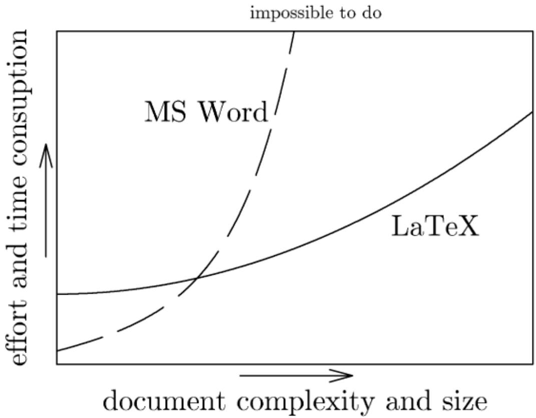

impossible to do
effort and time consuption
MS Word
LaTeX
document complexity and size

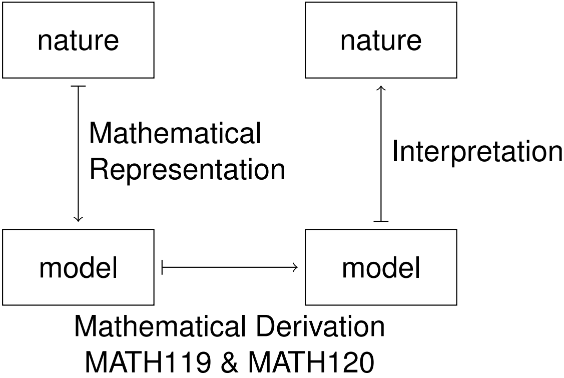
Learn LaTeX

$\boxed{\text{nature}}$

$$\boxed{\text{nature}} \xrightarrow{\text{Mathematical Representation}} \boxed{\text{model}}$$

$$\begin{array}{ccc} \boxed{\text{nature}} & & \\ \Big\downarrow \text{Mathematical Representation} & & \\ \boxed{\text{model}} & \longmapsto & \boxed{\text{model}} \end{array}$$

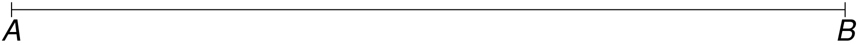
Mathematical Derivation
MATH119 & MATH120


nature
nature
Mathematical Representation
Interpretation
model
model
Mathematical Derivation
MATH119 & MATH120

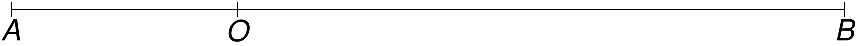
$$
\begin{tikzcd}
\boxed{\text{nature}} \arrow[d, mapsto, "\text{Mathematical Representation}"] \arrow[r, bend left] & \boxed{\text{nature}} \\
\boxed{\text{model}} \arrow[r, mapsto] & \boxed{\text{model}} \arrow[u, mapsto, "\text{Interpretation}"']
\end{tikzcd}
$$

Mathematical Derivation
MATH119 & MATH120

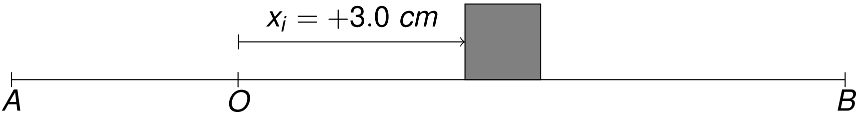
- Assume all the motion is along a given line.



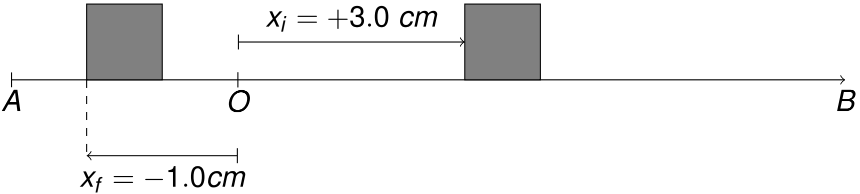
- Assume all the motion is along a given line.
- The position can be specified by a unique number: distance from origin O.



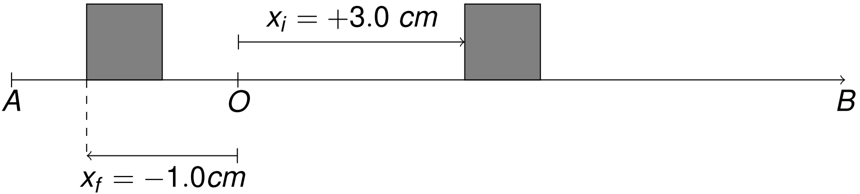
- Assume all the motion is along a given line.
- The position can be specified by a unique number: distance from origin O.
- One side is denoted as "+"



- Assume all the motion is along a given line.
- The position can be specified by a unique number: distance from origin O.
- One side is denoted as "+" , the other side "-"



- Assume all the motion is along a given line.
- The position can be specified by a unique number: distance from origin O.
- One side is denoted as "+" , the other side "-"
- The choice of O and the "+" side is completely arbitrary



Definitions:

- Displacement: the change in the position of an object Δx.

$$\begin{aligned} \Delta x &= \text{(final position)} - \text{(initial position)} \\ &= (3.0\ cm) - (-1.0\ cm) = 4.0\ cm \end{aligned} \tag{1}$$

- Average velocity: If Δt is the time that an object moves by Δx, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} \tag{2}$$

Department of Physics 1960 Fizik Bölümü

Definitions:

- Displacement: the change in the position of an object Δx.

ɔn)

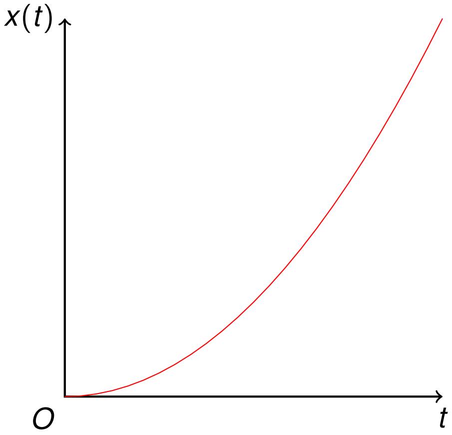
m (1)

- Average ve ves by Δx,
average vel

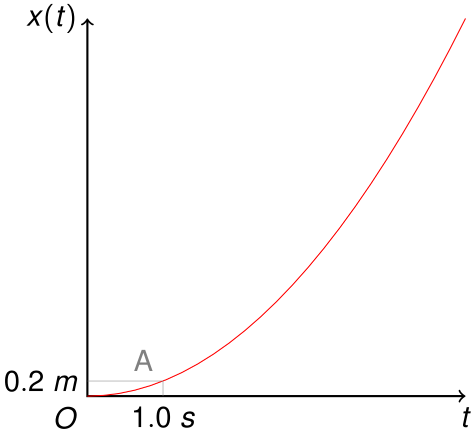
Greek Letters

Δ: Finite differences of any size
δ: Finite differences of small size
d: Infinitesimal difference (smaller than anything else)

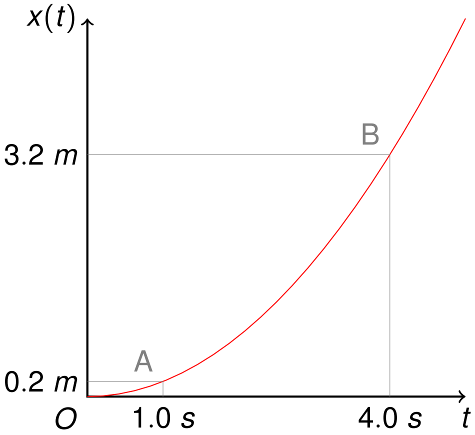
$$\bar{v} = \frac{\Delta x}{\Delta t} \tag{2}$$



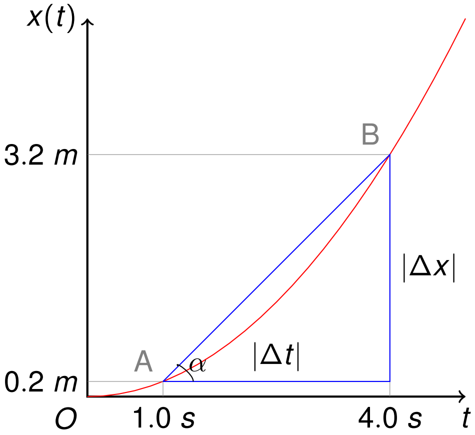
(3)



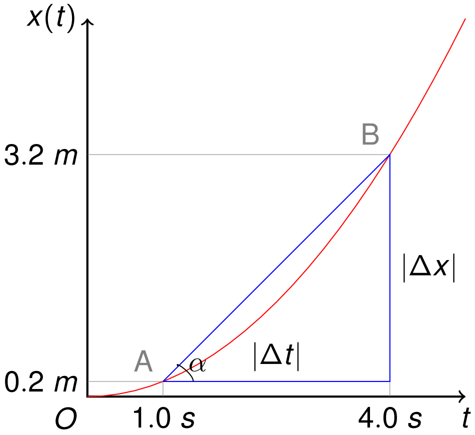
(3)



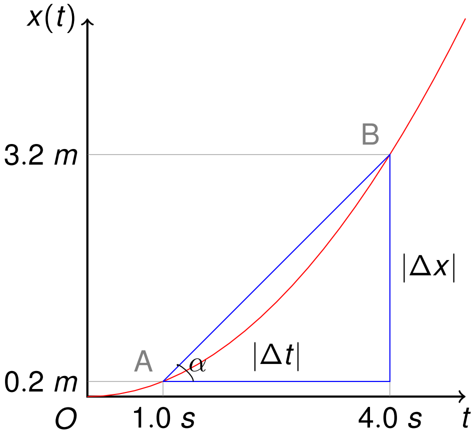
(3)



(3)



$$\bar{v}_{AB} = \frac{(3.2\ m) - (0.2\ m)}{(4.0\ s) - (1.0\ s)} = 1.0\ m/s \tag{3}$$



$$\bar{v}_{AB} = \frac{(3.2\ m) - (0.2\ m)}{(4.0\ s) - (1.0\ s)} = 1.0\ m/s = \tan\alpha \tag{3}$$

- As the final time moves closer to the initial time, i.e. the point B moves towards point A, we obtain the instantaneous velocity:

$$v_{inst} = \lim_{B \to A} \bar{v}_{AB} = \lim_{t_f \to t_i} \frac{\Delta x}{\Delta t} = \lim_{t_f \to t_i} \frac{x_f - x_i}{t_f - t_i} = \frac{dx}{dt} \tag{4}$$

- As the final time moves closer to the initial time, i.e. the point B moves towards point A, we obtain the instantaneous velocity:

$$v_{inst} = \lim_{B \to A} \bar{v}_{AB} = \lim_{t_f \to t_i} \frac{\Delta x}{\Delta t} = \lim_{t_f \to t_i} \frac{x_f - x_i}{t_f - t_i} = \frac{dx}{dt} \equiv v \quad (4)$$

- If δt is a sufficiently small amount of time, the displacement during this time is $\delta x = v\delta t$

$$x_f = x_i + v\delta t \quad (5)$$

Question

If $v(t)$ is know for all $t \in (t_i, t_f)$, and a particle is at the position $x(t_i) = x_0$ initially, how can we find $x(t)$ for any $t \in (t_i, t_f)$?

Question

If $v(t)$ is know for all $t \in (t_i, t_f)$, and a particle is at the position $x(t_i) = x_0$ initially, how can we find $x(t)$ for any $t \in (t_i, t_f)$?

A: Assume δt is sufficiently small and $t_f = t_i + N\delta t$.

$$\begin{aligned}
x(t_i + \delta t) - x(t_i) &= v(t_i)\delta t \\
x(t_i + 2\delta t) - x(t_i + \delta t) &= v(t_i + \delta t)\delta t \\
x(t_i + 3\delta t) - x(t_i + 2\delta t) &= v(t_i + 2\delta t)\delta t \\
&\cdots \\
x(t_i + N\delta t = t_f) - x(t_i + (N-1)\delta t) &= v(t_2 + (N-1)\delta t) \qquad (6)
\end{aligned}$$

$+$

$$x(t_f) - x_0 = \sum_{k=0}^{N-1} v(t_i + k\delta t)\delta t \qquad (7)$$

Read Zeno's paradox!

Question

If $v(t)$ is know for all $t \in (t_i, t_f)$, and a particle is at the position $x(t_i) = x_0$ initially, how can we find $x(t)$ for any $t \in (t_i, t_f)$?

A: Assume δt is sufficiently small and $t_f = t_i + N\delta t$.

$$\begin{aligned} x(t_i+\delta t) - x(t_i) &= v(t_i)\delta t \\ x(t_i+2\delta t) - x(t_i+\delta t) &= v(t_i+\delta t)\delta t \\ x(t_i+3\delta t) - x(t_i+2\delta t) &= v(t_i+2\delta t)\delta t \\ &\cdots \\ x(t_i+N\delta t = t_f) - x(t_i+(N-1)\delta t) &= v(t_2+(N-1)\delta t) \end{aligned} \tag{6}$$

$$x(t_f) - x_0 = \sum_{k=0}^{N-1} v(t_i + k\delta t)\delta t \xrightarrow{\delta t \to 0} \int_{t_i}^{t_f} v(t)dt \tag{7}$$

Read Zeno's paradox!

Special Case: Motion with constant velocity v_0:
In this case

$$x(t_f) - x_0 = \sum_{k=0}^{N-1} v(t_i + k\delta t)\delta t = \sum_{k=0}^{N-1} v_0 \delta t = v_0 N \delta t = v_0(t_f - t_i) \qquad (8)$$

Special Case: Motion with constant velocity v_0:
In this case

$$x(t_f) - x_0 = \sum_{k=0}^{N-1} v(t_i + k\delta t)\delta t = \sum_{k=0}^{N-1} v_0 \delta t = v_0 N \delta t = v_0(t_f - t_i) \tag{8}$$

$$x(t) = v_0(t - t_i) + x_0 \tag{9}$$

Note that for motion with constant velocity $\bar{v} = v_0$. Hence $\Delta x = v_0 \Delta t$

- The same steps can be repeated for the change of velocity.
  - $\bar{a} = \frac{\Delta v}{\Delta t}$. The unit of acceleration is m/s^2
  - $a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \equiv a$
  - $v(t) = v(t_0) + \int_{t_0}^{t_f} a(t')dt'$

Department of Physics 1960 Fizik Bölümü

- The same steps can be repeated for the change of velocity.
  - $\bar{a} = \frac{\Delta v}{\Delta t}$. The unit of acceleration is m/s^2
  - $a_{inst} = l$ ... Δv
  - $v(t) =$ ...

Acceleration is in the direction of Δv, **NOT** in the direction of v.

Department of Physics 1960 Fizik Bölümü

Example:

Motion with Constant Acceleration. Initial conditions: $x(0) = 0$, $v(0) = 0$. Realistic case: You stand at the top of a building. You are holding a mass m in your and release it from rest outside a window.

- Let a be the constant acceleration.

$$v(t) = v(0) + \int_0^t a dt' = at \tag{10}$$

- The position:

$$\begin{aligned} x(t) &= x(0) + \int_0^t v(t')dt' \\ &= \int_0^t (at')dt' = \frac{1}{2}at'^2\Big|_0^t = \frac{1}{2}at^2 \end{aligned} \tag{11}$$

Department of Physics Fizik Bölümü 1960

Dimensional Analysis

Most of the time, the final formula can be estimated unto overall coefficients using dimensions only. Denote the dimension of any quantity O by $[O]$

- Dimension of $x(t)$ is $[x(t)] = m$
- The dimensionful parameters in the problem are the acceleration a and the time t.
- Assume $x(t) = Aa^k t^l$ where A, k and l are numbers.

$$[Aa^m t^l] = [A][a]^k[t]^l = 1\left(\frac{m}{s^2}\right)^k s^l = m^k s^{l-2k} \tag{12}$$

- $x = Aa^k t^l \Longrightarrow k = 1$ and $l - 2k = 0 \Longrightarrow x(t) = Aat^2$
- Explicit calculation shows $A = \frac{1}{2}$.

- In principle these steps can be done for the change in acceleration, change in the change in the acceleration, etc.
- Newton's Laws tell us that this is not necessary
- The acceleration of an object is determined by external effects.

Compare

$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x(0) + \int_0^t v(t')dt' \tag{13}$$

$$a(t) = \frac{dv}{dt} \Longleftrightarrow v(t) = v(0) + \int_0^t a(t')dt' \tag{14}$$

Compare

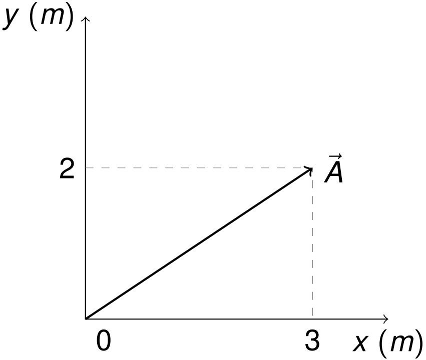
$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x(0) + \int_0^t v(t')dt' \qquad (13)$$

$$a(t) = \frac{dv}{dt} \Longleftrightarrow v(t) = v(0) + \int_0^t a(t')dt' \qquad (14)$$

Integration is the inverse of differentiation

Vectors

- For motion that is not confined to a line, more than a number is necessary to describe the direction.
- A vector is *a recipe* for how to go to the point *A* from the origin.
- A vector is a number and a direction
- Origin is arbitrarily chosen

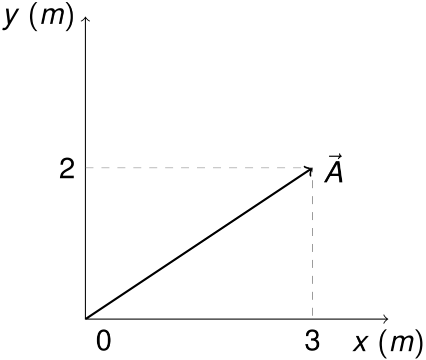


$$\vec{A} = (3, 2)\ m \tag{15}$$

$$\vec{A} = (3\ m)\hat{x} + (2\ m)\hat{y} \tag{16}$$

$$\vec{A} = (3\ m)\hat{i} + (2\ m)\hat{j} \tag{17}$$

$$\vec{A} = (\sqrt{13}\ m, \arctan\frac{2}{3}) \tag{18}$$



$$\vec{A} = (3,2)\ m \quad (15)$$

$$\vec{A} = (3\ m)\hat{x} + (2\ m)\hat{y} \quad (16)$$

$$\vec{A} = (3\ m)\hat{i} + (2\ m)\hat{j} \quad (17)$$

$$\vec{A} = (\sqrt{13}\ m, \arctan\frac{2}{3}) \quad (18)$$

$$\vec{A} = (2,3)\ m \quad (19)$$

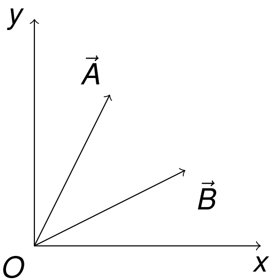
$$\vec{A} = (\sqrt{13}\ m, \arctan\frac{3}{2})$$

Vector Operations-Multiplication by a number

- A vector $\vec{A}$ is a number (the length of the vector, $|\vec{A}|$) and a direction.
- The vector $\lambda\vec{A}$ is another vector
  - The length of $\lambda\vec{A}$ is $|\lambda\vec{A}| = |\lambda||\vec{A}|$
  - The direction of $\lambda\vec{A}$ is the same as the direction of $\vec{A}$ if $\lambda > 0$, and opposite to $\vec{A}$ if $\lambda < 0$

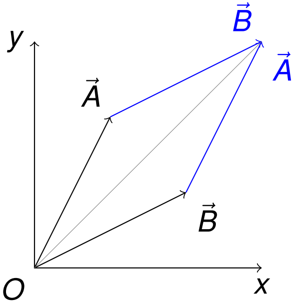
Vector Operations-Addition of Vectors

Geometrical Addition



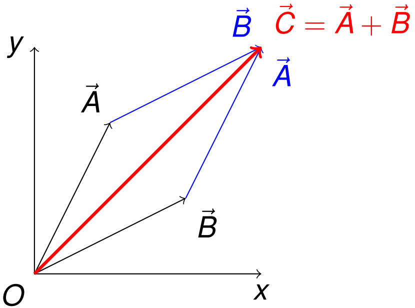
Vector Operations-Addition of Vectors

Geometrical Addition



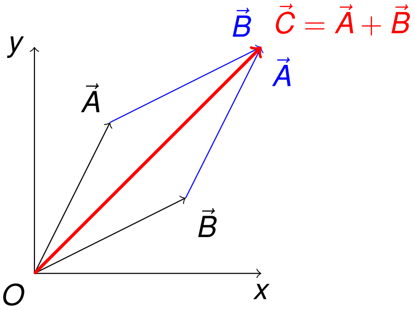
Vector Operations-Addition of Vectors

Geometrical Addition



Vector Operations-Addition of Vectors

Geometrical Addition

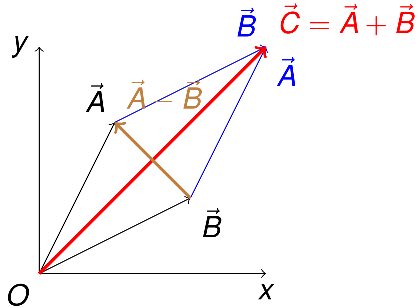


Componentwise Addition

- $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$
- $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$
- $\vec{C} = C_x\hat{x} + C_y\hat{y} + C_z\hat{z}$
- $C_x = A_x + B_x$, $C_y = A_y + B_y$ $C_z = A_z + B_z$
- $C_i = A_i + B_i$, $i = x,\ y$ or z

Vector Operations-Addition of Vectors

Geometrical Addition



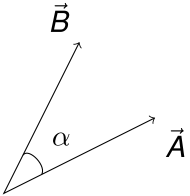
Componentwise Addition

- $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$
- $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$
- $\vec{C} = C_x\hat{x} + C_y\hat{y} + C_z\hat{z}$
- $C_x = A_x + B_x$, $C_y = A_y + B_y$ $C_z = A_z + B_z$
- $C_i = A_i + B_i$, $i = x,\ y$ or z

Subtraction

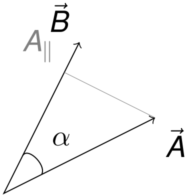
$\vec{A} - \vec{B} = \vec{A} + ((-1)\vec{B})$

Vector Operations: Scalar Product



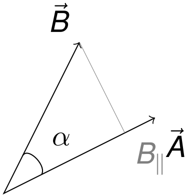
- Scalar product gives a number from two vectors
- $\vec{A} \cdot \vec{B} \equiv |\vec{A}||\vec{B}| \cos \alpha$

Vector Operations: Scalar Product



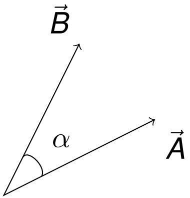
- Scalar product gives a number from two vectors
- $\vec{A} \cdot \vec{B} \equiv |\vec{A}||\vec{B}| \cos \alpha$
- $\vec{A} \cdot \vec{B} = A_{\|} B$

Vector Operations: Scalar Product



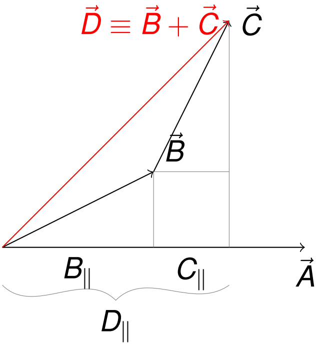
- Scalar product gives a number from two vectors
- $\vec{A} \cdot \vec{B} \equiv |\vec{A}||\vec{B}| \cos \alpha$
- $\vec{A} \cdot \vec{B} = AB_{\parallel}$

Vector Operations: Scalar Product



- Scalar product gives a number from two vectors
- $\vec{A}\cdot\vec{B} \equiv |\vec{A}||\vec{B}|\cos\alpha$
- Scalar product is linear: $\vec{A}\cdot(a\vec{B}+b\vec{C}) = a(\vec{A}\cdot\vec{B}) + b(\vec{A}\cdot\vec{C})$
- $\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1,$ $\hat{x}\cdot\hat{y} = \hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0$

Vector Operations: Scalar Product



$$\vec{A}\cdot\vec{D} = AD_{||} = A(B_{||}+C_{||}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

duct gives a number from

;

$\vec{B}| \cos\alpha$

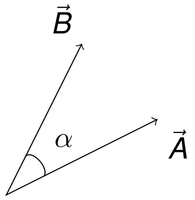
duct is linear:

$\vec{C}) = a(\vec{A}\cdot\vec{B}) + b(\vec{A}\cdot\vec{C})$

$= \hat{z}\cdot\hat{z} = 1,$

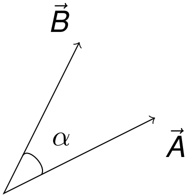
$= \hat{y}\cdot\hat{z} = 0$

Vector Operations: Scalar Product



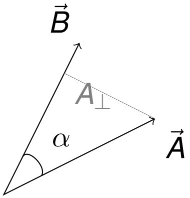
- Scalar product gives a number from two vectors
- $\vec{A} \cdot \vec{B} \equiv |\vec{A}||\vec{B}| \cos \alpha$
- $\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$,
  $\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$
- $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
- $A_x = \vec{A} \cdot \hat{x}$, $A_y = \vec{A} \cdot \hat{y}$, and $A_z = \vec{A} \cdot \hat{z}$

Vector Operations: Vector Product



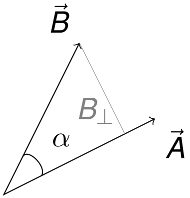
- Vector product gives a vector from two vectors
- $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \alpha$
- Direction of $\vec{A} \times \vec{B}$ is given by the right hand rule. ($\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$)

Vector Operations: Vector Product



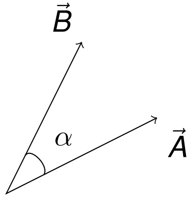
- Vector product gives a vector from two vectors
- $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \alpha$
- Direction of $\vec{A} \times \vec{B}$ is given by the right hand rule. ($\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$)
- $|\vec{A} \times \vec{B}| = A_\perp B$

Vector Operations: Vector Product



- Vector product gives a vector from two vectors
- $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \alpha$
- Direction of $\vec{A} \times \vec{B}$ is given by the right hand rule. ($\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$)
- $|\vec{A} \times \vec{B}| = AB_{\perp}$

Vector Operations: Vector Product



- Vector product gives a vector from two vectors
- $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \alpha$
- Direction of $\vec{A} \times \vec{B}$ is given by the right hand rule. ($\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$)
- Vector product is linear: $\vec{A} \cdot (a\vec{B} + b\vec{C}) = a(\vec{A} \cdot \vec{B}) + b(\vec{A} \cdot \vec{C})$
- $\hat{x} \times \hat{x} = \hat{y} \times \hat{y} = \hat{z} \times \hat{z} = 0$, $\hat{x} \times \hat{y} = \hat{z}$, $\hat{x} \times \hat{z} = -\hat{y}$, $\hat{y} \times \hat{z} = \hat{x}$

Vector Operations- Vector Division

Vector Operations- Vector Division

Division by a vector DOES NOT exist!