Outline of Topics Covered

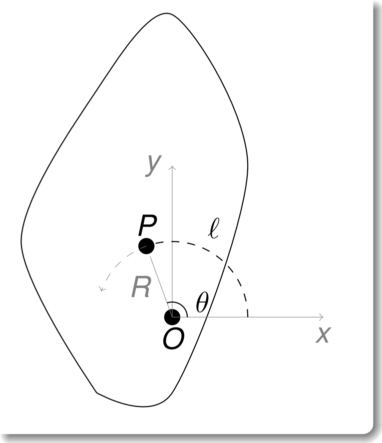
Will be Redone for Rotational Motion

- Kinematics-how to describe the state (position and velocity) of the system
- Dynamics–why the state of the system changes (acceleration)
- Work done by force
- Conserved quantities
  - Energy Conservation
  - Momentum Conservation

Rotational Variables

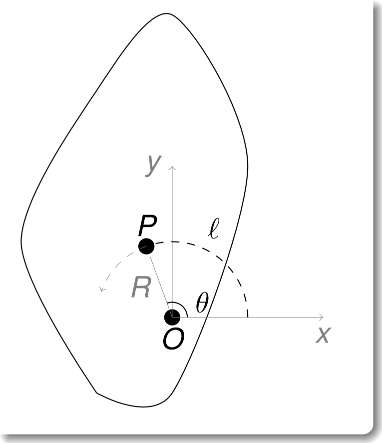
- **Rigid body:** The distances between parts of the object are fixed.
- General motion of a rigid body: translation+ rotation
- Pure rotation around a fixed axis: all the points on the object rotate around a given axis-**axis of rotation**
- The orientation of an object can be completely specified by specifying the position of a determined point.

Rotation Angle(Kinematics)



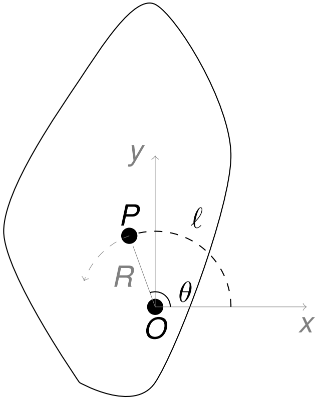
- In **radians** $\theta = \frac{\ell}{R}$, or $\ell = \theta R$ (θ = theta)
- $\Delta\theta$: change in θ, angular displacement
- Average angular velocity: $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$ (ω = omega)
- Average angular acceleration:$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$ (α = alpha)
- Instantaneous angular velocity: $\omega = \frac{d\theta}{dt}$
- Instantaneous angular acceleration: $\alpha = \frac{d\omega}{dt}$

Rotation Angle(Kinematics)



- The speed of point P is $v = \frac{d\ell}{dt} = \frac{d(R\theta)}{dt} = R\frac{d\theta}{dt} = R\omega$
- The further the point is from the axis of rotation, the faster it moves
- $a_{tan} = \frac{dv}{dt} = R\alpha$ (note that these are not vectors)
- **frequency**: How many full rotations the object completes in one second: $f = \frac{\omega}{2\pi} \implies w = 2\pi f$
- **Period:** How long one full rotation takes: $T = \frac{2\pi}{\omega} = \frac{1}{f}$

Rotation Angle(Kinematics)



- Angular velocity has a magnitude and a direction (the axis of rotation) hence it is a **vector.**
- The direction of angular velocity vector $\vec{\omega}$ can be found by the right hand rule.
- Angular acceleration vector $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
- $\vec{v} = \vec{\omega} \times \vec{r}$ (see ▸ vector products)
- $\vec{a}_{tan} = \vec{\alpha} \times \vec{r}$

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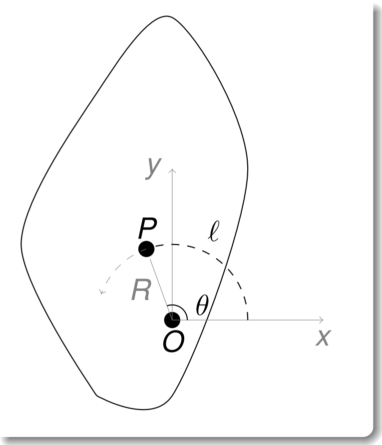
Rotation Angle(Kinematics)



For variable angular acceleration

$$\omega(t) = \omega_0 + \int_0^t \alpha(t')dt' \tag{125}$$

$$\Longleftrightarrow \vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t')dt' \tag{126}$$

$$\theta(t) = \theta_0 + \int_0^t \omega(t')dt' \tag{127}$$

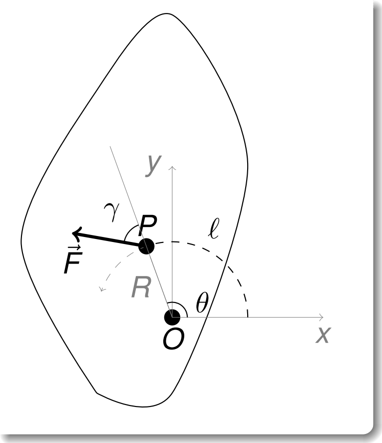
$$\Longleftrightarrow \vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')dt' \tag{128}$$

For constant angular acceleration:

$$\omega(t) = \omega_0 + \alpha t \tag{129}$$

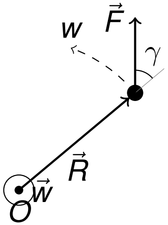
$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \tag{130}$$

Torque(Dynamics)



- If a force $\vec{F}$ is applied at the point P, such that it makes an angle γ with the line connecting P to O, the torque is defined as: $\tau = FR\sin\gamma = |\vec{F} \times \vec{R}|$

Torque on Point Mass



- $F_{tan} = F sin\theta$
- $ma_{tan} = F_{tan} \Longrightarrow mR\alpha = F_{tan}$
- $\tau = F_{tan}R = mR^2\alpha \equiv I\alpha$
- $I = mR^2$ is called the moment of inertia.

- Note that the net force acting on the mass *m* also contain the force due to tension. But this force does not have a tangential component, hence does not contribute to the tangential acceleration, and hence to angular acceleration.
- $\vec{\tau} = \vec{r} \times \vec{F}$ has the same magnitude as $I\vec{\alpha}$, and is in the same direction.
- Hence $\vec{\tau} = I\vec{\alpha}$

Torque on Continuous Mass

- For a system formed by m_i, the torque acting on m_i is

$$\vec{\tau}_i = \vec{R}_i \times \left(\sum_{j \neq i} \vec{F}_{ij} + \vec{F}_i^{ext} \right) = m_i^2 R_i \vec{\alpha}$$

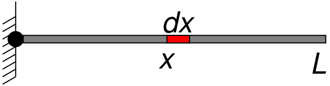
(Note that for a right body α is the same for all parts of the system)

- Total torque acting on the system

$$\vec{\tau} \equiv \sum_i \vec{\tau}_i = \sum_i m_i R_i^2 \vec{\alpha} = \left(\sum_i m_i R_i^2 \right) \vec{\alpha} \equiv I \vec{\alpha}$$

- $I = \sum_i m_i R_i^2$

Example: Thin Rod Rotating Around One End



- The net torque on the rod around the fixed axes:

$$\vec{\tau} = \sum_i \vec{r}_i \times (m_i \vec{g}) = \left(\sum_i m_i \vec{r}_i \right) \times \vec{g} = (M\vec{r}_{CM}) \times \vec{g} = \vec{r}_{CM} \times \vec{w} \quad (125)$$

  Hence the center of gravity for an object in uniform gravitational field is its CM.
- $\tau = \frac{MgL}{2}$
- $\alpha = \frac{\tau}{I} = \frac{\frac{MgL}{2}}{\frac{1}{3}ML^2} = \frac{3}{2}\frac{g}{L}$
- If the rod is initially at rest: $a^{CM} = a^{CM}_{tan} = \alpha\frac{L}{2} = \frac{3}{4}g < g$
- $a_{CM} = \frac{F^{ext}}{M}$. Which other force is acting on the rod?