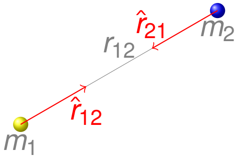
Newton's Law of Gravitation



- $\vec{F}_{12}$: Force acting on m_1 due to m_2

$$\vec{F}_{12} = G_N \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12} \tag{80}$$

- $\vec{F}_{21}$: Force acting on m_2 due to m_1

$$\vec{F}_{21} = G_N \frac{m_2 m_1}{r_{21}^2} \hat{r}_{21} \equiv \vec{F}_{12} \tag{81}$$

- On the surface of the Earth, the force acting on a mass m is:

$$|\vec{F}| = mg = G_N \frac{m m_E}{R_E}^2 \Longrightarrow g = G_N \frac{m_E}{R_E^2} \tag{82}$$

Measured value of g is position dependent:

- Shape of earth is not a sphere
- Mass density is not uniform
- Earth is rotating, i.e. any reference frame fixed on the surface of the Earth is non-inertial. At the poles, g would be measured larger than on the equator.

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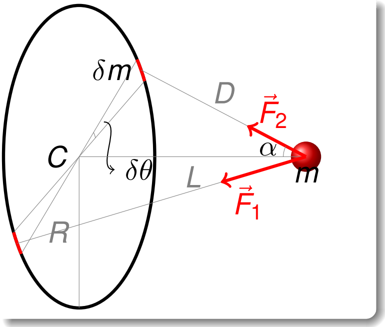
- $\vec{F}_{12} = G_N \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$ is valid for point masses
- If various masses m_i exert gravitational attraction on a mass M, the total force acting on M is:

$$\vec{F} = G_N \sum_i \frac{m_i M}{r_i^2} \hat{r}_i \tag{83}$$

where r_i is the distance of mass m_i from M, and $\hat{r}_i$ is the unit vector pointing from M towards m_i.

- Superposition of forces is valid only in Newton's Theory of gravity
- Superposition of forces is not valid on General Theory of Relativity

Gravitational Force of a Ring on a Mass



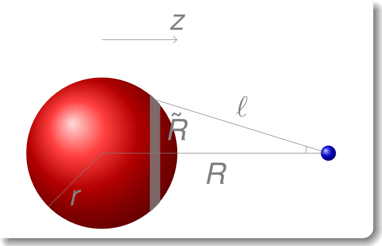
- $\delta m = M\frac{\delta\theta}{2\pi}$
- $|\vec{F}_1| = |\vec{F}_2| = G_N\frac{\delta m m}{D^2}$
- $\vec{F}_1 + \vec{F}_2$ will point from the mass m to the center of the ring with a magnitude $F_{\parallel} = G_N\frac{(2\delta m)m}{D^2}\cos\alpha$
- $F_{\parallel}$ does not depend on which δm along the ring is considered
- $\cos\alpha = \frac{L}{D}$

- Summing over all δm gives

$$|\vec{F}| = G_N\frac{Mm}{D^2}\cos\alpha = G_N\frac{MmL}{D^3} = G_N\frac{MmL}{(L^2+R^2)^{\frac{3}{2}}} \quad (84)$$

- $\vec{F}$ points towards the center of the ring

Gravitational Force of a Shell on a Mass



- The ring has has its center at $z = k$ and sees and angle $d\theta$.
- The area of the ring: $dA = 2\pi\tilde{R}rd\theta$
- $\tilde{R}^2 + (R-k)^2 = \ell^2$, $\tilde{R}^2 + k^2 = r^2$
- The mass of the ring: $dM = \frac{dA}{4\pi r^2}M = \frac{\tilde{R}M}{2r}d\theta$
- The force due to the ring:

$$d\vec{F} = -G_N dM \frac{mL}{\ell^3}\hat{z} = -G_N \frac{Mm}{r}\frac{(R-k)\tilde{R}}{2\ell^3}d\theta\hat{z} \quad (85)$$

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Gravitational Force of a Shell on a Mass

k

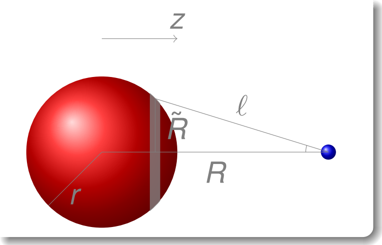
θ

ATTENTION

In deciding the sphere into rings, once can either assume that *dk* is constant, or $rd\theta$ is constant for each ring. When the ring is opened into an approximate rectangle, $rd\theta$ is the height of this rectangle. Hence it is better (and simpler) to assume that $rd\theta$ is constant for each ring. This implies that *dk*, (i.e. the distance between the centers of the inner and the outer circles of the ring) i not constant. If one takes *dk* constant, especially closer to the point $k = R$, when you open the ring, it will no longer look like a rectangle

$\ell^3 \quad r \quad 2\ell^3$ (85)

Gravitational Force of a Shell on a Mass



$$d\vec{F} = -G_N \frac{Mm}{r} \frac{(R-k)\tilde{R}}{2\ell^3} d\theta \hat{z} \qquad (85)$$

$$\ell^2 = r^2 + R^2 - 2rR\cos\theta \qquad (86)$$

$$k = r\cos\theta = \frac{R}{2} + \frac{r^2 - \ell^2}{2R} \qquad (87)$$

$$\tilde{R} = r\sin\theta \qquad (88)$$

- $\ell d\ell = rR\sin\theta d\theta = R\tilde{R}d\theta \Longrightarrow d\theta = \frac{\ell}{R\tilde{R}} d\ell$

$$d\vec{F} = -G_N \frac{Mm}{rR} \frac{(R-k)}{2\ell^2} d\ell \qquad (89)$$

$$= -G_N \frac{Mm}{2rR} \left(\frac{R}{2} - \frac{r^2 - \ell^2}{2R} \right) \frac{d\ell}{\ell^2} \qquad (90)$$

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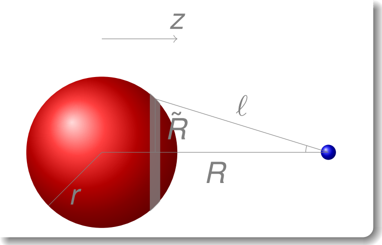
Gravitational Force of a Shell on a Mass



$$d\vec{F} = -G_N \frac{Mm}{2rR}\left(\frac{R}{2} - \frac{r^2-\ell^2}{2R}\right)\frac{d\ell}{\ell^2}\hat{z}$$

- If the point is outside the shell, $R - r \leq \ell \leq R + r$,
  $\vec{F} = \int_{R-r}^{R+r} d\ell(\cdots) = -G_N \frac{Mm}{R^2}\hat{z}$
- If the point is inside the shell, $r - R \leq \ell \leq R + r$,
  $\vec{F} = \int_{r-R}^{R+r} d\ell(\cdots) = 0$
- Inside the shell, zero gravitational attraction, outside the shell, shell acts as if all its mass in at its center

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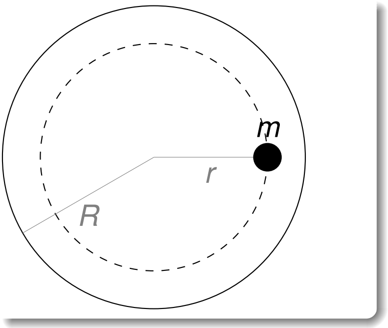
Planet Vulcan

Planet Vulcan was hypothesized to exist between the Sun and Mercury

Dark Matter

Consider a simple model of a galaxy as a sphere of uniform mass density. The galaxy will spiral around its center. consider a star on its equilateral plane at a distance r from the center. Calculate and sketch its speed as a function of r.



- If the star is inside the galaxy, block will feel the the attraction for only the mass inside the sphere of radius r: $m(r) = \frac{4\pi}{3}\rho r^3$
- This is the centripetal force:

$$G_N \frac{m\left(\frac{4\pi}{3}\rho r^3\right)}{r^2} = m\frac{v^2}{r} \quad (85)$$

$$\Longrightarrow v = r\left(G_N \frac{4\pi}{3}\rho\right)^{\frac{1}{2}} \quad (86)$$

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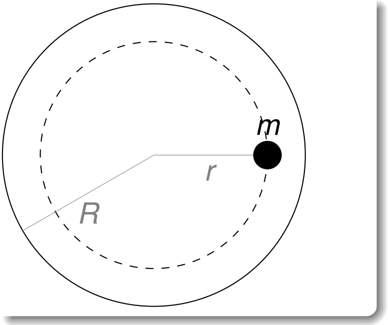

- If the star is outside the galaxy, then

$$G_N \frac{m\left(\frac{4\pi}{3}\rho R^3\right)}{r^2} = m\frac{v^2}{r} \tag{85}$$

$$\Longrightarrow v = \frac{1}{\sqrt{r}}\left(G_N \frac{4\pi}{3} R^3\right) \tag{86}$$

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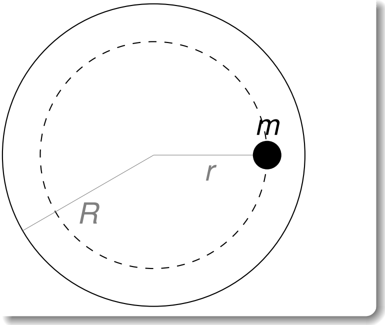

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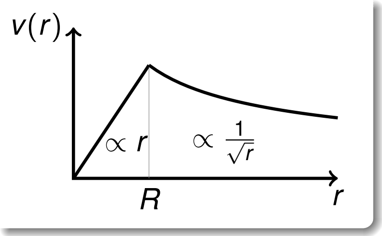
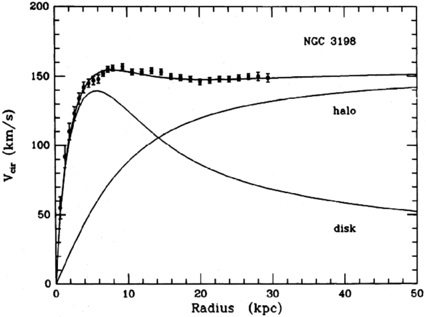
Dark Matter

The galaxy appears to have a larger mass than is seen





Gravitational Field

- According to Newton's Gravitation Law, gravity acts at a distance
- To avoid the concept of action at a distance, gravitational *field* is hypothesized
- Every mass, M, creates a field $\vec{g}$ around it given by

$$\vec{g} = -G_N \frac{M}{r^2}\hat{r} \tag{85}$$

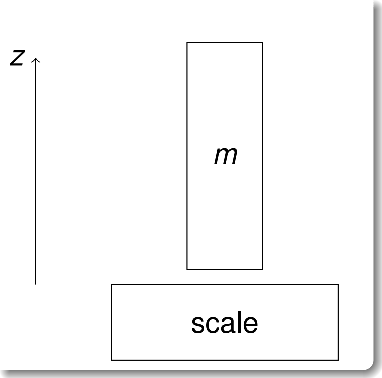
- Every other mass m placed in this field, feels a force due to the field at its location given by

$$\vec{F} = m\vec{g} \tag{86}$$

Equivalence Principle

- Mass appears in Newton's second Law: $\vec{F} = m\vec{a}$: *inertial* mass
- Mass appears in Newton's law of gravity: $\vec{g} = -G_N \frac{m}{r^2}\hat{r}$: *gravitational* mass
- Equivalence principle: gravitational mass and inertial mass are equal. WHY?
- Einstein's theory of relativity relies on this equality

Weightlessness



- The forces acting on the mass m are:

$$\vec{w} = -mg\hat{z} \tag{87}$$

$$\vec{N} = N\hat{z} \tag{88}$$

$$\vec{F}_T = (N - mg)\hat{z} \tag{89}$$

where g is the gravitational acceleration at the point of the mass m.

- In a satellite, whole system accelerates. If the acceleration is also in the $\hat{z}$ direction, $\vec{F}_T = m\vec{a} \equiv ma\hat{z}$
- Then $N - mg = ma \Longrightarrow N = m(g + a)$
- If $a = -g$, the object appears massless

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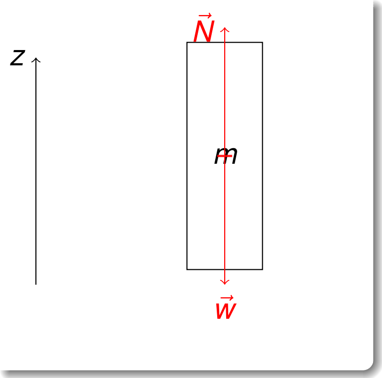
Weightlessness



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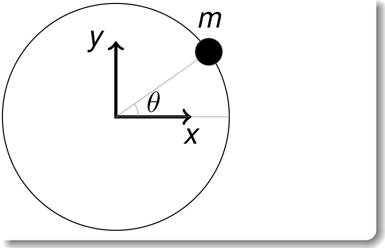
QUIZ 3

Q: Consider a mass m attached at the end of a massless string. The string has a length L. The mass is moving uniformly around a circle of radius R. (ignore gravity and friction)



1. Draw the free body diagram at the shown instant.
2. Write the force(s) acting on the mass m in terms of their components, i.e. in the form $\vec{F} = F_x\hat{x} + F_y\hat{y}$. Use the given coordinate axes.
3. Find a relation between T, L and v.