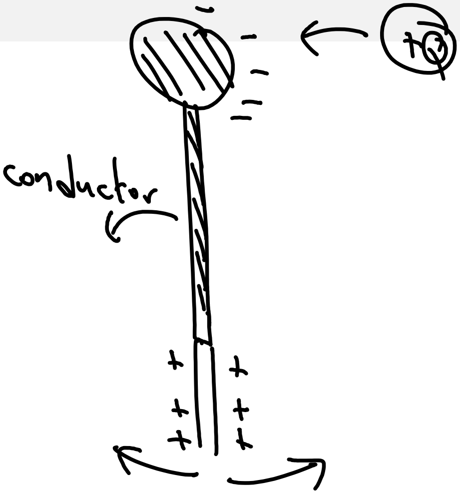
February 19, 2015

$$\hat{r}_{12} = \vec{r}_1 - \vec{r}_2$$

$q_1 \longleftarrow r \longrightarrow q_2$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

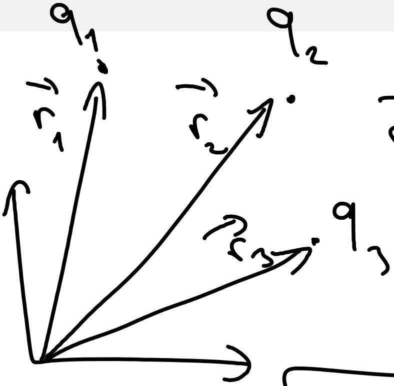
action at a distance


conductor
+Q

insulators

semi-conductors (Si, Ge, ...)

conductors



$$\vec F_1 = \vec F_{12} + \vec F_{13}$$

$$\vec F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat r_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat r_{13}$$

superposition principle

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}$$

$$\vec{F}_1 = q_1 \underbrace{\left(\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{12}^2}\hat{r}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{13}^2}\hat{r}_{13}\right)}_{\vec{E}(\vec{r}_1)}$$

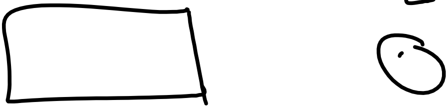
$$\vec{F}_1 = q_1 \vec{E}(\vec{r}_1) \Longrightarrow \boxed{\vec{E}(\vec{r}_1) = \frac{\vec{F}}{q}}$$

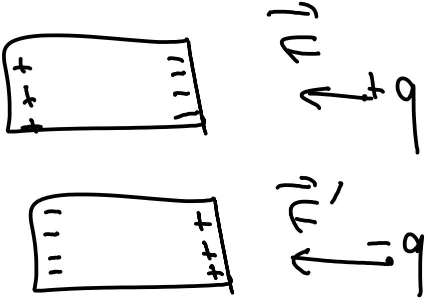
$$t = \frac{d}{c}$$

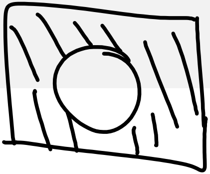
c: speed of light

$$\vec{E}(\vec{r}) = \lim_{q \to 0} \frac{\vec{F}}{q}$$

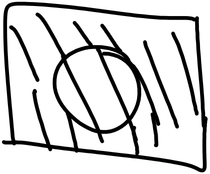
conductor $\vec{E} = 0$







$\vec{E} = ?$



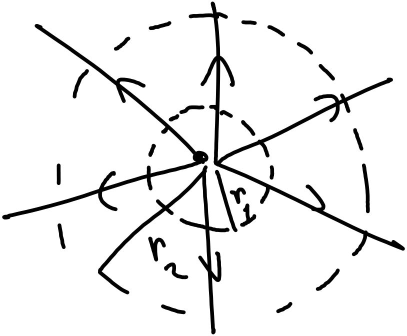
$\vec{E} = ?$

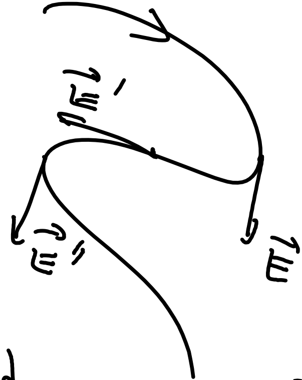
$$\vec{E} = \lim_{q' \to 0} \frac{1}{q'} \vec{F}$$

$$= \lim_{q' \to 0} \frac{1}{q'} \frac{1}{4\pi\varepsilon_0} \frac{q q'}{r^2} \hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}}$$

Electric Field Lines

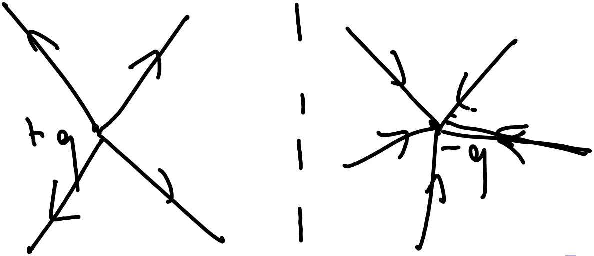




$$\text{density of field lines} = \frac{\#\text{ of field lines}}{\text{area}} \propto \frac{1}{r^2} \propto |\vec{E}|$$

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\hat{r} = q\left(\frac{1}{4\pi\varepsilon_0 r^2}\hat{r}\right)$$

$$\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)\left(4\pi r^2\right) = \frac{q}{\varepsilon_0} = \text{const!}$$

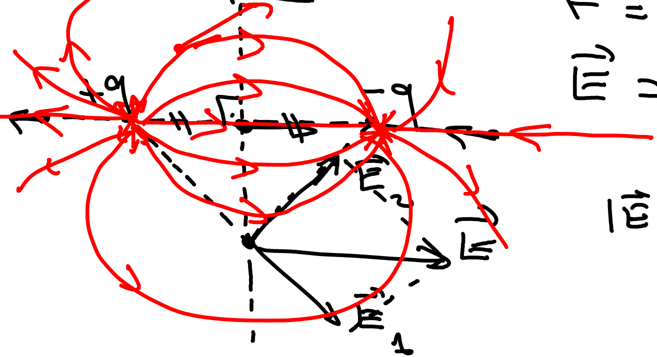


Electric field lines:

i) They are tangent to the electric field at every point

ii) Their density is proportional to the magnitude of electric field.

Example

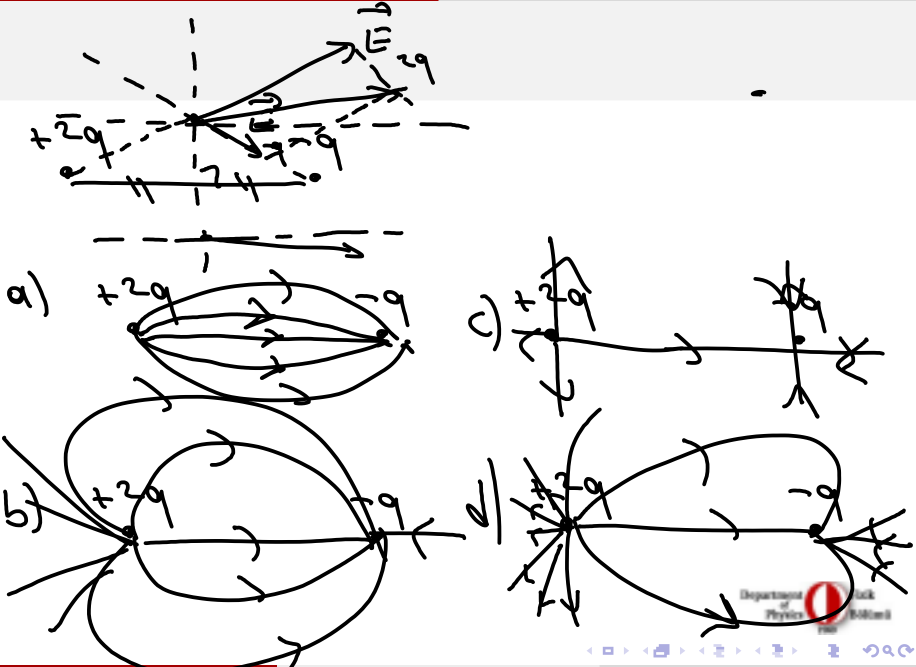


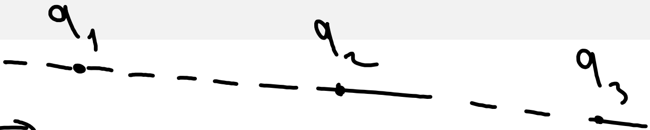
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$$

$$|\vec{E}_1| = |\vec{E}_2|$$

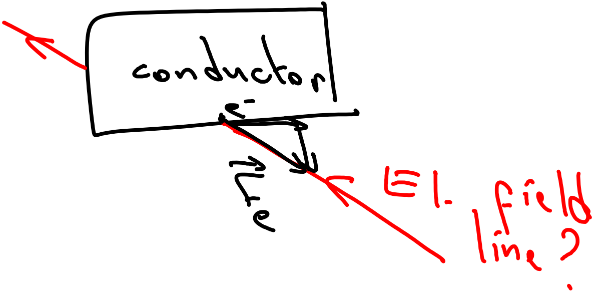
$$|\vec{E}| = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

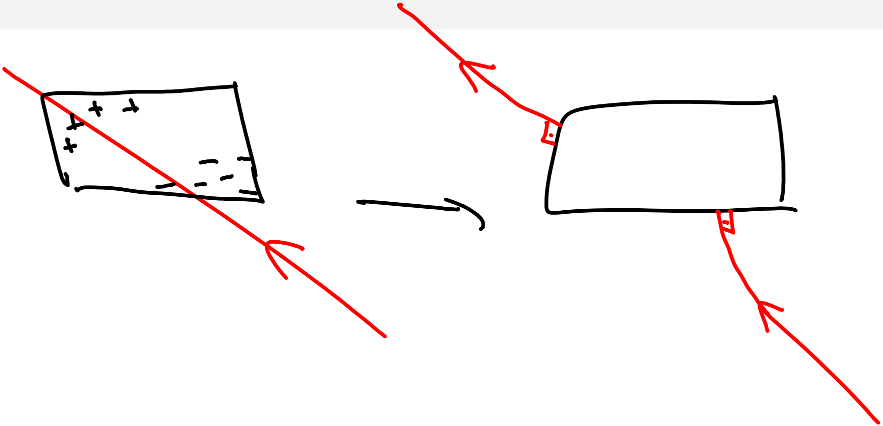

+2q
-q
a)
b)
c)
d)



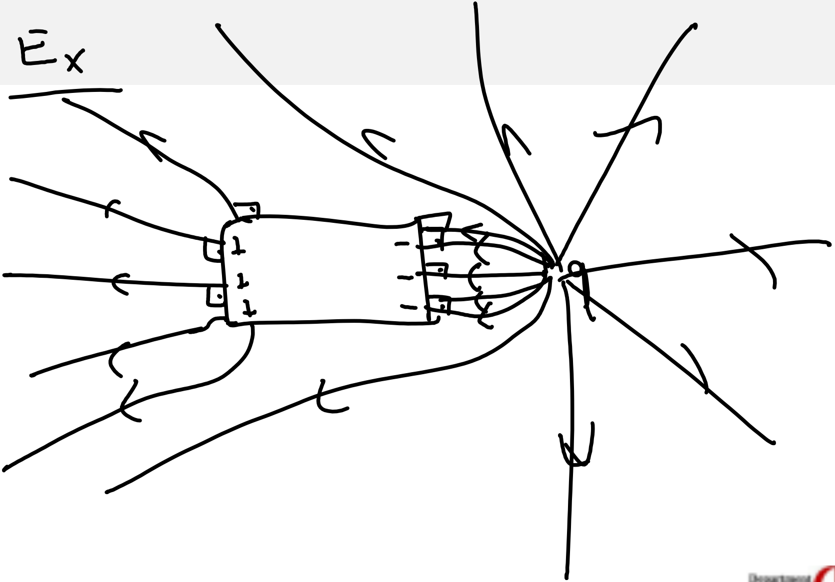
$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}$$

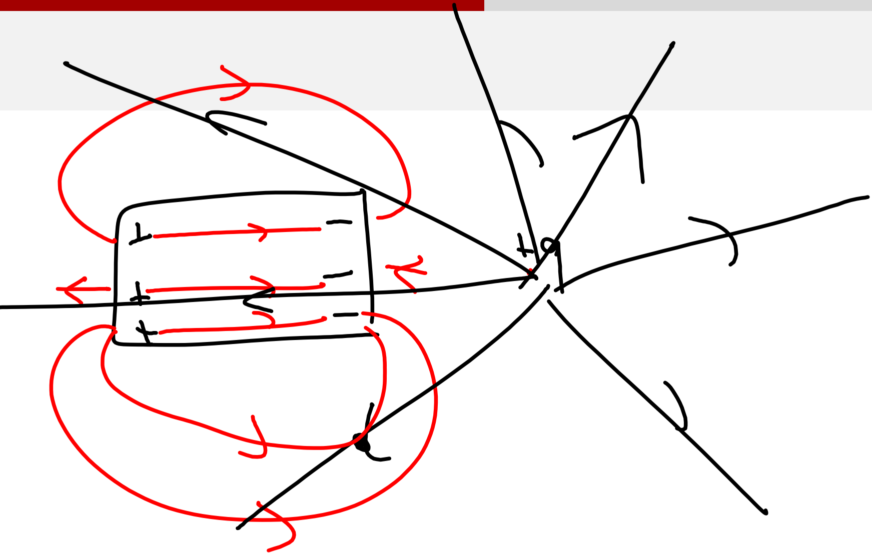
electro STATICS

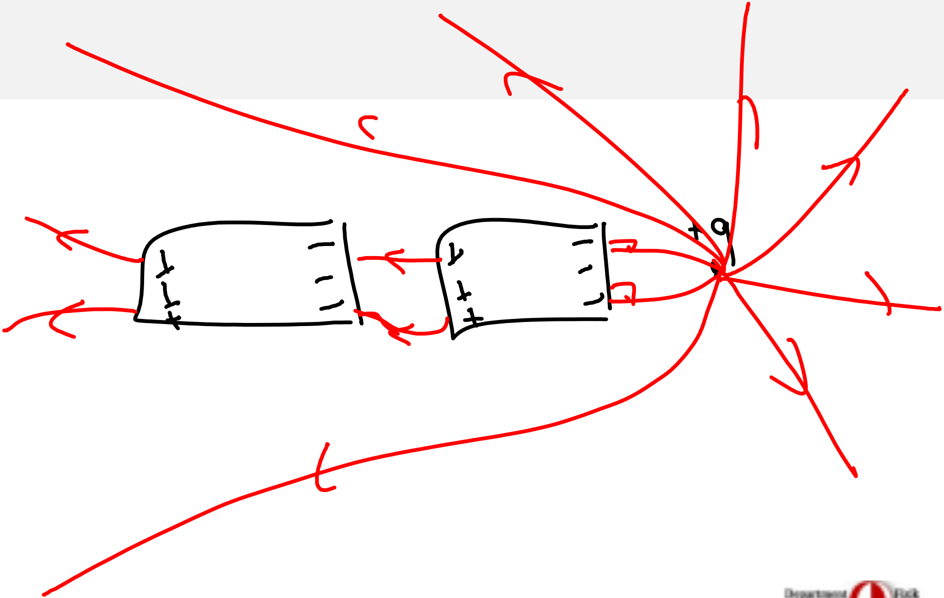


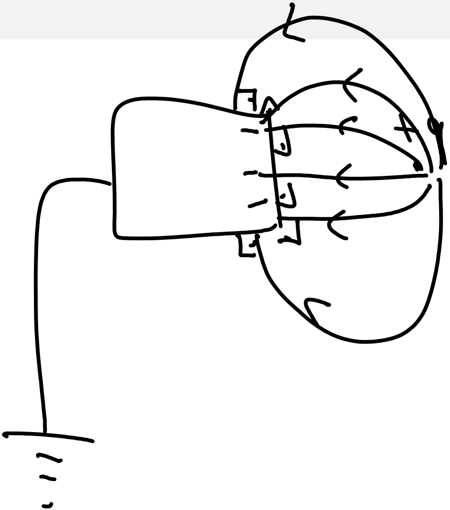


Ex






+q

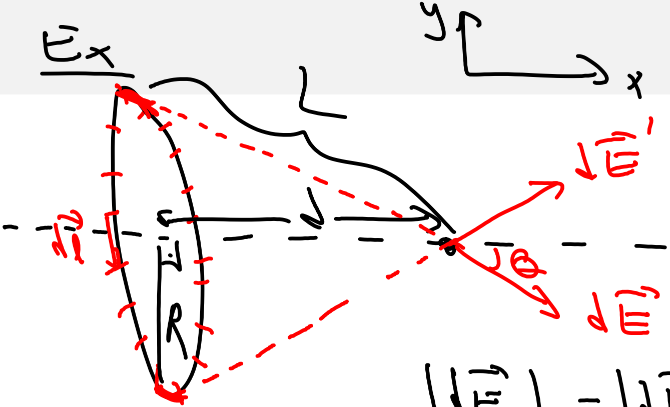


Electric Field ✗ strawberry fields

Elektrik Alan ✗ area

vector field : a function that assigns a vector to every point in space.

E_x

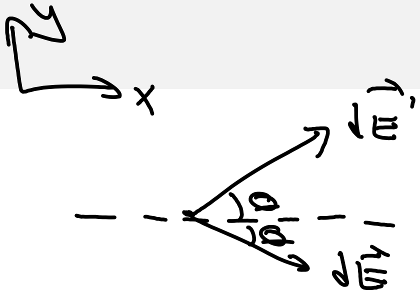


$$|d\vec{E}| = |d\vec{E}'| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{L^2}$$

Q

$$\vec{E} = E_x\,\hat{x}$$

$$dq = \frac{Q}{2\pi R}\,dl$$



$$d\vec{E} = dE\cos\theta\,\hat{x} - dE\sin\theta\,\hat{y}$$

$$d\vec{E}' = dE\cos\theta\,\hat{x} + dE\sin\theta\,\hat{y} \quad (\text{since } dE' = dE)$$

$$+ \quad \overline{\hphantom{xxxxxxxxxxxx}}$$

$$d\vec{E} + d\vec{E}' = 2\,dE\cos\theta\,\hat{x}$$

$$dq \neq d \times q$$

$$E_x = \sum (d\vec{E})_x$$

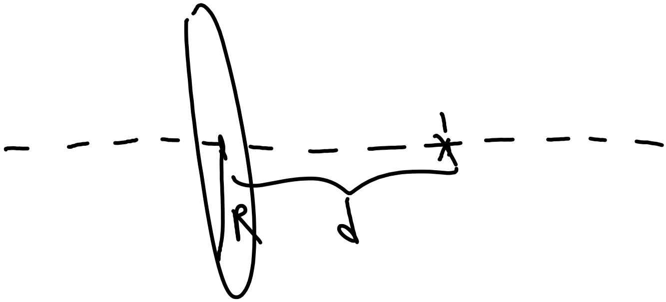
$$= \sum \frac{1}{4\pi\varepsilon_0} \frac{dq}{L^2} \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\cos\theta}{L^2} \sum dq$$

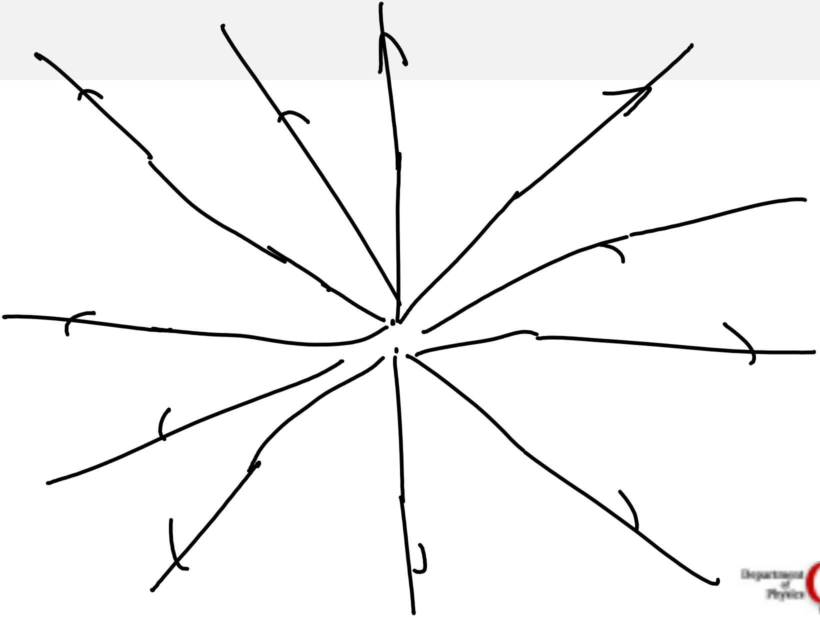
$$= \frac{1}{4\pi\varepsilon_0} \frac{Q\cos\theta}{L^2} = \frac{1}{4\pi\varepsilon_0} Q \frac{d}{L^3}$$

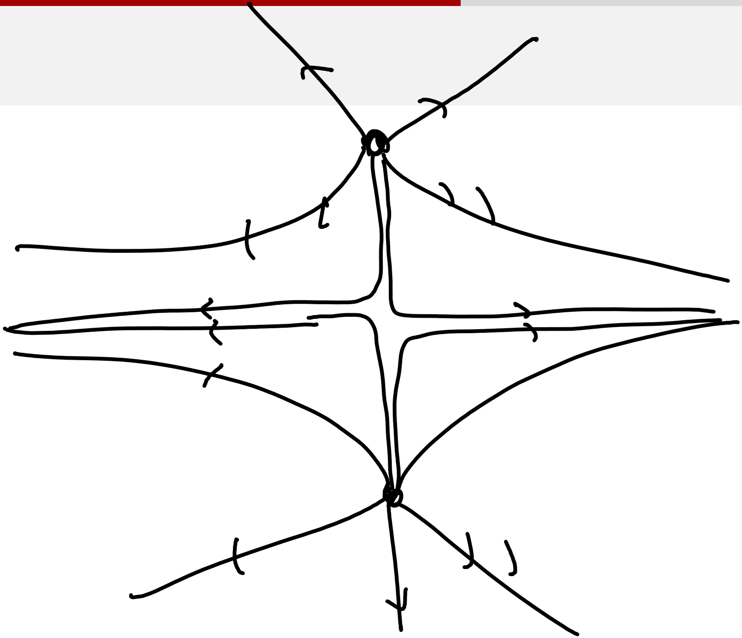
$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{d}{(d^2+R^2)^{3/2}} \hat{x}}$$

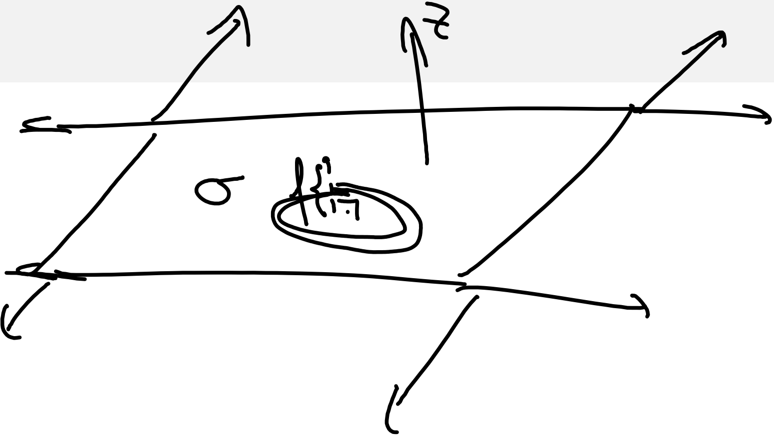
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{d}{(d^2+R^2)^{3/2}}\hat{x}$$



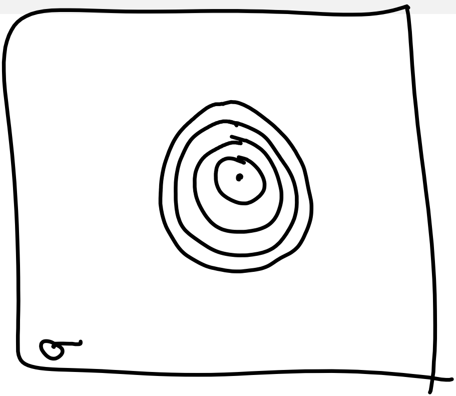
$$\vec{E} \xrightarrow{d \gg R} \frac{Q}{4\pi\varepsilon_0}\frac{1}{d^2}\hat{x}$$

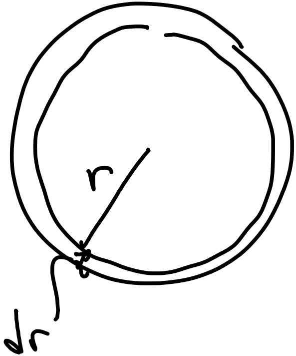






σ : surface charge density (charge per unit surface area)





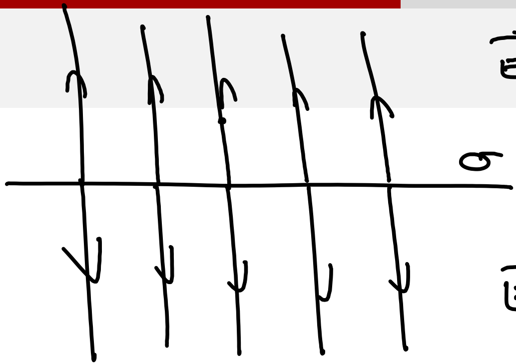
$$dA = (2\pi r)\, dr$$

$$dq = \sigma\, dA = 2\pi \sigma\, r\, dr$$

$$d\vec{E} = \frac{(2\pi\sigma r\,dr)}{4\pi\varepsilon_0}\frac{l}{(l^2+r^2)^{3/2}}\hat{z}$$

$$\vec{E} = \sum d\vec{E} = \int_0^{\infty}\frac{2\pi\sigma r\,dr}{4\pi\varepsilon_0}\frac{l}{(l^2+r^2)^{3/2}}\hat{z}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}}$$



$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z}$$

$$\vec{E} = -\frac{\sigma}{2\epsilon_0}\hat{z}$$