Dynamics-Newton's Laws of Motion

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This is a definition of an inertial reference frame

Dynamics-Newton's Laws of Motion

Inertial Reference Frame

- To test if a given reference frame is inertial, consider a test object
  - Eliminate all external influecens.
  - Check to see if the object accelerates or not
  - If the object is not accelerating, that reference frame is an inertial reference frame
- Given one inertial reference frame, any other frame that moves at constant velocity relative to the inertial reference frame is inertial:

$$\vec{v} = \vec{V} + \vec{v}' \Longrightarrow \vec{a} = \vec{A} + \vec{a}' \tag{33}$$

- If a given reference frame is an inertial reference frame, all objects obey Newton's 1^{st} law in that frame

Dynamics-Newton's Laws of Motion

2^{nd} **Law:** In an inertial reference frame, the acceleration of an object is proportional to the force acting on the object. The proportionality constant is $\frac{1}{m}$ where m is the mass of the object

$$\vec{a} = \frac{\vec{F}}{m} \tag{34}$$

3^{rd} **Law:** If an object A exerts a force $\vec{F}_{AB}$ on another object B, then object B also exerts a force $\vec{F}_{BA}$ on object A whose magnitude is equal to the magnitude of $\vec{F}_{AB}$, but opposite in direction:

$$\vec{F}_{AB} = -\vec{F}_{BA} \tag{35}$$

Dynamics-Newton's Laws of Motion

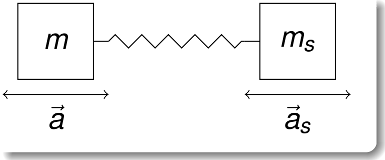
2^{nd} and 3^{rd} laws define the mass of an object

- By the 3^{rd} law, the magnitudes of the force acting on the standard mass and the unknown mass are equal:
- Using 2^{nd} law:

$$ma = m_s a_s \tag{36}$$

- Accelerations can be measured experimentally. Hence the unknown mass can be obtained as:

$$m = m_s \frac{a_s}{a} \tag{37}$$



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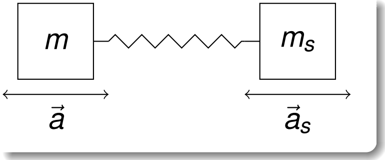

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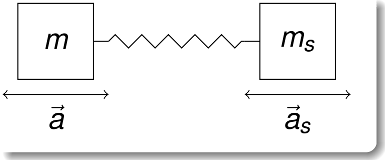

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Dynamics-Newton's Laws of Motion

- Once the mass is defined, 2^{nd} Law can be considered as the definition of the force.
- Also, if the force is given (by some means), the second law can be used to obtain acceleration.
- Unit of Force:

$$[\vec{F}] = [m\vec{a}] = [m][\vec{a}] = kg\frac{m}{s^2} \equiv N(\text{Newton}) \tag{38}$$

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Dynamics-Newton's Laws of Motion

- Once the mass is defined, 2^{nd} Law can be considered as the definition of the force.
- Also, if ATTENTION law can be used to
- Unit of

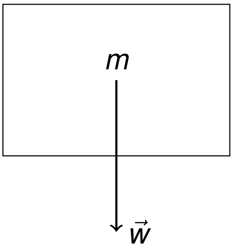
ATTENTION

There is no force *due to* acceleration!

The force is the *cause* of acceleration!

$$[\vec{F}] = [m\vec{a}] = [m][\vec{a}] = kg\frac{m}{s^2} \equiv N(\text{Newton}) \qquad (38)$$

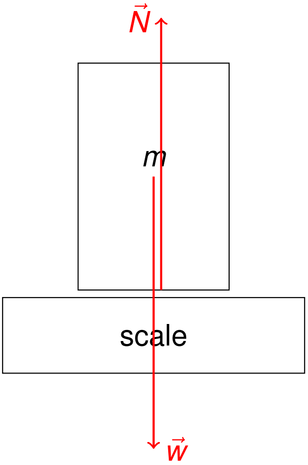
Weight



- Weight, $\vec{w}$, is the force acting on an object due to gravity.
- Near Earth, all object accelerate with the same acceleration $\vec{g}$.
- By Newton's second law, the force acting on an object of mass m is

$$\vec{w} = m\vec{g} \tag{39}$$

Example 1: Mass on a scale



- $\vec{N}$: unknown force acting on the body by the scale
- $\vec{w}$: force of gravity acting on the body

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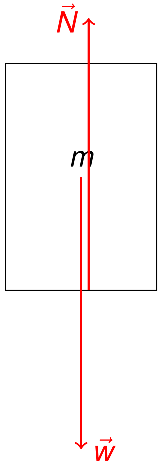
Example 1: Mass on a scale



- $\vec{N}$: unknown force acting on the body by the scale
- $\vec{w}$: force of gravity acting on the body
- Free body diagram: A diagram of masses only with the forces acting on each body shown separately

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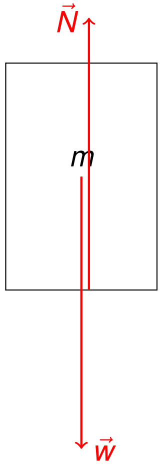
Example 1: Free Body Diagram

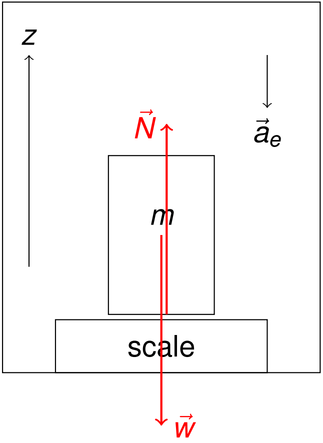


- Object is not accelerating:

$$\vec{F}_{net} = \vec{N} + \vec{w} \equiv 0$$
$$\Longrightarrow \vec{N} = -\vec{w} \qquad (40)$$

- The scale shows the magnitude of the force acting on it: $|-\vec{N}| = |\vec{w}| = mg$

Example 2: Mass on a scale inside an Elevator



- $\vec{a}_e$ is the acceleration of the elevator
- The vectors in the problem are:

$$\vec{N} = N\hat{z}(\text{Unknown}) \quad (41)$$

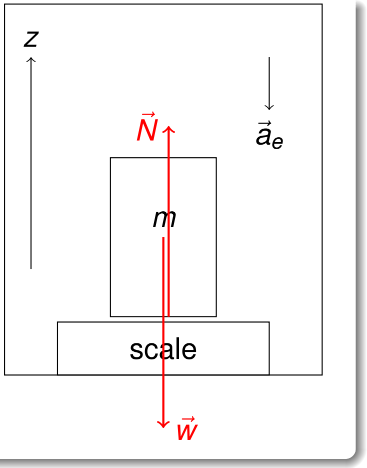
$$\vec{w} = -mg\hat{z} \quad (42)$$

$$\vec{a}_e = a_e\hat{z} \quad (43)$$

(In drawing the figure, it is assumed that $a_e < 0$)

- If the mass m stays on the scale, $\vec{a} = \vec{a}_e = a_e\hat{z}$

Example 2: Mass on a scale inside an Elevator



- Net force acting on the mass:

$$\vec{F}_T = (N - mg)\hat{z} = ma_e\hat{z}$$
$$\implies N = m(g + a_e) \quad (41)$$

- The force acting on the scale is $-\vec{N}$, Scale will show a weight $m(g + a_e)$.

14^{th} Century Bologna University

Learners and Learning

Herb Simon Nobel laureate, Social Scientist, one of the founders of AI

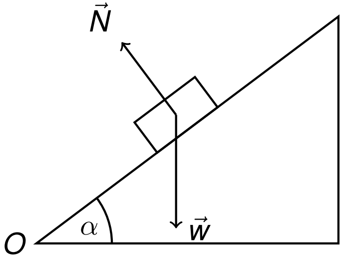
> *Learning results from what the student does and thinks and only from what the student does and thinks. The teacher can advance learning only by influencing what the student does to learn.*

Dylan William renowned UK expert on maths education

> *... teachers do not create learning, and yet most teachers behave as if they do. Learners create learning. Teachers create the conditions under which learning can take place.*

Example 3: Mass on an Inclined Plane

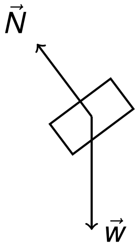
The object on the inclined surface



- A block sits on a frictionless incline as shown in the figure
- The forces acting on the mass are its weight and the normal force

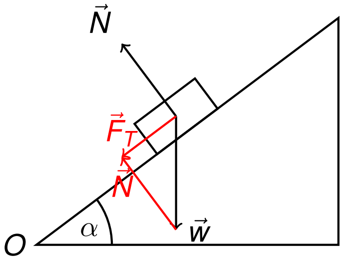
Example 3: Mass on an Inclined Plane

Free Body Diagram



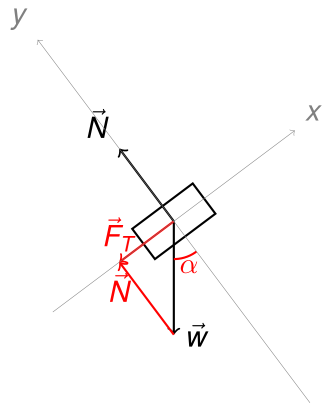
- Free body diagram includes only the mass and the forces

Example 3: Mass on an Inclined Plane



- The net force has to be along the surface of the inclined plane

Example 3: Mass on an Inclined Plane



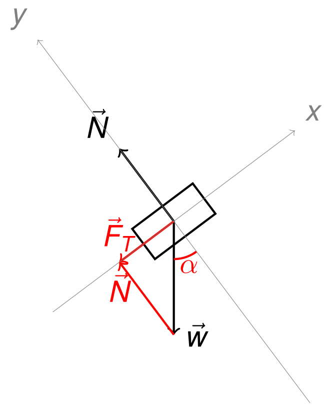
- In terms of their components, the forces can be written as:

$$\vec{N} = N\hat{y} \text{ (Unknown)} \quad (42)$$
$$\vec{w} = -mg\cos\alpha\hat{y} - mg\sin\alpha\hat{x} \quad (43)$$

- The net force is:

$$\vec{F}_T = (N - mg\cos\alpha)\hat{y} - mg\sin\alpha\hat{x} \quad (44)$$

Example 3: Mass on an Inclined Plane



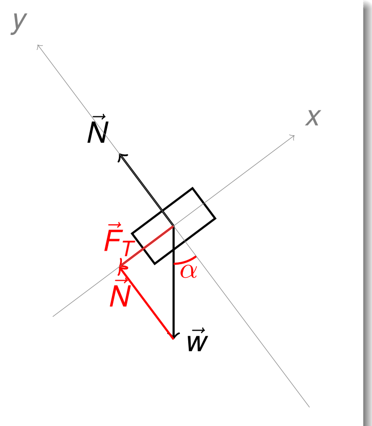
- The net force is:

$$\vec{F}_T = (N - mg\cos\alpha)\hat{y} - mg\sin\alpha\hat{x} \tag{42}$$

- The acceleration along the y direction should be zero, hence
$a_y = 0 \Longrightarrow F_{Ty} = 0$

$$N - mg\cos\alpha = 0 \Longrightarrow N = mg\cos\alpha \tag{43}$$

Example 3: Mass on an Inclined Plane



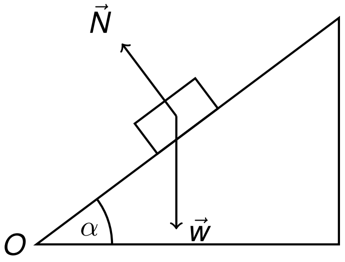
- The net force is:

$$\vec{F}_T = -mg\sin\alpha\hat{x} \tag{42}$$

- Using Newton's second law:

$$\vec{a} = \frac{\vec{F}_T}{m} = -g\sin\alpha\hat{x} \tag{43}$$

Example 3: Mass on an Inclined Plane



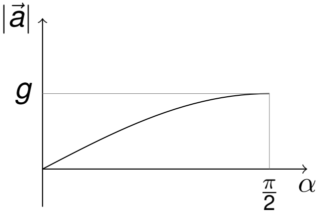
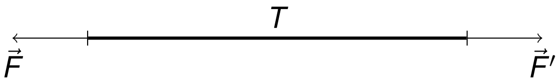


Figure : $|\vec{a}| = g \sin\alpha$

Tension of a String

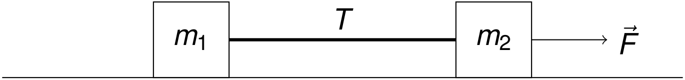


- Tension is the magnitude of the force acting on a string
- In the above figure, if the string has negligible mass ($m = 0$), than

$$\vec{F}_T = \vec{F} + \vec{F}' = m\vec{a} = 0 \tag{42}$$

- A massless string transfers force along its length without changing its magnitude.

Example: Two masses attached by a massless string



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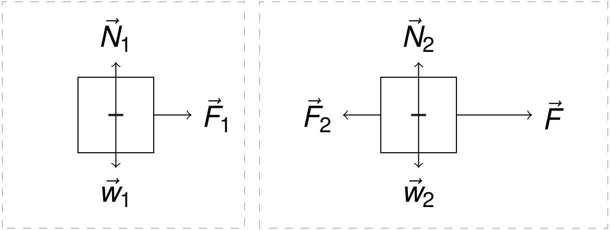
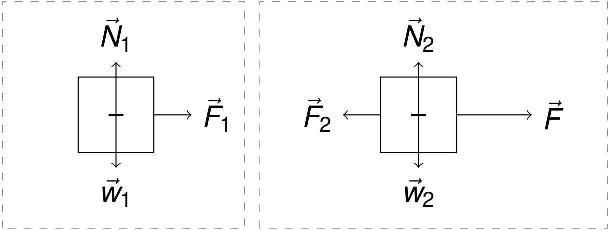


- No motion in the vertical direction: $\vec{w}_1 + \vec{N}_1 = 0$ and $\vec{w}_2 + \vec{N}_2 = 0$
- The string is massless $(-\vec{F}_1) + (-\vec{F}_2) = 0 \implies \vec{F}_2 = -\vec{F}_1$
- If the elasticity of the string is neglected, both masses should have the same acceleration: $\vec{F}_1 = m_1\vec{a}$, $\vec{F} + \vec{F}_2 = m_2\vec{a}$

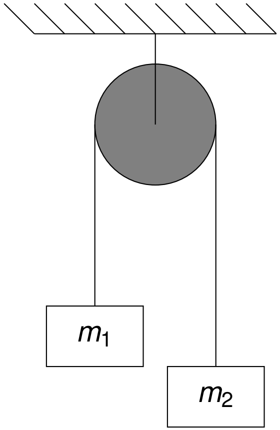
Example: Two masses attached by a massless string



- Since all the forces and accelerations are in the horizontal direction, I will only write the horizontal components of each vector

$$\left.\begin{aligned} F_1 &= m_1 a \\ F + F_2 &= F - F_1 = m_2 a \end{aligned}\right\} F = (m_1 + m_2) a \Longrightarrow a = \frac{F}{m_1 + m_2} \tag{43}$$

Atwood's Machine



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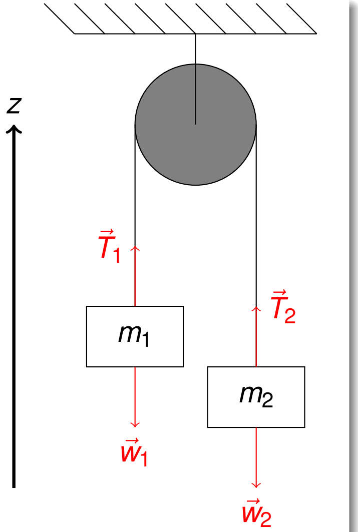
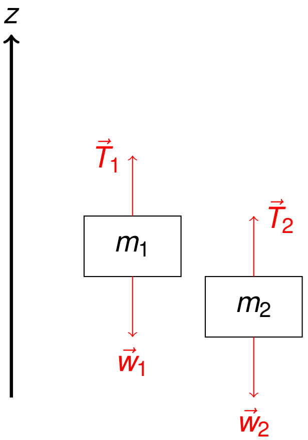
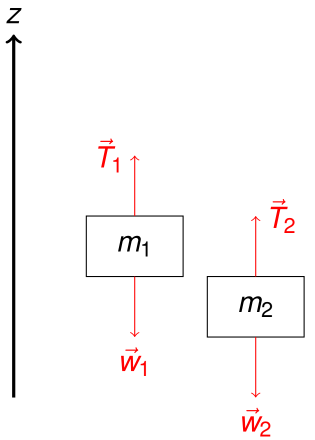


Atwood's Machine



- Let accelerations be $\vec{a}_1 = a_1\hat{z}$, and $\vec{a}_2 = a_2\hat{z}$.
- $\vec{T}_1 = T\hat{z}$, $\vec{T}_1 = T\hat{z}$ where T is the (unknown) tension of the string
- $\vec{a}_1 = a_1\hat{z}$, $\vec{a}_2 = a_2\hat{z}$ (a_i's are unknown)
- $\vec{w}_1 = -m_1 g\hat{z}$, $\vec{w}_2 = -m_2 g\hat{z}$
- For the masses m_1 and m_2: $\vec{T}_i + \vec{w}_i = m_i\vec{a}_i \rightarrow T - m_i g = m_i a_i$
- The velocities of the masses have to have equal magnitudes but opposite direction: $\vec{v}_1 = -\vec{v}_2 \rightarrow \vec{a}_1 = -\vec{a}_2$

Atwood's Machine

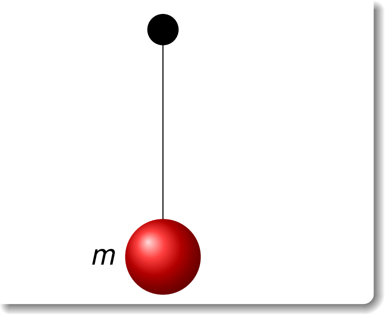


$$a_2 = -a_1 \tag{44}$$
$$T - m_1 g = m_1 a_1 \tag{45}$$
$$T - m_2 g = m_2 a_2 \tag{46}$$

- Subtracting the second equation from the third and using the first:

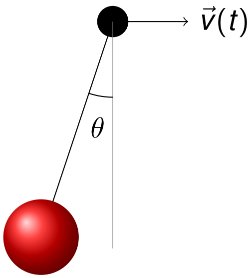
$$(m_1 - m_2)g = m_2 a_2 - m_1 a_1$$
$$= -(m_2 + m_1)a_1$$
$$\Longrightarrow a_1 = -\frac{m_1 - m_2}{m_1 + m_2} g \tag{47}$$

Accelerometer



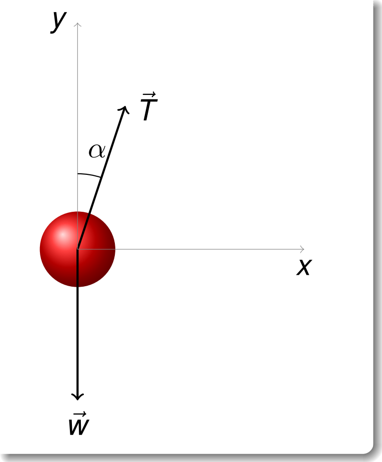
- A ball of mass m is suspended from a point by a massless string.

Accelerometer



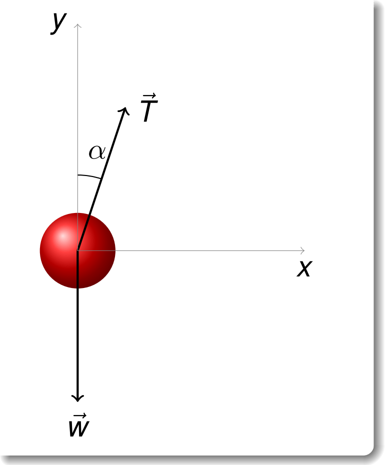
- A ball of mass m is suspended from a point by a massless string.
- If the suspension point starts to move with constant acceleration $\vec{a}$, what is the relation between the angle θ and $|\vec{a}| \equiv a$?

Accelerometer



- Once the oscillations settle down, the acceleration of the ball is equal to the acceleration of the suspension point $a\hat{x}$

Accelerometer



- $\vec{w} = -mg\hat{y}$, $\vec{T} = T\cos\alpha\hat{y} + T\sin\alpha\hat{x}$

$$\vec{F}_T = (T\cos\alpha - mg)\hat{y} + T\sin\alpha\hat{x} \tag{48}$$

- By Newton's second law: $\vec{F}_T = ma\hat{x}$:

$$T\cos\alpha - mg = 0 \Longrightarrow T = \frac{mg}{\cos\alpha} \tag{49}$$

$$T\sin\alpha = ma \Longrightarrow mg\tan\alpha = ma \tag{50}$$

- Hence $\tan\alpha = \frac{a}{g}$