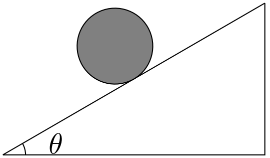
Rolling Without Sliding



- If the object rolls without sliding, the point contact is at rest
- Suppose the object starts at rest at a height h: $ME = Mgh$
- When it reaches the bottom of the incline, it has a velocity $\vec{v}$.
- Then, its angular speed is $\omega = \frac{v}{R}$
- Its final mechanical energy is:

$$ME = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left(M + \frac{I}{R^2}\right)\omega^2$$

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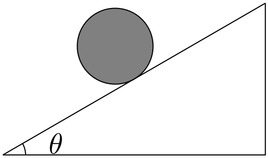

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- Conservation of Energy:

$$Mgh = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v^2 \tag{130}$$

$$\implies v = \sqrt{2gh\frac{M}{M + \frac{I}{R^2}}} \tag{131}$$

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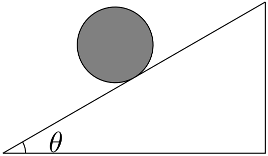

Using concepts of torque:

- The weight of the rolling object creates a torque $\tau = MgR\sin\theta$
- The moment of inertia of the object around the point of contact $I' = I + MR^2$
- The angular acceleration: $\alpha = \frac{MgR\sin\theta}{I+MR^2}$
- The linear acceleration along the incline is $a = \frac{MR^2}{I+MR^2} g\sin\theta (< g\sin\theta)$

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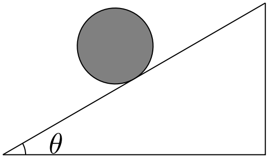

Using concepts of torque:

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  $\alpha = \frac{MgR\sin\theta}{I+MR^2}$
- The linear acceleration along the incline is $a = \frac{MR^2}{I+MR^2} g \sin\theta (< g\sin\theta)$
- Along the incline, its position as a function of time is $x(t) = \frac{1}{2}at^2$.
- The time it takes to reach the bottom from a height h is
  $\frac{h}{\sin\theta} = \frac{1}{2}at^2 \Longrightarrow t = \sqrt{\frac{2h}{a\sin\theta}}$

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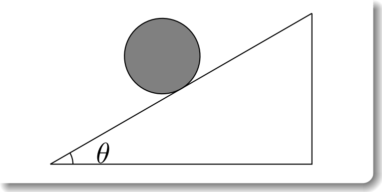

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  $\frac{h}{\sin\theta} = \frac{1}{2}at^2 \Longrightarrow t = \sqrt{\frac{2h}{a\sin\theta}}$
- Its translational speed at the bottom is:

$$v = at = \sqrt{\frac{2ha}{\sin\theta}} = \sqrt{2gh\frac{M}{M + \frac{I}{R^2}}}$$

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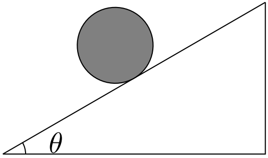

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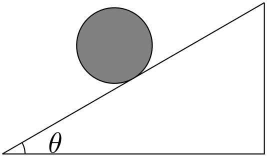


Q: We have shown that $a = \frac{MR^2}{I+MR^2} g \sin\theta (< g \sin\theta)$ for linear acceleration of the CM. We also know that the acceleration of the CM for any system of particles is determined completely by the total force acting on the system, independent of whether the object is rotating or not. If the object was not rotating, its acceleration would have been given by $a = g \sin\theta$. Which force causes its acceleration to further reduce?

Concept Questions

Object A sits at the outer edge (rim) of a merry-go-round, and object B sits halfway between the rim and the axis of rotation. The merry-go-round makes a complete revolution once every thirty seconds. The magnitude of the angular velocity of Object B is

- A half the angular speed of Object A .
- B the same as the angular speed of Object A .
- C twice the angular speed of Object A .
- D impossible to determine

Concept Questions

Which has the smallest I about its center?

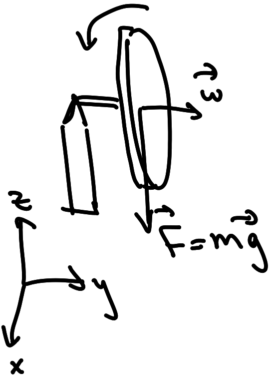
- A Ring (mass m , radius R)
- B Disc (mass m , radius R)
- C Sphere (mass m , radius R)
- D All have the same I.

Concept Questions

In this problem ignore any friction/drag. Suppose that you release (from rest) an object from a very high building. Where does it fall?

- A straight down
- B a bit to the north
- C a bit to the south
- D a bit to the east
- E a bit to the west

1 Gyroscope

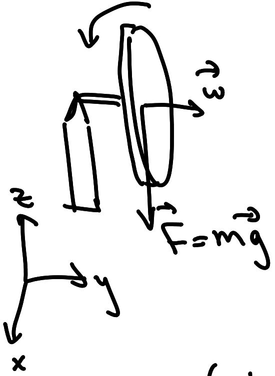


$$\vec{w} = w\hat{y}$$
$$\vec{F} = -mg\hat{z}$$

$$\left.\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F} \\ \vec{r} &= L\hat{y} \end{aligned}\right.$$

$$\vec{\tau} = (L\hat{y}) \times (-mg\hat{z}) = -mgL\,\hat{x}$$

2 Gyroscope



$$\vec{\tau} = -mgL\,\hat{y} = I\vec{\alpha}$$

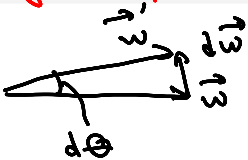
$$\Rightarrow \vec{\alpha} = -\frac{mgL}{I}\,\hat{y}$$

$$d\vec{w} = \vec{\alpha}\,dt$$

$$d\vec{w} = -\frac{mgL}{I}\,dt\,\hat{y}$$

Note that $d\vec{w} \perp \vec{w}$

(similar to uniform circular motion)



Since $d\vec{w} \perp \vec{w}$,

$$|\vec{w}'| = |\vec{w}|$$

$$d\theta = \frac{|d\vec{w}|}{w} = \frac{\frac{mgL}{I}dt}{w}$$

$$\frac{d\theta}{dt} = \frac{mgL}{Iw}$$

$\vec{w}$ rotates around the $\hat{z}$ axis with angular velocity $\frac{d\theta}{dt} = \frac{mgL}{Iw}$

$\vec{w}$ is along the axes of rotation