

### 3.3.3 The Gaussian CDF

The random variable  $X$  is Gaussian, in other words, Normal, with parameters  $(\mu, \sigma^2)$  if it has the PDF:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

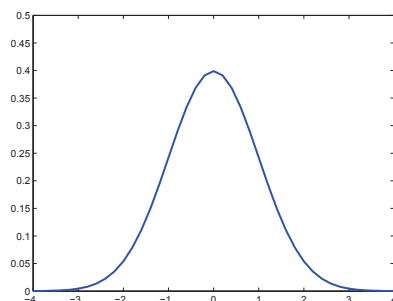


Figure 3.4: Gaussian (normal) PDF

$X$  is said to be a Standard Normal if it's Normal (i.e. Gaussian) with mean 0 and variance 1. That is,

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

The CDF of the standard Gaussian is defined as follows:

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

Note that this is the area under the standard Gaussian curve, up to point  $x$ . We often use the function  $\Phi(x)$  to make calculation involving general Gaussian random variables.

Normality is preserved by linear transformations: If  $X$  is Normal( $\mu, \sigma^2$ ) and  $Y = aX + b$ , then  $Y$  is Normal( $a\mu + b, a^2\sigma^2$ ) (We can prove this after we learn about Transforms of PDFs.) So, we can obtain any Gaussian by making a linear transformation on a standard Gaussian. That is, letting  $X$  be a standard Gaussian, if we let  $Y = \sigma X + \mu$ , then  $Y$  is Normal with mean  $\mu$  and variance  $\sigma^2$ .

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

Then we have:

$$P(Y \leq y) = P((Y-\mu)/\sigma \leq (y-\mu)/\sigma) = P(X \leq (y-\mu)/\sigma) = \Phi((y-\mu)/\sigma)$$

**Ex:** (Adapted from Ex 3.7 from the textbook.) The annual snowfall at Elmadağ is modeled as a normal random variable with a mean of  $\mu = 150$  cm and a standard deviation of 50 cm. What is the probability that next year's snowfall will be at least 200 cm? (Note that from the standard normal table,  $\Phi(1) = 0.8413$ .)

**Ex:** Signal Detection (Adapted from Ex 3.7 from the textbook.) A binary message is transmitted as a signal  $S$ , which is either +1 or -1 with equal probability. The communication channel corrupts the transmission with additive Gaussian noise with mean  $\mu = 0$  and variance  $\sigma^2$ . The receiver concludes that the signal +1 (or -1) was transmitted if the value received is not negative (or negative, respectively). Find the probability of