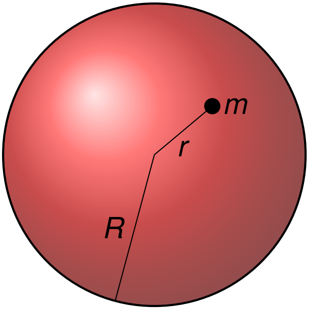
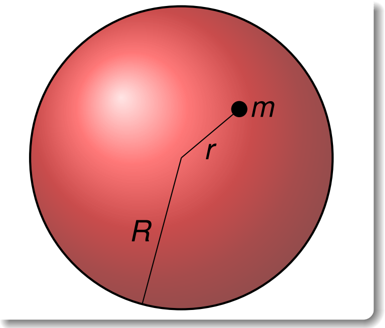
Example: A mass Inside a Spherical Mass



The shown sphere has a radius R and a mass M uniformly distributed over its surface. Another mass m is placed at a distance $r < R$ from the center. What will be the gravitational force that the object will feel?

Example: A mass Inside a Spherical Mass



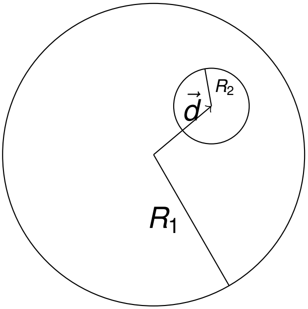
- Divide the sphere into an inner sphere with radius r and outer shell.
- Outer shell will not exert any force.
- Inner shell will have a mass:

$$M(r) = \frac{M}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = M\left(\frac{r}{R}\right)^3$$

- The force exerted by the inner shell is

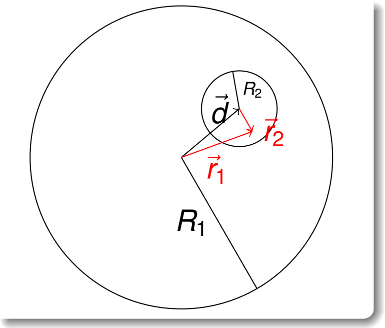
$$\vec{F} = -G_N M\left(\frac{r}{R}\right)^3 \frac{m}{r^2}\hat{r} = -G_N\frac{Mm}{R^3}\vec{r}$$

Example: A sphere with another sphere carved out



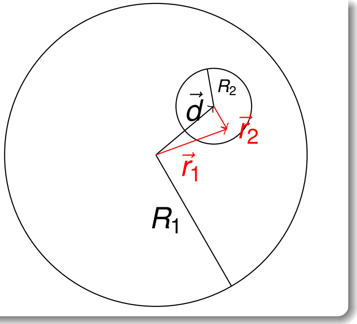
A sphere of radius R_2 is carved out of another sphere of radius R_1. The position of the center of the carved sphere is denoted by $\vec{d}$. The mass density of the system is ρ. If a mass m is placed inside the cavity, what will be the force that this mass m will feel?

Example: A sphere with another sphere carved out



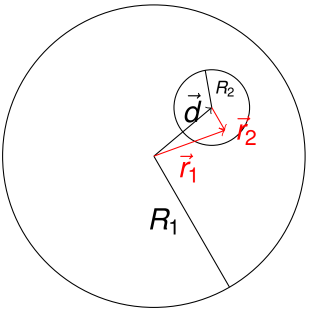
- Let $\vec{r}_1$ ($\vec{r}_2$) be the position of the mass m relative to the center of the large (small) sphere.

Example: A sphere with another sphere carved out



- Let $\vec{r}_1$ ($\vec{r}_2$) be the position of the mass m relative to the center of the large (small) sphere.
- $\vec{F}_{full\ sphere} = \vec{F}_T + \vec{F}_{carved\ out\ mass} \implies$ $\vec{F}_T = \vec{F}_{full\ sphere} - \vec{F}_{carved\ out\ mass}$
- The cavity can be modeled as a mass with mass density $-\rho$.

Example: A sphere with another sphere carved out



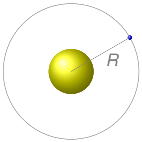
- $\vec{F}_{full\ sphere} = \vec{F}_T + \vec{F}_{carved\ out\ mass} \Longrightarrow$
  $\vec{F}_T = \vec{F}_{full\ sphere} - \vec{F}_{carved\ out\ mass}$
- The cavity can be modeled as a mass with mass density $-\rho$.
- Large sphere:
  $\vec{F}_L = -G_N \frac{\rho \frac{4}{3}\pi r_1^3}{r_1^2}\hat{r}_1 = -\frac{4\pi}{3}G_N \rho \vec{r}_1$
- Small sphere: $\vec{F}_s = -\frac{4\pi}{3}G_N(-\rho)\vec{r}_2$

$\vec{F}_T = \vec{F}_L + \vec{F}_s = -\frac{4\pi}{3}G_N\rho(\vec{r}_1 - \vec{r}_2) = \frac{4\pi}{3}G_N\rho\vec{d}$

Gravitational attraction is uniform inside the cavity.

CHALLENGE: Can you prove this without using vectors? (not recommended)

Example: Circular Orbits



- Let M_E (M_s) be the mass of Earth (satellite)
- What is the speed of the satellite?
- $m\frac{v^2}{R} = G_N\frac{Mm}{R^2} \Longrightarrow v = \sqrt{\frac{G_N M}{R}}$
- The closer the satellite is to the Earth, the faster it should be.
- The period of the satellite is

$$T = \frac{2\pi R}{v} = \frac{2\pi}{\sqrt{G_N M}} R^{\frac{3}{2}} \Longrightarrow \frac{T^2}{R^3} = \frac{2\pi}{\sqrt{G_N M}}$$

- Measuring the ratio T^2/R^3, it is possible to determine the mass of the sun.

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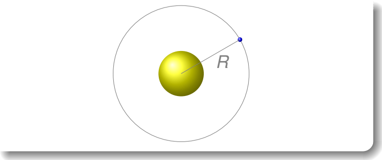

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- Geocentric orbits are those for which the relative position of the satellite is fixed with respect to the surface of the planet.