Rotational Kinetic Energy

- Assume that a rigid body is rotating around a fixed axis with angular velocity ω.
- m_i located at a distance R_i from the axis will have a speed ωR_i.
- The kinetic energy of m_i is $K = \frac{1}{2}m_i(R_i\omega)^2 = \frac{1}{2}m_iR_i^2\omega^2$.
- Summing the kinetic energy of all the masses, the kinetic energy of the rigid body is:

$$K = \sum_i \frac{1}{2}(m_iR_i^2)\omega^2 \equiv \frac{1}{2}I\omega^2$$

Kinetic Energy of A Rotating Object That also Has Translational Motion

- Let $\vec{r}_{CM}$ be the position of the CM.
- Let $\vec{r}_i$ and $\vec{R}_i$ be the position of mass m_i in the rigid body relative to a fixed coordinate axis and relative to the CM respectively: $\vec{r}_i = \vec{r}_{CM} + \vec{R}_i$
- $\vec{v}_i = \vec{v}_{CM} + \vec{V}_i$, where $\vec{V}_i$ is the velocity relative to the CM.
- Total Kinetic Energy of the rigid body is:

$$
\begin{aligned}
K &= \sum_i \frac{1}{2} m_i \vec{v}_i^2 = \sum_i \frac{1}{2} m_i (\vec{v}_{CM}^2 + 2\vec{v}_{CM} \cdot \vec{V}_i + \vec{V}_i^2) \\
&= \frac{1}{2} (\sum_i m_i) V_{CM}^2 + \frac{1}{2} \sum_i m_i \vec{V}_i^2 + \vec{v}_{CM} \cdot \sum_i m_i \vec{V}_i
\end{aligned}
$$

- The first terms is the translational kinetic energy
- The second term is the rotational kinetic energy around the CM.

Kinetic Energy of A Rotating Object That also Has Translational Motion

- Total Kinetic Energy of the rigid body is:

$$K = \sum_i \frac{1}{2} m_i \vec{v}_i^2 = \sum_i \frac{1}{2} m_i (\vec{v}_{CM}^2 + 2\vec{v}_{CM} \cdot \vec{V}_i + \vec{V}_i^2)$$

$$= \frac{1}{2} (\sum_i m_i) V_{CM}^2 + \frac{1}{2} \sum_i m_i \vec{V}_i^2 + \vec{v}_{CM} \cdot \sum_i m_i \vec{V}_i$$

- The first terms is the translational kinetic energy
- The second term is the rotational kinetic energy around the CM.
- The third term is zero since $\sum_i m_i \vec{V}_i$ is the total momentum relative the the *CM* which is zero.
- $K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I \omega^2$
- Note that this simple form is valid only if one considers a rotation axis passing through the CM.

Work Done On a Rotating Object

- $W = \int_{P_i}^{P_f} \vec{F} \cdot d\vec{\ell}$
- $d\ell = R d\theta$
- $\vec{F} \cdot d\vec{\ell} = FRd\theta \cos\gamma$, where γ is the angle between $\vec{F}$ and $d\vec{\ell}$
- $F\cos\gamma$ is the component of $\vec{F}$ along $d\vec{\ell}$
- $d\ell$ is perpendicular to $\vec{R}$.
- Hence $F\cos\gamma = F_\perp$

$$W = \int_{\theta_i}^{\theta_f} F_\perp R d\theta = \int_{\theta_i}^{\theta_f} \tau d\theta$$

- The power is:

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

Department of Physics Fizik Bölümü 1960

Work Energy Principle For Rotations

$$
\begin{aligned}
W &= \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{t_i}^{t_f} I \frac{d\omega}{dt} \frac{d\theta}{dt} dt \\
&= \int_{t_i}^{t_f} \frac{d\omega}{dt} I\omega dt = \int_{t_i}^{t_f} \frac{d}{dt} \left(\frac{1}{2} I\omega^2 \right) dt \\
&= \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2
\end{aligned}
$$

Concept Questions

Object A sits at the outer edge (rim) of a merry-go-round, and object B sits halfway between the rim and the axis of rotation. The merry-go-round makes a complete revolution once every thirty seconds. The magnitude of the angular velocity of Object B is

A half the angular speed of Object A .

B the same as the angular speed of Object A .

C twice the angular speed of Object A .

D impossible to determine

Concept Questions

Which has the smallest I about its center?

A Ring (mass m , radius R)

B Disc (mass m , radius R)

C Sphere (mass m , radius R)

D All have the same I.

Concept Questions

In this problem ignore any friction/drag. Suppose that you release (from rest) an object from a very high building. Where does it fall?

- A straight down
- B a bit to the north
- C a bit to the south
- D a bit to the east
- E a bit to the west

QUIZ 6

A mass $m_1 = 2\ kg$ that has a velocity $\vec{v}_1 = (3\ m/s)\hat{x} + (4\ m/s)\hat{y}$ collides with a mass $m_2 = 3\ kg$ that moves with a velocity $\vec{v}_2 = (2\ m/s)\hat{y}$. The two masses stick together.

1. What is their common velocity after the collision?
2. How much kinetic energy is converted into internal energy in this collision?