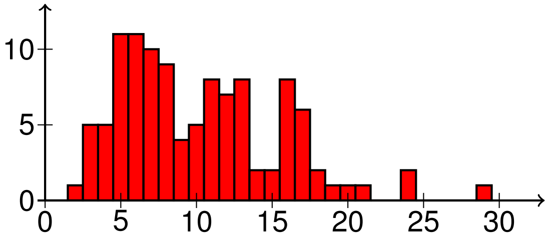
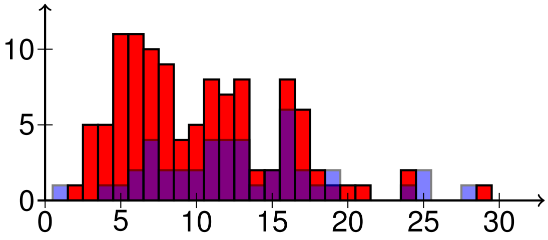


There seems to be three different groups of students:

- A group around 6
- A group around 12
- A group around 16



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- A group around 6
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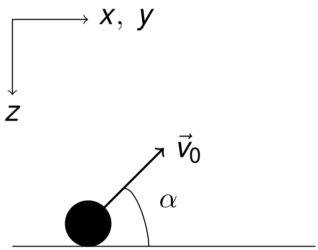
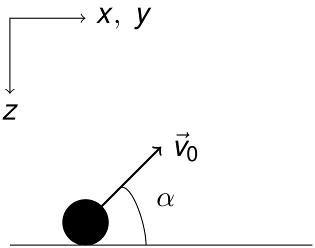
Trajectory of a Football



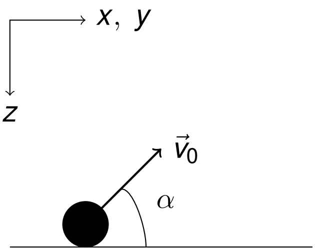
- Assume that you hit a football lying on the ground.
- It's initial speed is v_0 making an angle α with the ground.
- Choose the origin of time such that $t_i = 0$ and origin of coordinate axis such that $\vec{x}(0) = 0$

Trajectory of a Football



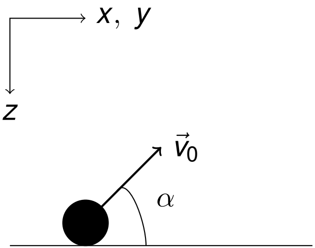
- $\vec{x}(t) = \vec{v}_0 t + \frac{1}{2} g t^2 \hat{z}$
- $z(t) = v_{0z} t + \frac{1}{2} g t^2$
- $z(t) = 0$ when $t = 0$(initial time) and at $t = -\frac{2v_{0z}}{g}$ (when the ball hits the ground)

Trajectory of a Football



- The flight time of the ball is $t = -\frac{2v_{0z}}{g}$.
- The only acceleration is along the z axis.
- $-\frac{v_{0z}}{g}$ is the time it take for the z component of the velocity to become zero, i.e. the time it takes to reach maximum height
- The time it takes to fall down is the same (in the absence of air friction)

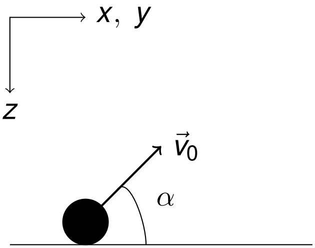
Trajectory of a Football



- Assume that x and y axis are chosen such that $\vec{v} = -v_0 \sin\alpha \hat{z} + v_0 \cos\alpha \hat{x}$
- $v_y(t) = 0,\ y(t) = 0$ for all times
- $x(t) = v_{0x} t$.
- Range is the distance the ball covers during its flight, i.e. $R = |x(t_f)|$

$$\begin{aligned} R &= v_{0x}\left(-\frac{2v_{0z}}{g}\right) \\ &= v_0 \cos\alpha \left(-\frac{2(-v_0 \sin\alpha)}{g}\right) \\ &= \frac{v_0^2 \sin 2\alpha}{g} \end{aligned} \tag{29}$$

Trajectory of a Football



- $R = \frac{v_0^2 \sin(2\alpha)}{g}$
- $\sin 2\alpha$ has maximum value of 1 when $\alpha = 45°$
- Increasing v_0 by a factor of 2 increases the range by 4.
- If $\alpha_1 + \alpha_2 = \frac{\pi}{2}$, their ranges are the same

Trajectory

- Trajectory is a relationship between the components of the position of a particle that does not involve time.
- When the particle is at the horizontal distance x, the time that has passed is $t(x) = x/v_{0x}$.
- The z coordinate of the particle at that time is

$$\begin{aligned} z(x) &= v_{0z}t(x) + \frac{1}{2}gt(x)^2 \\ &= v_{0z}\left(\frac{x}{v_{0x}}\right) + \frac{1}{2}g\left(\frac{x}{v_{0x}}\right)^2 \\ &= \frac{g}{2v_0^2\cos^2\alpha}x(x-R) \quad (29) \end{aligned}$$

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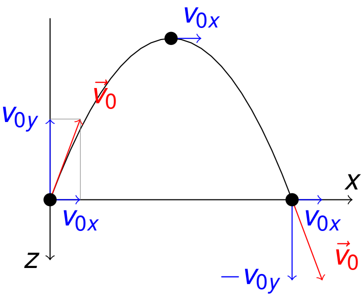
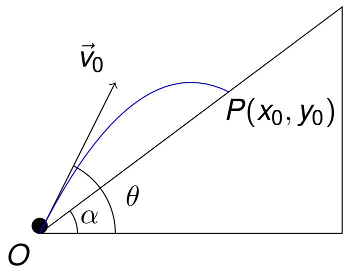
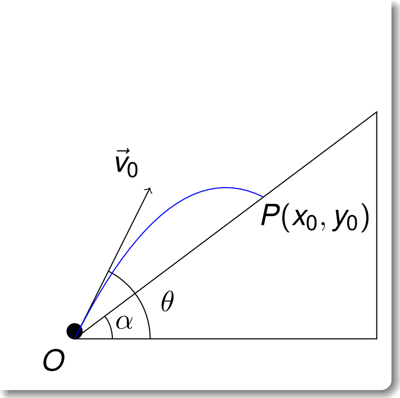


Example



- **Q:** What is the distance $|OP|$?

Example



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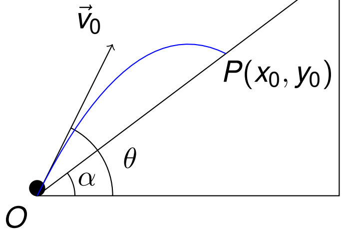
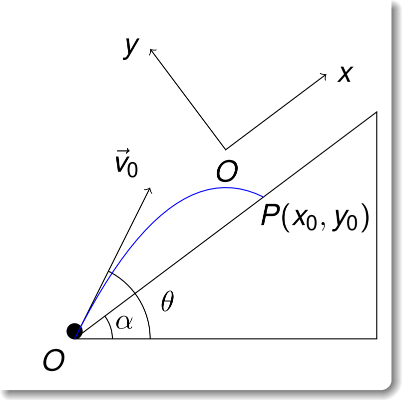


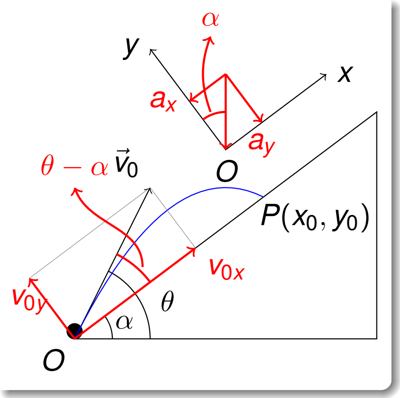
- **Q:** What is the distance $|OP|$?
- To find the point P, we will use the fact that point P is both on the parabola describing the trajectory, and also on the line that describes the hill.

Example



- **Q:** What is the distance $|OP|$?
- First choose a coordinate axis.

Example

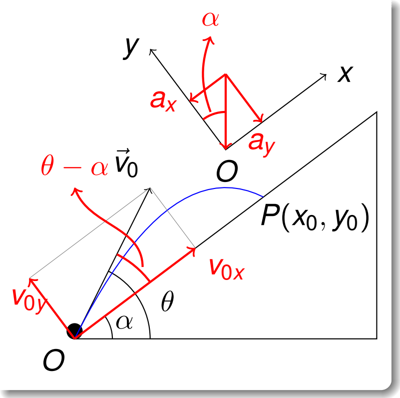


- **Q:** What is the distance $|OP|$?
- First choose a coordinate axis.
- The initial velocity and acceleration in these new coordinate axes are:

$$\vec{v}_0 = v_0 \cos(\theta - \alpha)\hat{x} + v_0 \sin(\theta - \alpha)\hat{y}$$

$$\vec{a} = g \cos\alpha(-\hat{y}) + g \sin\alpha(-\hat{x})$$

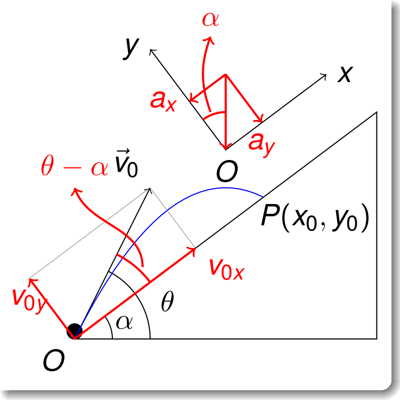
Example



- **Q:** What is the distance $|OP|$?
- The velocity at time t can be obtained as

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t')dt' = \vec{v}_0 + t\vec{a}$$
$$= [v_0 \cos(\theta - \alpha) - gt \sin\alpha]\,\hat{x}$$
$$+ [v_0 \sin(\theta - \alpha) - gt \cos\alpha]\,\hat{y}$$

Example



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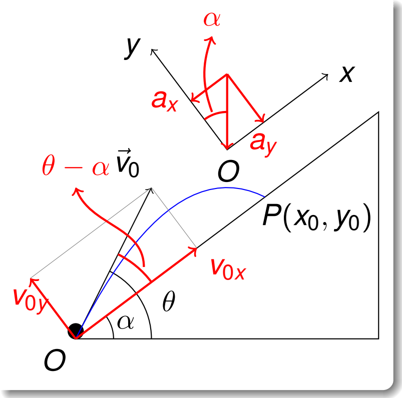
- The position at time t is

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')dt' \tag{30}$$

Example



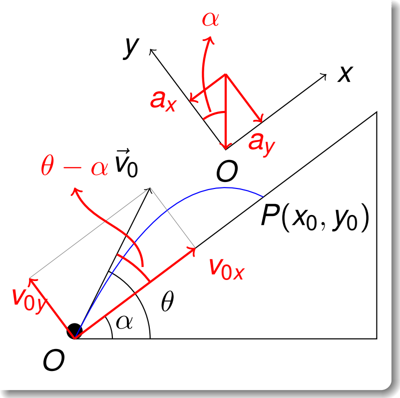
- **Q:** What is the distance $|OP|$?
- The position at time t is

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')dt'$$

$$= \left[v_0 t \cos(\theta-\alpha) - \frac{1}{2}gt^2 \sin\alpha\right]\hat{x}$$

$$+ \left[v_0 t \sin(\theta-\alpha) - \frac{1}{2}gt^2 \cos\alpha\right]\hat{y}$$

Example



- **Q:** What is the distance $|OP|$?
- Hence, if the object reaches the point P at time t_0,

$$x_0 = v_0 t_0 \cos(\theta - \alpha) - \frac{1}{2} g t_0^2 \sin\alpha$$
$$y_0 = v_0 t_0 \sin(\theta - \alpha) - \frac{1}{2} g t_0^2 \cos\alpha \quad (30)$$

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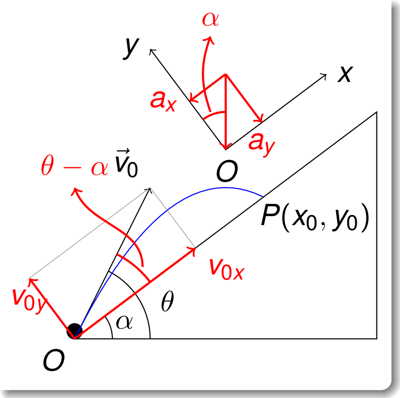
Example

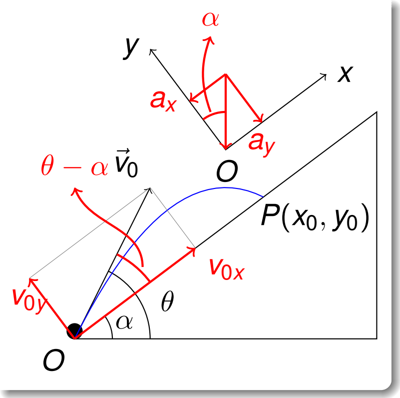


- **Q:** What is the distance $|OP|$?
- At point P, $y_0 = 0$

$$v_0 t_0 \sin(\theta-\alpha) - \frac{1}{2} g t_0^2 \cos\alpha = 0 \quad (30)$$

which has solutions $t_0 = 0$ or $t_0 = \frac{2v_0 \sin(\theta-\alpha)}{g\cos\alpha}$

Example



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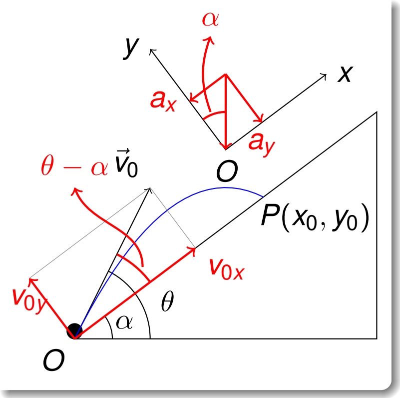
which has solutions $t_0 = 0$ or $t_0 = \frac{2v_0 \sin(\theta - \alpha)}{g \cos\alpha}$

- $t_0 = 0$ is the beginning of motion. The second solution is the solution we are looking for.

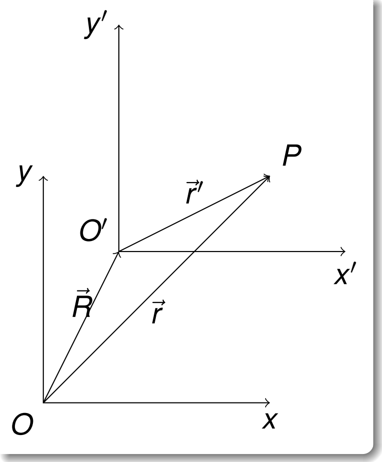
Example



- **Q:** What is the distance $|OP|$?
- The distance $|OP| = x_0$.
- Using $t_0 = \frac{2v_0 \sin(\theta-\alpha)}{g\cos\alpha}$

$$x_0 = v_0 \left(\frac{2v_0 \sin(\theta - \alpha)}{g \cos\alpha} \right) \sin(\theta - \alpha) - \frac{1}{2} g \left(\frac{2v_0 \sin(\theta - \alpha)}{g \cos\alpha} \right)^2 \cos\alpha \quad (30)$$

Relative Motion



- From the definition of vector addition $\vec{r} = \vec{R} + \vec{r}'$.
- The displacement of the point P in a time Δt is

$$\Delta\vec{r} = \Delta\vec{R} + \Delta\vec{r}' \tag{31}$$

- The velocities in the two reference frames are related by $\vec{v} = \vec{v}' + \vec{V}$

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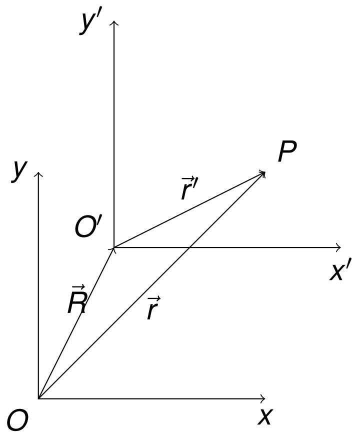
Relative Motion



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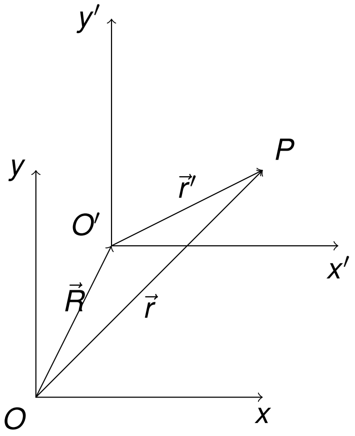

- From the definition of vector addition $\vec{r} = \vec{R} + \vec{r}'$.
- The displacement of the point P in a time Δt is

$$\frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta \vec{R}}{\Delta t} + \frac{\Delta \vec{r}'}{\Delta t} \tag{31}$$

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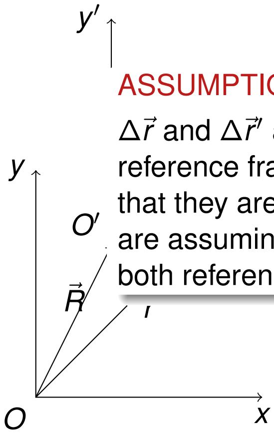
Relative Motion



- From the definition of vector addition

ASSUMPTIONS

$\Delta\vec{r}$ and $\Delta\vec{r}'$ are measured in different reference frames. We are assuming that they are equal. Furthermore, we are assuming the Δt is the same in both reference frames.

ent of the point P in a

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Example

Question 3.78

Raindrops make an angle θ with the vertical when viewed through a moving train window. If the speed of the train is $\vec{v}_T$, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?

Solution:

Let $\vec{v}_R = -v\hat{z}$ be the speed of the raindrops in the reference frame of Earth, $\vec{v}_E$ be the velocity of the Earth relative to the train, i.e. $\vec{v}_E = -\vec{v}_T$.

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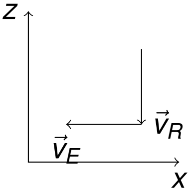

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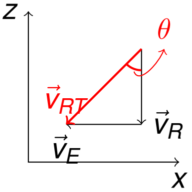

- Let $\vec{v}_{RT}$ be the velocity of the raindrops relative to train

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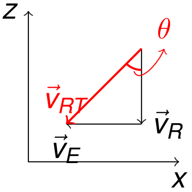

- It is given that $\vec{v}_{RT}$ makes θ radians with respect to the vertical
- From the figure, it is seen that

$$\tan\theta = \frac{v_E}{v_R} \implies v_R = v_T \cot\theta \qquad (32)$$

Reference Frames

- event: position+time
- A reference frame is a coordinate axis (to measure the position of an event)
- And a clock at each point of space (to measure the time of an event)

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Dynamics-Newton's Laws of Motion

1^{st} **Law:** In an inertial reference frame, in the absence of any external influences, the velocity of an object is constant

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Dynamics-Newton's Laws of Motion

1st **Law:** In an inertial reference frame, in the absence of any external influences, the velocity of an object is constant

This is a definition of an inertial reference frame

Dynamics-Newton's Laws of Motion

Inertial Reference Frame

- To test if a given reference frame is inertial, consider a test object
  - Eliminate all external influecens.
  - Check to see if the object accelerates or not
  - If the object is not accelerating, that reference frame is an inertial reference frame
- Given one inertial reference frame, any other frame that moves at constant velocity relative to the inertial reference frame is inertial:

$$\vec{v} = \vec{V} + \vec{v}' \tag{33}$$

- If a given reference frame is an inertial reference frame, all objects obey Newtons 1^{st} law in that frame

Dynamics-Newton's Laws of Motion

2^{nd} **Law:** In an inertial reference frame, the acceleration of an object is proportional to the force acting on the object. The proportionality constant is $\frac{1}{m}$ where m is the mass of the object

$$\vec{a} = \frac{\vec{F}}{m} \tag{34}$$

3^{rd} **Law:** If an object A exerts a force $\vec{F}_{AB}$ on another object B, then object B also exerts a force $\vec{F}_{BA}$ on object A whose magnitude is equal to the magnitude of $\vec{F}_{AB}$, but opposite in direction:

$$\vec{F}_{AB} = -\vec{F}_{BA} \tag{35}$$

Dynamics-Newton's Laws of Motion

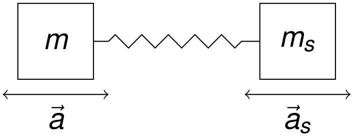
2^{nd} and 3^{rd} laws define the mass of an object

- By the 3^{rd} law, the magnitudes of the force acting on the standard mass and the unknown mass are equal:
- Using 2^{nd} law:

$$ma = m_s a_s \quad (36)$$

- Accelerations can be measured experimentally. Hence the unknown mass can be obtained as:

$$m = m_s \frac{a_s}{a} \quad (37)$$



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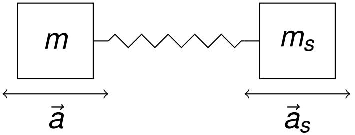

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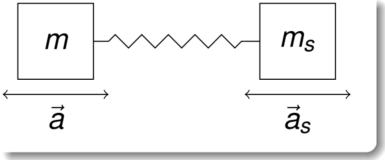

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Dynamics-Newton's Laws of Motion

- Once the mass is defined, 2^{nd} Law can be considered as the definition of the force.
- Also, if the force is given (by some means), the second law can be used to obtain acceleration.