- Ask questions: let me know how you perceive nature so that if there is a misperception, we can correct it
- Ask simple questions
- if you have any doubt, repeat what you have understood. It need not be in the form of a question
- Don't try to guess the type of questions that I can ask, try to understand nature
- Bad habits that you have learned in years takes more than a few months to correct!

The only forces that we will study in this year are gravity, and EM force. EM force represents itself through

- Friction
- Any pull or a push (e.g. Normal Force, forced due to the tension on a string, force acting by a spring)

TRACKER Software (http://www.cabrillo.edu/ dbrown/tracker/)

# Review of Work Done By Gravity

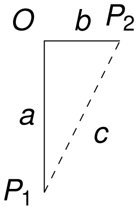
- $W_{tot} = \Delta T$ where $T = \frac{1}{2}mv^2$
- $W_G = -mg\Delta h$ depends only on the height difference, and on nothing else
- If an object moves under the influence of gravity only then throughout the motion

$$\frac{1}{2}mv^2 + mgh = const \tag{97}$$

# Work Done By Gravity

- The force acting on an object of mass $m$ is $\vec{F}_w = -mg\hat{z}$ ($\hat{z}$ points upwards)
- If the object is displaced by $d\vec{\ell}$, then $dW = \vec{F}_w \cdot d\vec{\ell} = -mgdz$, i.e. the work done by its weight is proportional to the change in its $z$ coordinate (its height)
- As the object goes from $P_1$ to $P_2$, the to calculate the total work, just sum the changes in its height. Hence total work is $W_{tot} = -mg\Delta h$

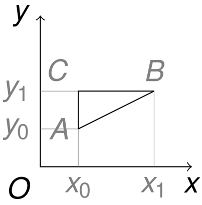
# Work Done By Friction




- Assume a constant friction force of magnitude $F_f$.
- $a$, $b$ and $c$ are the corresponding length.
- $W_{P_1 \to O} = -F_f a$
- $W_{O \to P_2} = -F_f b$
- $W_{P_1 \to O \to P_2} = -F_f(a+b)$
- $W_{P_1 \to P_2} = -F_f c \neq -F_f(a+b)$

- Hence the work done by friction depends on how one goes from the initial point to the final point

# Example: $\vec{F} = ax\hat{x} + by\hat{y}$




$\vec{F} = ax\hat{x} + by\hat{y}$

Work done by the force as one goes from $A$ to $B$ through $C$:

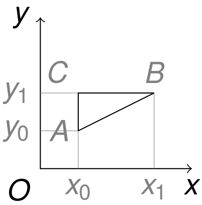
$W_{A\to C\to B} = W_{A\to C} + W_{C\to B}$

- Along the path $A \to C$, $x = x_0$ and hence $\vec{F} = ax_0\hat{x} + by\hat{y}$, $d\vec{\ell} = dy\hat{y}$
- $dW = \vec{F}\cdot d\vec{\ell} = bydy$

$$\int_0^{W_{A\to C}} dW = \int_{y_0}^{y_1} bydy \tag{98}$$

$$W_{A\to C} = \frac{1}{2}by_1^2 - \frac{1}{2}by_0^2 \tag{99}$$

# Example: $\vec{F} = ax\hat{x} + by\hat{y}$




$\vec{F} = ax\hat{x} + by\hat{y}$

$W_{A\to C} = \frac{1}{2}by_1^2 - \frac{1}{2}by_0^2$

Work done by the force as one goes from $A$ to $B$ through $C$:

$W_{A\to C\to B} = W_{A\to C} + W_{C\to B}$

- Along the path $A \to C$, $y = y_1$ and hence $\vec{F} = ax\hat{x} + by_1\hat{y}$, $d\vec{\ell} = dx\hat{x}$
- $dW = \vec{F} \cdot d\vec{\ell} = axdx$

$$\int_0^{W_{C\to B}} dW = \int_{x_0}^{x_1} axdx \tag{98}$$

$$W_{C\to B} = \frac{1}{2}ax_1^2 - \frac{1}{2}ax_0^2 \tag{99}$$

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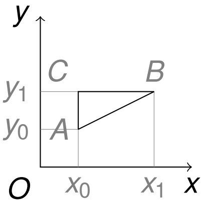
# Example: $\vec{F} = ax\hat{x} + by\hat{y}$

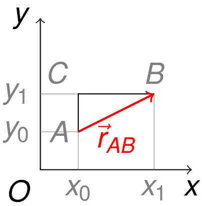



$$\vec{F} = ax\hat{x} + by\hat{y}$$
$$W_{A\to C} = \tfrac{1}{2}by_1^2 - \tfrac{1}{2}by_0^2$$
$$W_{C\to B} = \tfrac{1}{2}ax_1^2 - \tfrac{1}{2}ax_0^2$$

Work done by the force as one goes from $A$ to $B$ through $C$:

$$W_{A\to C\to B} = W_{A\to C} + W_{C\to B}$$
$$W_{A\to C\to B} = \left(\tfrac{1}{2}ax_1^2 + \tfrac{1}{2}by_1^2\right) - \left(\tfrac{1}{2}ax_0^2 + \tfrac{1}{2}by_0^2\right)$$

# Example: $\vec{F} = ax\hat{x} + by\hat{y}$




$\vec{F} = ax\hat{x} + by\hat{y}$

$W_{A\to C} = \frac{1}{2}by_1^2 - \frac{1}{2}by_0^2$

$W_{C\to B} = \frac{1}{2}ax_1^2 - \frac{1}{2}ax_0^2$

Work done by the force as one goes from $A$ to $B$ through a straight line:

- Let $\vec{r}_A = x_0\hat{x} + y_0\hat{y}$, $\vec{r}_B = x_1\hat{x} + y_1\hat{y}$, $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$
- Any point on the trajectory can be written as $\vec{r} = \vec{r}_A + \lambda\vec{r}_{AB}$; $0 \le \lambda \le 1$
- $d\vec{\ell} = (d\lambda)\vec{r}_{AB}$, $dW = \vec{F}\cdot d\vec{\ell} = d\lambda\vec{F}\cdot\vec{r}_{AB} = d\lambda(a(x_1\lambda + x_0(1-\lambda))(x_1 - x_0) + b(y_1\lambda + y_0(1-\lambda)(y_1 - y_0))$
- $W_{A\to B} = \int_0^{W_{A\to B}} dW = \int_0^1 d\lambda(\cdots) = \left(\frac{1}{2}ax_1^2 + \frac{1}{2}by_1^2\right) - \left(\frac{1}{2}ax_0^2 + \frac{1}{2}by_0^2\right)$

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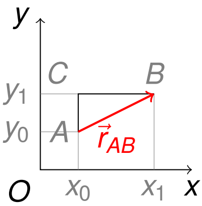
# Example: $\vec{F} = ax\hat{x} + by\hat{y}$




$\vec{F} = ax\hat{x} + by\hat{y}$
$W_{A\to C} = \frac{1}{2}by_1^2 - \frac{1}{2}by_0^2$
$W_{C\to B} = \frac{1}{2}ax_1^2 - \frac{1}{2}ax_0^2$

Work done by the force as one goes from $A$ to $B$ through a straight line:

- Let $\vec{r}_A = x_0\hat{x} + y_0\hat{y}$, $\vec{r}_B = x_1\hat{x} + y_1\hat{y}$, $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$
- Any point on the trajectory can be written as $\vec{r} = \vec{r}_A + \lambda\vec{r}_{AB}$; $0 \leq \lambda \leq 1$
- $d\vec{\ell} = (d\lambda)\vec{r}_{AB}$, $dW = \vec{F}\cdot d\vec{\ell} = d\lambda\vec{F}\cdot\vec{r}_{AB} = d\lambda(a(x_1\lambda + x_0(1-\lambda))(x_1 - x_0) + b(y_1\lambda + y_0(1-\lambda)(y_1 - y_0))$
- $W_{A\to B} = \int_0^{W_{A\to B}} dW = \int_0^1 d\lambda(\cdots) = \left(\frac{1}{2}ax_1^2 + \frac{1}{2}by_1^2\right) - \left(\frac{1}{2}ax_0^2 + \frac{1}{2}by_0^2\right)$

Work done as one goes from $A$ to $B$ is the same in both paths. And can be written as $W_{A\to B} = U(x_0, y_0) - U(x_1, y_1)$

# Conservative Forces

- **Definition:** A force is conservative if the work done by that force as an object moves from a point $P_1$ to a point $P_2$ is independent of the path that the object takes.
- Friction is an example of a ***non-conservative*** force.
- Gravity (any constant force in general) is a ***conservative*** force.
- $\vec{F} = ax\hat{x} + by\hat{y}$ is a conservative force

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- If $\vec{F}$ is conservative $W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{\ell}$ is independent of how one goes from $P_1$ to $P_2$.
- To evaluate $W$, one can *choose* any path
- Define a function $U(P)$ such that $U(P) = U(P_0) - \int_{P_0}^{P_1} \vec{F} \cdot d\vec{\ell}$.
- $W = \Delta T$
- Suppose the object moves from $P_1$ to $P_2$ under the influence of the conservative force $\vec{F}$.
- The work done by $\vec{F}$ is:

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{\ell} = \int_{P_1}^{P_0} \vec{F} \cdot d\vec{\ell} + \int_{P_0}^{P_2} \vec{F} \cdot d\vec{\ell} \tag{98}$$

$$= (U(P_1) - U(P_0)) + (U(P_0) - U(P_2)) = U(P_1) - U(P_2) \tag{99}$$

- $W = T_2 - T_1$:

$$T_2 - T_1 = U(P_1) - U(P_2) \tag{100}$$

$$T_1 + U(P_1) = T_2 + U(P_2) \tag{101}$$

# Conservation of Energy

- $U$ is called the potential energy
- $T + U$ is called the mechanical energy
- For an object moving under the influence of a conservative force only $T + U$ is always conserved.
- An object that is raised by $h$, has ***potential*** to do work, it has a larger ***potential*** energy
- Potential energy is **NOT** a property of a single object, but a property of the ***system*** as a whole.

- Newton's three laws are vector relations
- Conservation of energy is a scalar relation
- If the expression for potential energy is known, speed at any point can be determined without solving any differential equation or integrals.

# Force from Potential

- Consider two nearby points separated by the vector $d\vec{r}$.
- The difference in their potential energies is:

$$U(\vec{r}+d\vec{r})-U(\vec{r})=-\vec{F}(\vec{r})\cdot d\vec{r} \tag{102}$$

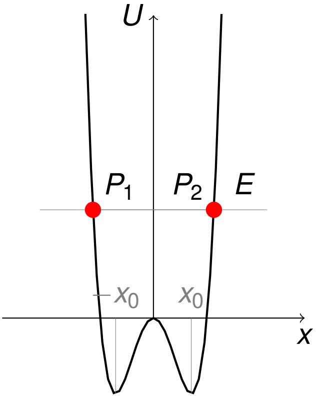
- Above is valid for any $d\vec{r}$. Denote $U(\vec{r})\equiv U(x,y,z)$ if $\vec{r}=x\hat{x}+y\hat{y}+z\hat{z}$
- If $d\vec{r}=dx\hat{x}$, then

$$U(x+dx,y,z)-U(x,y,z)=-F_x(x,y,z)dx \tag{103}$$

$$\Longrightarrow F_x(x,y,z)=-\frac{U(x+dx,y,z)-U(x,y,z)}{(x+dx)-x}\equiv-\frac{\partial U}{\partial x} \tag{104}$$

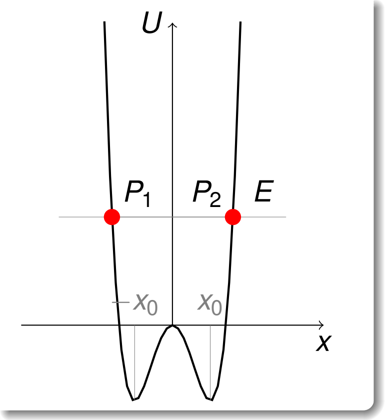
- Similarly $F_y=-\frac{\partial U}{\partial y}$ and $F_z=-\frac{\partial U}{\partial z}$
- Hence $\vec{F}=-(\hat{x}\frac{\partial U}{\partial x}+\hat{y}\frac{\partial U}{\partial y}+\hat{z}\frac{\partial U}{\partial z})\equiv-\vec{\nabla}U$ where $\vec{\nabla}$ is the *nabla* operator.

# Interpreting Potential Graphs



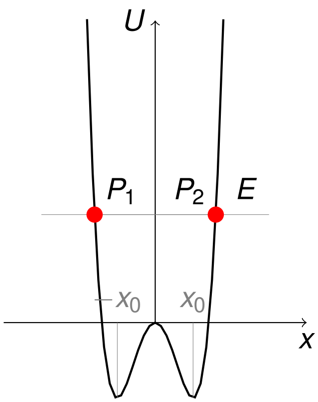

- Consider one dimensional example $F_x = -\frac{dU}{dx}$, i.e. $F_x$ points in the direction that $U(x)$ is decreasing
- Consider a particle with total energy $E$. Then $K = E - U > 0$, i.e. The particle can only be at points for which $U(x) < E$.
- points $x$ such that $U(x) = E$ are called turning points

# Interpreting Potential Graphs




- At points $x = \pm x_0$, and $x = 0$, the force acting on an object is zero: they are called equilibrium points.
- If an object is slightly displaced from $x = \pm x_0$, they try to move towards $\pm x_0$: they are called stable equilibrium points
- If an object is slightly displaced from $x = 0$, they try to move away from $x = 0$: $x = 0$ is an unstable equilibrium point.

# Interpreting Potential Graphs




- If an object is slightly displaced from $x = \pm x_0$, they try to move towards $\pm x_0$: they are called stable equilibrium points
- If an object is slightly displaced from $x = 0$, they try to move away from $x = 0$: $x = 0$ is an unstable equilibrium point.
- If an object is displaced slightly from an equilibrium point, it neither goes towards nor away from the equilibrium point, it is called neutral equilibrium point.

# Dissipative Forces

- Dissipative forces in fact convert mechanical energy to internal energy
- The total energy of the universe is constant
- $\Delta(T+U) = W_{dissipative\ forces}$

# Gravitational Potential Energy and Escape Velocity

- Gravitational force acting on a mass *m*, due to a mass *M* is:

$$\vec{F} = -G_N \frac{mM}{r^2}\hat{r} \tag{105}$$

- The work done when *m* is displaced by $d\vec{\ell}$ is

$$dW = \vec{F}\cdot d\vec{r} = -G_N \frac{mM}{r^2}\hat{r}\cdot d\vec{\ell} = -G_N \frac{mM}{r^2}dr \tag{106}$$

  where *dr* is the change in the radial distance, i.e. radial component if $d\vec{\ell}$.
- Potential energy difference is

$$U(P) - U(\infty) = -\int_\infty^P \left(-G_N \frac{mM}{r^2}dr\right) \tag{107}$$

$$= -G_N \frac{mM}{r}\Big|_{r=\infty}^{r=r_P} = -G_N \frac{mM}{r_P} \tag{108}$$

- In general $U(\infty)$ is chosen to be zero $U(\infty) = 0$

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# Gravitational Potential Energy and Escape Velocity

**Q:** What should be the initial speed of an object on the surface of a planet of mass $M$ and radius $R$, if it is to go until infinity?

- Let $v_0$ and $v_\infty$ be the initial and final speeds.
- Initial mechanical energy is $E = \frac{1}{2}mv_0^2 + G_N\frac{mM}{R}$.
- Final mechanical energy is $E = \frac{1}{2}mv_\infty^2$
- Conservation of mechanical energy:

$$\frac{1}{2}mv_0^2 - G_N\frac{mM}{R} = \frac{1}{2}mv_\infty^2 \tag{109}$$

$$\Longrightarrow v_0^2 = \frac{2}{m}\left(\frac{1}{2}mv_\infty^2 + G_N\frac{mM}{R}\right) \tag{110}$$

- The minimum possible speed is called the escape velocity:

$$v_{esc} = \sqrt{G_N\frac{2M}{R}};\ v_{esc,Earth} = 11.2\ km/s = 40320\ km/h \tag{111}$$

# Potential Energy of a Spring

- The work done by a spring on an object as it moves from $x = 0$ to $x = L$ was calculated as $W = -\frac{1}{2}kL^2$.
- The potential energy of a spring that is stretched by $L$ is $U(L) = \frac{1}{2}kL^2$