Terminal Velocity

- In liquids and gases, friction is not constant, but velocity dependent.
- For small velocities $\vec{F}_D = -b\vec{v}$
- Consider a mass m left from rest at some height. (1D motion)
- Newton's second law:

$$mg - bv = ma \equiv m\frac{dv}{dt} \tag{71}$$

$$\tag{72}$$

- If $v = mg/b$, $a = 0$. The terminal velocity is $v_t = mg/b$

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- Integration both sides from t_i to t (v_i to $v(t)$)

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$$\Longrightarrow (t - t_i) = \frac{m}{b}\log\frac{mg - bv_i}{mg - bv(t)} \tag{73}$$

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- Solving for $v(t)$:

$$v(t) = v_t - e^{-\frac{b}{m}(t-t_i)}(v_t - v_i) \tag{73}$$

$$= v_i e^{-\frac{b}{m}(t-t_i)} + v_t \left(1 - e^{-\frac{b}{m}(t-t_i)}\right) \tag{74}$$

Kepler's Laws

Kepler's laws are based on observation only:

1. The orbit of planets around the sun are ellipses with the Sun positioned at one of the centers
2. The vector from the sun to the planet, sweeps equal areas at equal times
3. Let s_i and T_i, $i = 1, 2$ be the semi major axis and the period of rotation respectively, of two planets. Then

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^3 \tag{75}$$

or

$$\frac{T^2}{s^3} \tag{76}$$

is the same for every planet.

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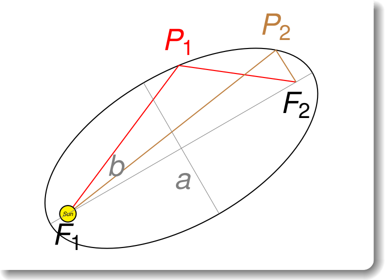
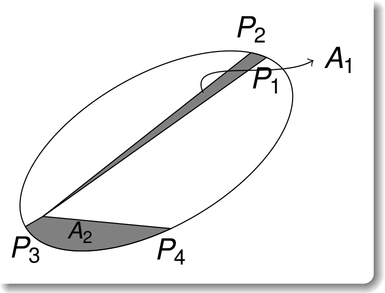
Kepler's First Law



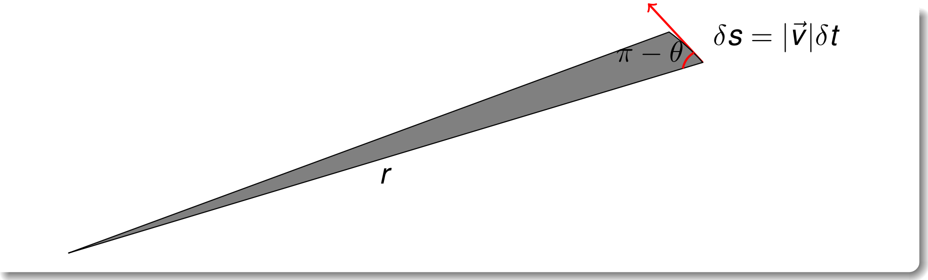
- a (b) is the semi minor (major) axis
- Definition of an ellipse: $|F_1P_1| + |P_1F_2| = |F_1P_2| + |P_2F_2| \equiv 2b$

Kepler's Second Law



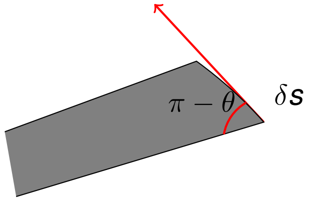
- t_{12} (t_{34}) time it takes for the planet to go from P_1 (P_3) to P_2 (P_4)
- If $t_{12} = t_{34}$ then $A_1 = A_2$.

Kepler's Second Law



- The area covered in time interval δt is $\delta A = \frac{1}{2} r \delta s \sin(\pi - \theta) = \frac{1}{2} r v \sin\theta \delta t$
- δA is the same independent of where the planet is on its orbit
- As the planet moves, $rv\sin\theta$ is constant.
- $rv\sin\theta = |\vec{r} \times \vec{v}|$

Kepler's Second Law



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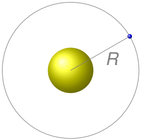

- $\frac{T^2}{s^3}$ is constant
- Consider a circular orbit $s = R$
- $T = \frac{2\pi R}{v}$
- Kepler's Law:

$$\left(\frac{2\pi R}{v}\right)^2 \left(\frac{1}{R^3}\right) = \frac{2\pi}{Rv^2} = \frac{2\pi}{R^2 \frac{v^2}{R}} \Longrightarrow \frac{v^2}{R} R^2 = constant \tag{77}$$

$$\Longrightarrow |\vec{F}| R^2 = constant \tag{78}$$

Kepler's second law implies that the central force decreases with the square of the distance

Newton's Law of Gravitation

- Kepler's third Law $\Longrightarrow F \propto \frac{1}{r^2}$
- Law of uniform gravitational acceleration $\Longrightarrow F = mg \propto m$
- Symmetry of forces (action reaction pairs) $\longrightarrow F \propto m_E$

$$|\vec{F}| = G_N \frac{m m_E}{r^2} \tag{79}$$

where m and m_E are the masses of two gravitating objects, r is the distance between their centres.

- $G_N = 6.67384 \times 10^{-11} N(m/kg)^2$

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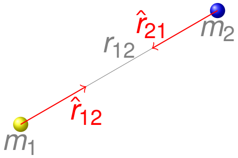
Newton's Law of Gravitation



- $\vec{F}_{12}$: Force acting on m_1 due to m_2

$$\vec{F}_{12} = G_N \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12} \qquad (80)$$

- $\vec{F}_{21}$: Force acting on m_2 due to m_1

$$\vec{F}_{21} = G_N \frac{m_2 m_1}{r_{21}^2} \hat{r}_{21} \equiv \vec{F}_{12} \qquad (81)$$

- On the surface of the Earth, the force acting on a mass m is:

$$|\vec{F}| = mg = G_N \frac{m m_E}{R_E}^2 \Longrightarrow g = G_N \frac{m_E}{R_E^2} \qquad (82)$$

- $\vec{F}_{12} = G_N \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$ is valid for point masses
- If various masses m_i exert gravitational attraction on a mass M, the total force acting on M is:

$$\vec{F} = G_N \sum_i \frac{m_i M}{r_i^2} \hat{r}_i \qquad (83)$$

where r_i is the distance of mass m_i from M, and $\hat{r}_i$ is the unit vector pointing from M towards m_i.