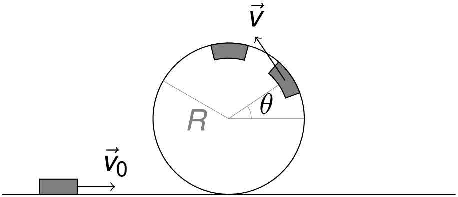
Roller Coaster

Track of a Roller Coaster



$$\frac{1}{2}mv^2 + mgh = const, \vec{F}_{fr} = \vec{0}$$

Q: Assume that the cart is going with a very slow speed v. What is the maximum height that it can reach on the circular track?

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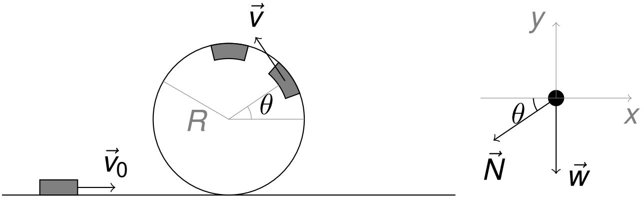

$$\frac{1}{2}mv^2 + mgh = const,\ \vec{F}_{fr} = \vec{0}$$

Q: Assume that the cart is going with a very slow speed v. What is the maximum height that it can reach on the circular track?

- At the maximum height, its speed is zero $v = 0$:
  $$\frac{1}{2}mv_0^2 + mg0 = \frac{1}{2}m0^2 + mgh_{max} \Longrightarrow h_{max} = \frac{v_0^2}{2g}$$
- Same as if it was thrown vertically upwards!

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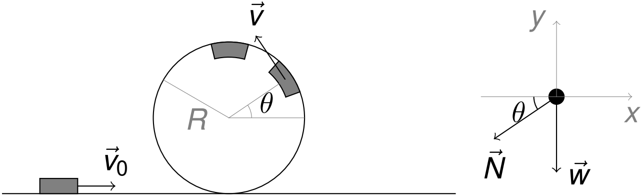

$$\frac{1}{2}mv^2 + mgh = const, \ \vec{F}_{fr} = \vec{0}$$

Q: What is the minimum speed with which the cart should have at the bottom so that it can go around the top of the loop?

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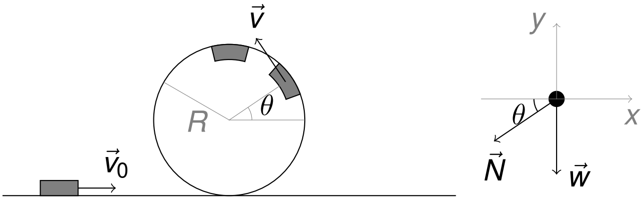

$$\frac{1}{2}mv^2 + mgh = const,\ \vec{F}_{fr} = \vec{0}$$

Q: What is the minimum speed with which the cart should have at the bottom so that it can go around the top of the loop?

- $h_{max} = 2R \Longrightarrow v_0^2 = 2g(2R) = 4gR$

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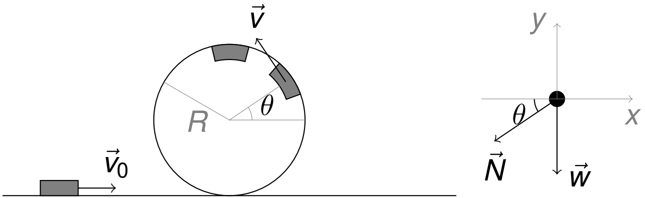

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- WRONG

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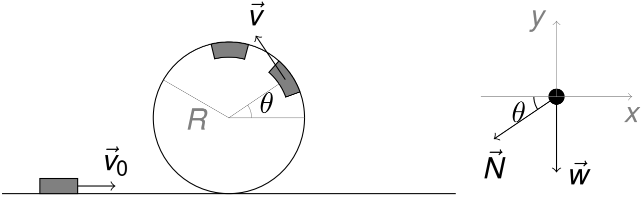

$$\frac{1}{2}mv^2 + mgh = const, \vec{F}_{fr} = \vec{0}$$

Q: What is the minimum speed with which the cart should have at the bottom so that it can go around the top of the loop?

- $h_{max} = 2R \Longrightarrow v_0^2 = 2g(2R) = 4gR$
- WRONG
- If $v_0^2 = 4gR$, the cart reaches the top with zero velocity. It can not go round!

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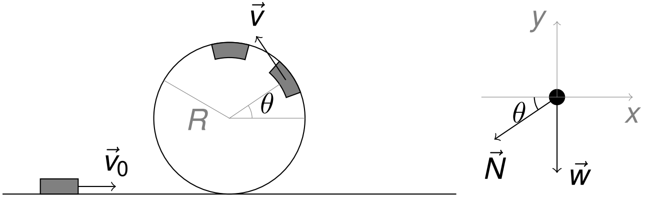

$$\frac{1}{2}mv^2 + mgh = const,\ \vec{F}_{fr} = \vec{0}$$

Q: What is the minimum speed with which the cart should have at the bottom so that it can go around the top of the loop?

- At the top $\vec{N} = -N\hat{y},\ \vec{w} = -mg\hat{y} \Longrightarrow \vec{F}_T = -(N+mg)\hat{y}$
- For circular motion at the top
  $\vec{F} \equiv -m\frac{v^2}{R}\hat{y} \Longrightarrow v^2 = (N+mg)R/m \geq gR$
- At the threshold of falling off the track, $N = 0 \Longrightarrow v_{min}^2 = gR$.
- At the bottom of the roller coaster $\frac{1}{2}mv_{0min}^2 = \frac{1}{2}mv_{min}^2 + mg(2R)$
  $\Longrightarrow v_{0min}^2 = 5gR$

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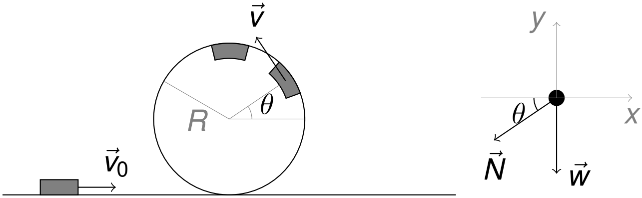

$$\frac{1}{2}mv^2 + mgh = const, \vec{F}_{fr} = \vec{0}$$

Q: If the cart is moving with a speed less then the minimum speed, at what point will it leave the track?

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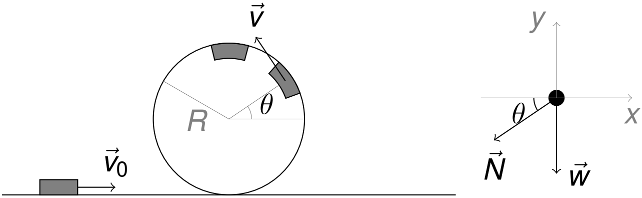

$$\frac{1}{2}mv^2 + mgh = const, \; \vec{F}_{fr} = \vec{0}$$

Q: If the cart is moving with a speed less then the minimum speed, at what point will it leave the track?

- Assume it falls at angle $\theta > 0$.
- At that point, the central force is only due to $\vec{w}$, $w_r = mg\sin\theta$
- For circular motion: $m\frac{v^2}{R} = w_r = mg\sin\theta \implies v^2 = gR\sin\theta$.
- $\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgR(1+\sin\theta) = \frac{1}{2}mgR(3\sin\theta + 2)$
  $\implies \sin\theta = \frac{v_0^2 - 2gR}{3gR}$

MOON

It is known that moon always shows its same face to the Earth. This is because the period of rotation of the moon around its axes is equal to its period of rotation around Earth. Can you come up with an explanation of this equality?
Keywords to consider: tides, friction, work