3$

$j = 3$

$\sum_{i=1}^{3} f_i = 4$ (2R+GP*)

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$$F = 3(3 - 3 - 1) + 4$$

$$F = 1$$

Permanent critical form because R_{12} and R_{13} are centers of gears **AND** $|R_{12}R_{13}| = r_2 + r_3$

Degree of Freedom of Mechanisms with Gear Pairs

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

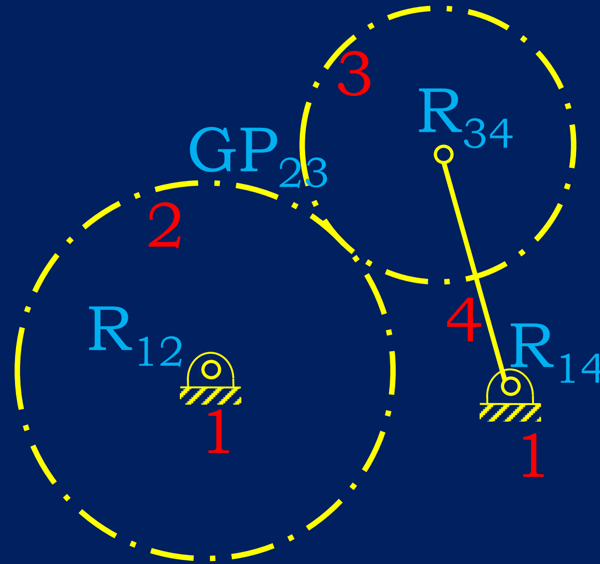
Force-closed gear pair:

$\lambda = 3$ (co-planar)

$\ell = 4$

$j = 4$

$\sum_{i=1}^{4} f_i = 4$ (3R+GP)



$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(4 - 4 - 1) + 4$

$F = 1$

Degree of Freedom of Mechanisms with Gear Pairs

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

Force-closed gear pair:

$\lambda = 3$ (co-planar)

$\ell = 4$

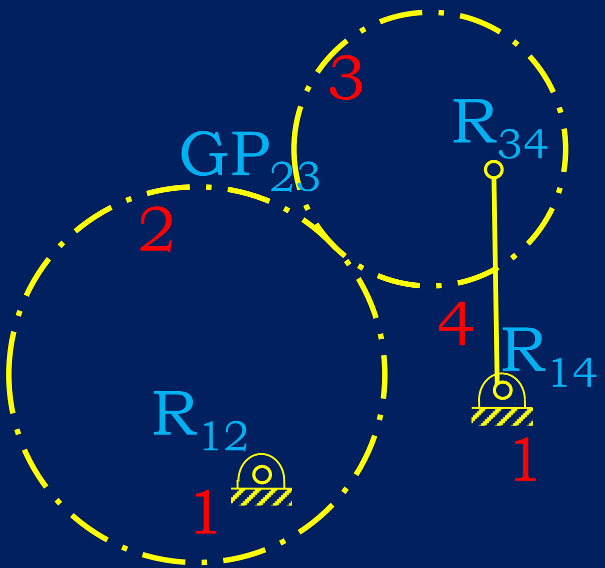
j = 4

$\sum_{i=1}^{4} f_i = 4$ (3R+GP)

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

$F = 3(4 - 4 - 1) + 4$

$F = 1$



Degree of Freedom of Mechanisms with Gear Pairs

Gear Pair
- Force Closed → $f = 1$
- Form Closed
  - *Irregular* → $f = 1$
  - *Regular* → $f = 2$

This concise classification is due to Prof. Dr. M. Kemal Özgören, METU-ME

Constrained – Unconstrained Mechanisms

Constrained mechanism may mean:

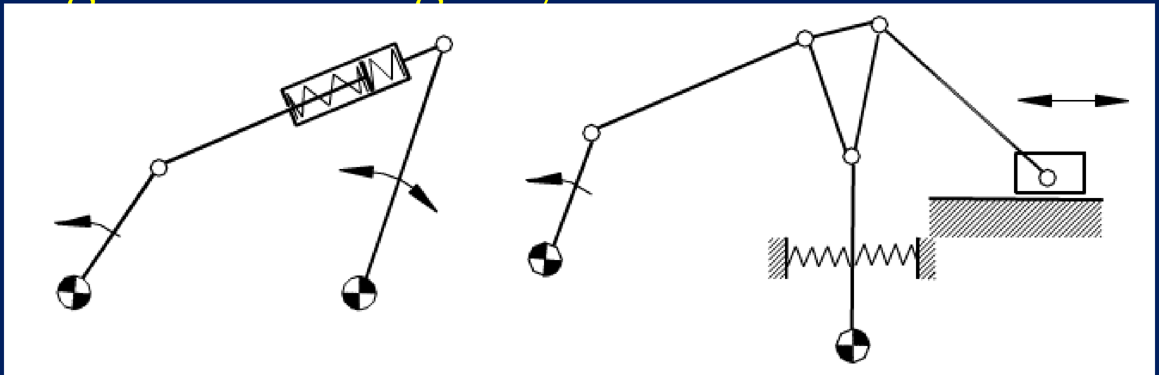
1. $F = 1$
2. $F > 1$ **and** number of actuators is equal to F

Unconstrained (underactuated) mechanisms have number of inputs less than F therefore their motion cannot be determined kinematically. Forces determine the motion.

Typical applications:

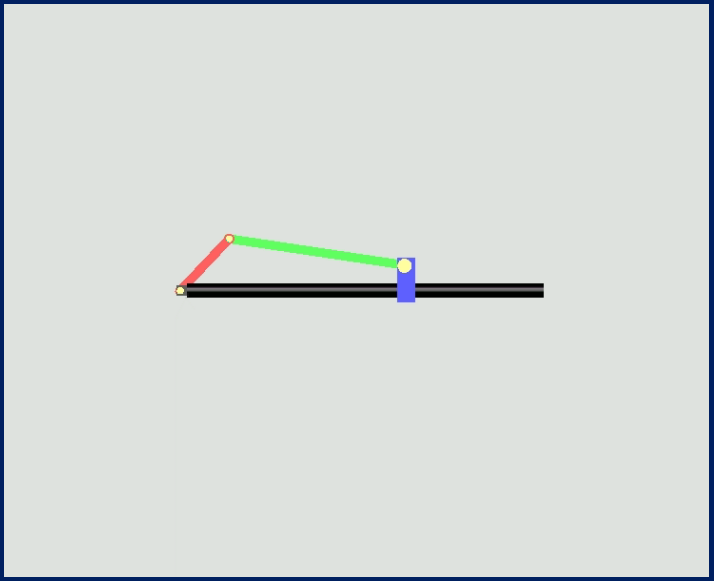
Car differential (F = 2 only drive is engine so that on straight roads two wheel share the same speed but during turns inner wheel is slower)

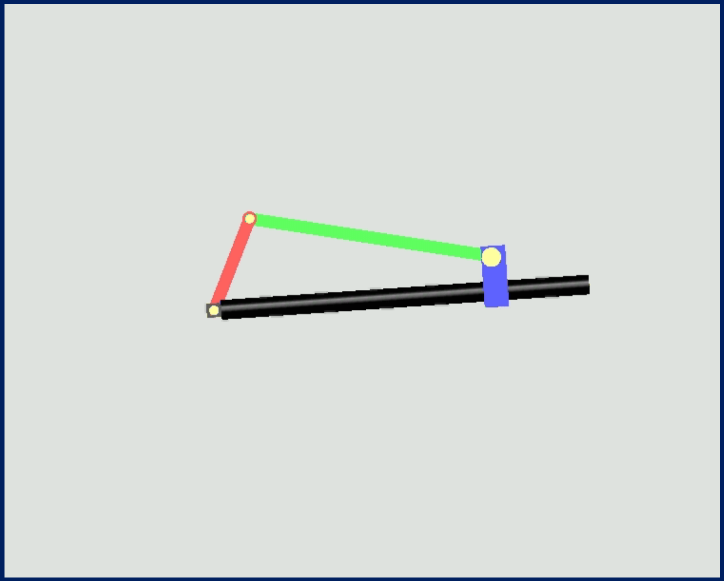
Force limiting under emergency conditions



Kinematic Inversion

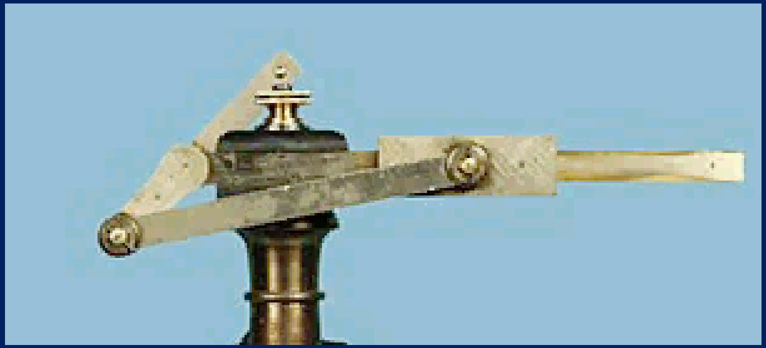
Let the fixed link move and fix another link for the purpose of obtaining another mechanism.

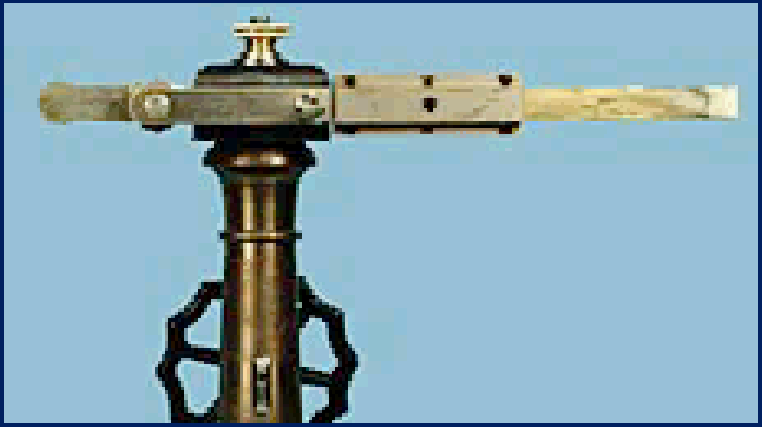




Kinematic Inversion

Let the fixed link move and fix another link with the purpose of obtaining another mechanism.





Grübler's Equation

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

For:

F = 1 (Single degree of freedom / constrained mechanism)

λ = 3 : Planar mechanism

ℓ: Number of links of the mechanism

j : Number of joints of the mechanism

Let $f = 1$ (only revolute and prismatic joints so $\sum_{i=1}^{j} f_i = j$)

$1 = 3(\ell - j - 1) + j$

$\boxed{3\ell - 2j = 4}$: Grübler's (Gruebler's) Equation

Grübler's equation is a special form of the general degree of freedom equation for planar mechanisms having $F = 1$ with the single degree of freedom joints.

Grübler's Equation

$$3\ell - 2j = 4$$

Consequences:

- ℓ must be even

  $3\ell = 2j + 4$, $2j + 4$ is even whatever j is. For 3ℓ to be even ℓ must be even!

- $\ell_2 \geq 4$

  $\ell = \ell_2 + \ell_3 + \ell_4 + \cdots + \ell_n$

  Number of kinematic elements in a mechanism is $2j$

  $2j = 2\ell_2 + 3\ell_3 + 4\ell_4 + \cdots + n\ell_n$

  Substituting into Grübler's equation:

  $3(\ell_2 + \ell_3 + \ell_4 + \cdots + \ell_n) - (2\ell_2 + 3\ell_3 + 4\ell_4 + \cdots + n\ell_n) = 4$

  $\ell_2 - [\ell_4 + 2\ell_5 \ldots + (n-3)\ell_n] = 4$

  $\ell_4 + 2\ell_5 \ldots + (n-3)\ell_n = P \geq 0$

  For $P = 0$, $\ell_2 = 4$, for $P > 0$, $\ell_2 > 4$

Grübler's Equation

$3\ell - 2j = 4$

Consequences (cont'ed):

- Number of kinematic elements in one link cannot exceed half the number of links in the mechanism, $\ell_k, k \leq \ell/2$

  Consider type (a) link with maximum possible kinematic elements on it.

  Type (b) links (ternary) links are connected to type (a) links.

  Type (c) links (binary) connect type (b) links.

  Type (a) link has k kinematic elements on it and there are k type (b) links, (k-1) type (c) links.

  Total number of links in the mechanism for this case is

  $\ell = 1 + k + (k - 1) = 2k$

  or

  $k_{max} = \frac{\ell}{2}$

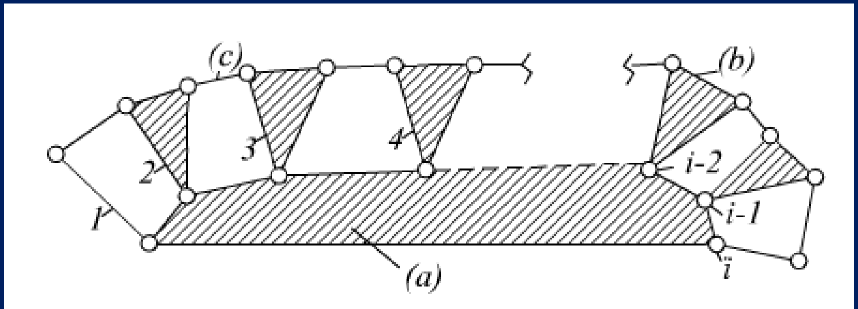


Figure from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Enumeration of Kinematic Chains and Mechanisms

Enumeration: A complete, ordered listing of all the items in a collection (*Wikipedia*)

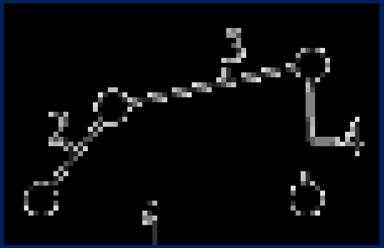
1. the act or process of making or stating a list of things one after another also: the list itself
2. the act or process of counting something or a count made of something Merriam-Webster

Determine all possible kinematic chains that satisfy certain predetermined criteria.

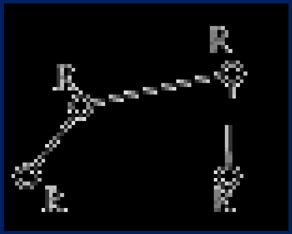
Enumeration of Kinematic Chains and Mechanisms

$3\ell - 2j = 4$

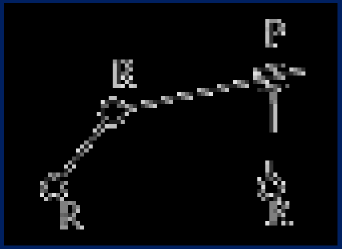
List all planar $F = 1$ mechanisms with 4 links:



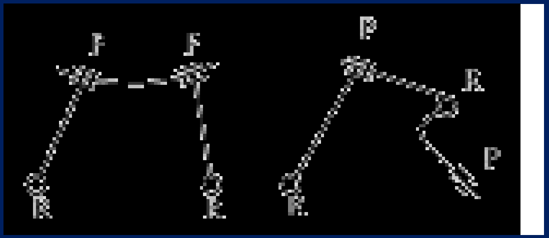
With revolute joints only (RRRR or 4R)



With three revolute and one prismatic joints (RRRP)



With two revolute and two prismatic joints (RPRP & RRPP)



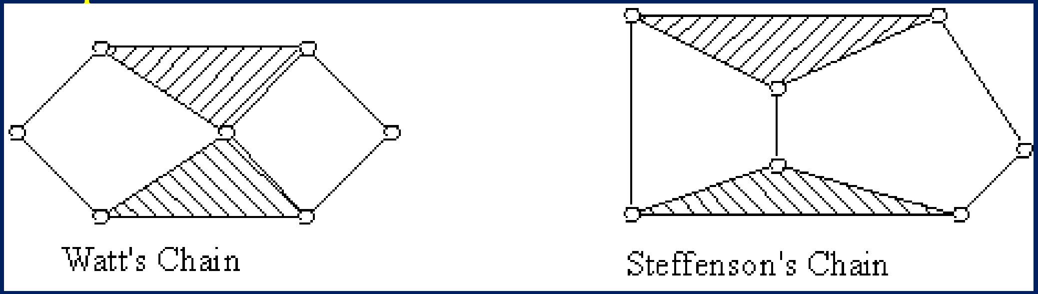
Three prismatic joints is not allowed (why?)

Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Enumeration of Kinematic Chains and Mechanisms

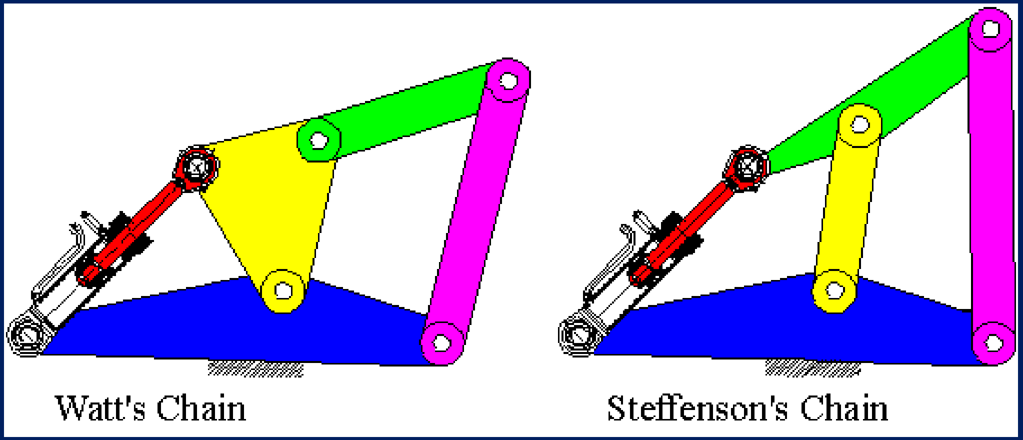
$3\ell - 2j = 4$

List all planar $F = 1$ mechanisms with 6 links:



With revolute joints only

With one prismatic joint, others revolute



Figures from electronic lecture notes of Eres Söylemez https://ocw.metu.edu.tr/course/view.php?id=132

Mechanism

1. Kinematics of machinery,

2. Group of rigid bodies connected by rigid kinematic elements (joints),

3. One of the links of a kinematic chain is fixed.