3.4 Conditional PDFs

The conditional PDF of a continuous random variable X, given an event $\{X = A\}$, with $\mathrm{P}(\{X = A\}) > 0$ is defined by

$$f_{X|A}(x) = \begin{cases} \frac{f_X(x)}{\mathrm{P}(\{X=A\})}, & \text{if } x \in A \\ 0, & \text{otherwise.} \end{cases}$$

Consequently, for any set B,

$$P(X \in B | X \in A) = \int_B f_{X|A}(x)dx$$

Ex: The exponential random variable is memoryless: Let X be the lifetime of a lightbulb, exponential with parameter λ. Given $X > t$, find $\mathrm{P}(X > t + x)$.)

Conditional Expectation:

$E(X|A) =$

$E(g(X)|A) =$

Ex: The voltage across the terminals of a power source is known to be between 4.8 to 5.2 Volts, uniformly distributed. The DC current supplied to the system is nonzero only when the source branch voltage exceeds 5 V, and then, it is linearly proportional to the voltage with a constant of proportionality a . Given that the system is working, compute the expected value of the power generated by the source.

Divide and Conquer Principle: Let A_1, A_2, ... A_n be disjoint partition of the sample space, with $\mathrm{P}(\{X = A_i\}) > 0$ for each i. We can find $f_X(x)$ by

$f_X(x) =$

and

$E(X) =$

as well as

$E(g(X)) =$

Ex: The metro train arrives at the station near your home every quarter hour starting at 6:00 a.m. You walk into the station every morning between 7:10 and 7:30, with your arrival time being random and uniformly distributed in this interval. What is the PDF of the time that you have to wait for the first train to arrive? Also find the expectation and variance of your waiting time.

3.5 Multiple Continuous Random Variables

Two random variables defined for the same sample space are said to be *jointly continuous* if there is a **joint probability density function** $f_{XY}(x, y)$ such that for any subset B of the two-dimensional plane,

$$\mathrm{P}((X,Y) \in B) = \int\int_{(x,y)\in B} f_{XY}(x,y)dxdy$$

When B is a rectangle:

When B is the entire real plane:

The joint pdf at a point can be approximately interpreted as the "probability per unit area" near the vicinity of that point. Just like the joint

PMF, the joint PDF contains all possible information about the individual random variables in consideration, and their dependencies. For example, the marginals are found as:

$f_X(x) =$

and

$f_Y(y) =$

Ex: Finding marginals from given two dimensional PDF: The joint PDF of the random variables X and Y is equal to a constant on the set S sketched on the board. Find the value of the constant c and the marginals of X and Y. Also compute the expectation of $X + 2Y$.