

Impulse

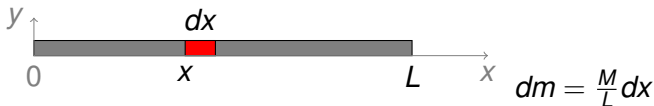
- $d\vec{p} = \vec{F}dt \implies \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \vec{F}dt \equiv \vec{J}$
- \vec{J} is called the impulse
- The average force acting on a system is defined as

$$\vec{F}_{av} = \frac{\vec{J}}{\Delta T} \quad (123)$$

where ΔT is the length of time during which the impulse is given to the system.

CM of a Continuous Mass Distribution

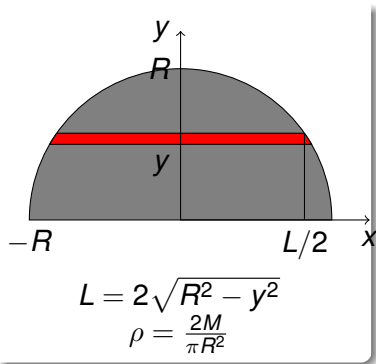
Q: What is the CM of a thin rod of length L and mass M uniformly distributed over its length?



- $\vec{r}_{CM} = \frac{1}{M} \sum_i m_i \vec{r}_i = \frac{1}{M} \sum_i m_i x_i \hat{x}$
- Each segment has mass $dm = \frac{M}{L} dx$.
- $\vec{r}_{CM} = \frac{1}{M} \int_0^L \left(\frac{M}{L} dx \right) x \hat{x} = \frac{L}{2} \hat{x}$

CM of a Continuous Mass Distribution

Q: Where is the CM of a semi disk with radius R and total mass M ?

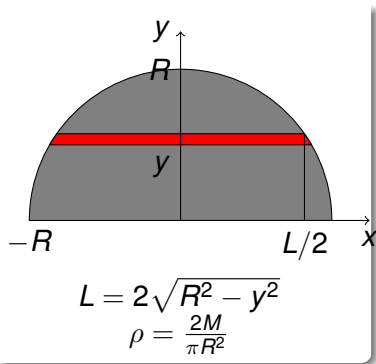


- The mass of the red strip: $dm = \rho L dy$
- The CM of red strip: $\vec{r}_i = y \hat{y}$
- CM of the semi disk:

$$\begin{aligned}
 \vec{r}_{CM} &= \frac{1}{M} \sum_i dm \vec{r}_i = \frac{1}{M} \sum_i (\rho L dy) (y \hat{y}) \\
 &= \frac{1}{M} \int_0^R \rho 2\sqrt{R^2 - y^2} y dy \hat{y}
 \end{aligned}
 \tag{124}$$

CM of a Continuous Mass Distribution

Q: Where is the CM of a semi disk with radius R and total mass M ?



- CM of the semi disk:

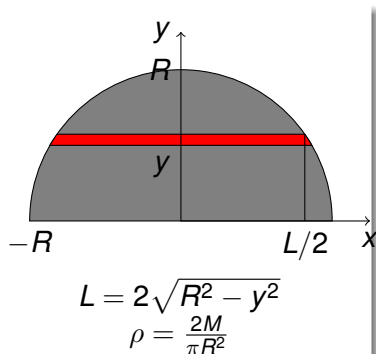
$$\begin{aligned}
 \vec{r}_{CM} &= \frac{1}{M} \sum_i dm \vec{r}_i = \frac{1}{M} \sum_i (\rho L dy) (y \hat{y}) \\
 &= \frac{1}{M} \int_0^R \rho 2\sqrt{R^2 - y^2} y dy \hat{y}
 \end{aligned}
 \tag{124}$$

- Make a change of variables:
 $y = R \sin \theta$, $dy = R \cos \theta d\theta$:

$$\vec{r}_{CM} = \frac{2R^3 \rho}{M} \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta \hat{y}$$

CM of a Continuous Mass Distribution

Q: Where is the CM of a semi disk with radius R and total mass M ?



- Make a change of variables:
 $y = R \sin \theta$, $dy = R \cos \theta d\theta$:

$$\begin{aligned}
 \vec{r}_{CM} &= \frac{2R^3 \rho}{M} \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta \hat{y} \\
 &= \frac{2R^3}{M} \frac{2M}{\pi R^2} \frac{1}{3} \hat{y} = \frac{4}{3\pi} R \hat{y} \quad (124)
 \end{aligned}$$

Systems of Variable Mass

Rocket Propulsion

Q: A rocket can expel exhaust at a speed u relative to the rocket. Suppose the it starts with a mass M and expels exhaust at a rate of r (in units of kg/s). What will be its speed when its mass becomes M_f ?

- At the moment that the rocket has a mass m , suppose it expels a mass dm of exhaust.
- Assume that initially it has a speed v , and after it expels the exhaust, it has a velocity $v + dv$.

- Initial momentum: mv

- Final momentum

$$(m - dm)(v + dv) + (-dm)(v - u) = mv + mdv - dmdv - dm u$$

$(dm < 0)$

Systems of Variable Mass

Rocket Propulsion

Q: A rocket can expel exhaust at a speed u relative to the rocket. Suppose the it starts with a mass M and expels exhaust at a rate of \dot{m} (in units of kg/s). What will be its speed when its mass becomes M_f ?

- Conservation of momentum:

$$mv + mdv + dmdv + dm u = mv \implies mdv + dm u = 0$$

- $dv/u = dm/m$. Integrating both sides from t_i (when the speed is v_i and mass M) until t_f (when speed is v_f and mass M_f)

$$\frac{v_f - v_i}{u} = -\ln \frac{M_f}{M}$$

- $v_f = v_i + u \ln \frac{M}{M_f}$

Discussion Questions

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Drop a stone from the top of a high cliff. Consider the earth and the stone as a system. As the stone falls, the momentum of the system

- A increases in the downward direction.
- B decreases in the downward direction.
- C stays the same.
- D not enough information to decide.

Discussion Questions

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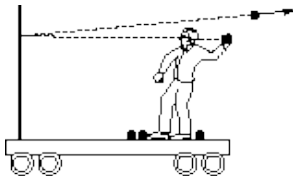
Consider yourself and the Earth as one system. Now jump up. Does the momentum of the system

- A increase in the downward direction as you rise?
- B increase in the downward direction as you fall?
- C stay the same?
- D dissipate because of friction?
- E Not enough information is given to decide.

Discussion Questions

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Suppose you are on a cart, initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart



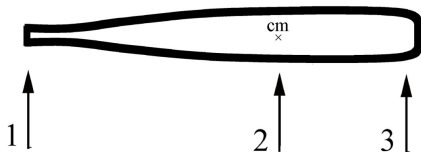
put in motion?

- A Yes, it moves to the right.
- B Yes, it moves to the left.
- C No, it remains in place.
- D Not enough information is given to decide.

Discussion Questions

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The greatest acceleration of the center of mass of a baseball bat will be produced by pushing with a force F at



- A Position 1
- B Position 2
- C Position 3
- D All the same
- E Not enough information is given to decide.

Discussion Questions

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A compact car and a large truck collide head on and stick together. Which undergoes the larger momentum change?

- A car
- B truck
- C The momentum change is the same for both vehicles.
- D Can't tell without knowing the final velocity of combined mass.

Discussion Questions

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Two balls that are dropped from a height h_i above the ground, one on top of the other. Ball 1 is on top and has mass m_1 , and ball 2 is underneath and has mass m_2 with $m_2 \gg m_1$. Ball 2 first collides with the ground and rebounds with speed v_0 . Then, as ball 2 starts to move upward, it collides elastically with the ball 1 which is still moving downwards also with speed v_0 . The final relative speeds after ball 1 and ball 2 collide is

- A Zero
- B v_0
- C $2v_0$
- D $3v_0$
- E None of the above

Discussion Questions

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An explosion splits an object initially at rest into two pieces of unequal mass. Which piece has the greater kinetic energy?

- A The more massive piece.
- B The less massive piece.
- C They both have the same kinetic energy.
- D There is not enough information to tell.

Discussion Questions

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A spacecraft with speed v_{1i} approaches Saturn which is moving in the opposite direction with a speed v_s . After interacting gravitationally with Saturn, the spacecraft swings around Saturn and heads off in the opposite direction it approached. What is the final speed of the spacecraft v_{1f} after it is far enough away from Saturn to be nearly free of Saturn's gravitational pull?

Discussion Questions

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Suppose rain falls vertically into an open cart rolling along a straight horizontal track with negligible friction. As a result of the accumulating water, the speed of the cart

- A increases.
- B does not change.
- C decreases.
- D not sure.
- E not enough information is given to decide.

LEARNING

Measurable and relatively permanent change in behavior through experience, instruction, or study. . . . Learning itself cannot be measured, but its results can be. In the words of Harvard Business School psychologist Chris Argyris, learning is "detection and correction of error" where an error means "any mismatch between our intentions and what actually happens."

Read more: <http://www.businessdictionary.com/definition/learning.html>

LEARNING

There are three components to the definition of Learning ¹:

- **“Learning is a process, not a product.”**

Exam scores and term papers are measures of learning, but they are not the process of learning itself.

¹ <http://www.cidde.pitt.edu/ta-handbook/teaching-and-learning-principles/definition-learning>

LEARNING

There are three components to the definition of Learning ¹:

- **“Learning is a change in knowledge, beliefs, behaviors or attitudes.”**

This change requires time, particularly when one is dealing with changes to core beliefs, behaviors, and attitudes. Don't interpret a lack of sea change in your students' beliefs or attitudes immediately following a lesson as a lack of learning on their part, but instead, consider that such a change will take time – perhaps a few weeks, perhaps until the end of the term, or even longer.

¹ <http://www.cidde.pitt.edu/ta-handbook/teaching-and-learning-principles/definition-learning>

LEARNING

There are three components to the definition of Learning ¹:

- **“Learning is not something done to students, but something that students themselves do.”**

If you have ever carefully planned a lesson, only to find that your students just didn't “get it,” consider that your lesson should be designed not just to impart knowledge but also to lead students through the process of their own learning (Ambrose 2010:3).

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