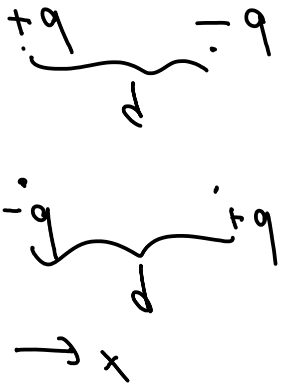
February 26, 2015

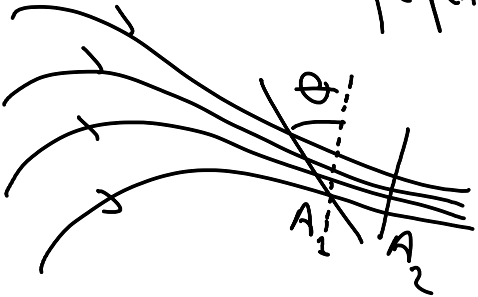


$$Q = (+q) + (-q) + (-q) + (+q) = 0$$

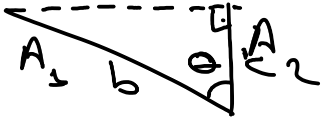
$$\vec{p} = (qd)(-\hat{x}) + (qd)(\hat{x}) = 0$$

$$\boxed{\text{quadrupole moment}}$$

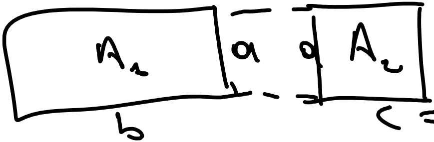
$$\text{density of field lines} = \frac{\#\text{ of field lines}}{\text{perpend. area}} = \frac{\#\text{ of field lines}}{\text{area } (\cos\Theta)}$$



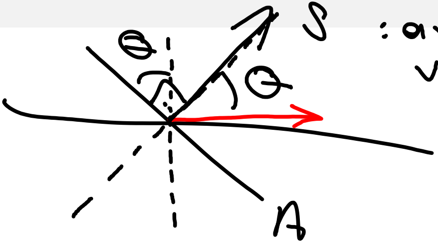
$$A_1 \cos\Theta = A_2$$



$$A_1 = ab$$
$$A_2 = ac$$
$$= A_1 \cos\theta$$



$$c = b\cos\theta$$



$\vec{S}$: area vector

$$\frac{\#\text{ of lines}}{\text{Area } \cos\theta} \propto |\vec{E}| \Rightarrow \#\text{ of lines} \propto |\vec{E}|(\cos\theta)\,\text{area}$$

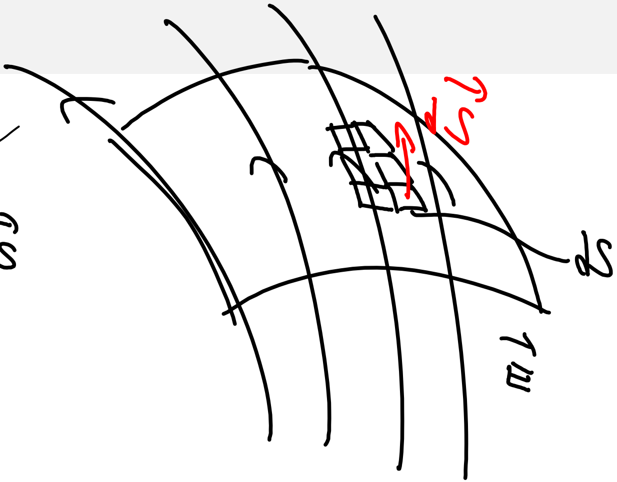
$$\text{electric flux} \equiv \Phi = |\vec{E}|(\cos\theta)\ \text{area} \equiv \vec{E}\cdot\vec{S}$$

$$[\vec{E}] = \left[\frac{\vec{F}}{q}\right] = \frac{N}{C} = \frac{kg\, m}{C s^2}$$

$$[\Phi] = [\vec{E}\ \text{area}] = \frac{kg\, m^3}{C s^2}$$

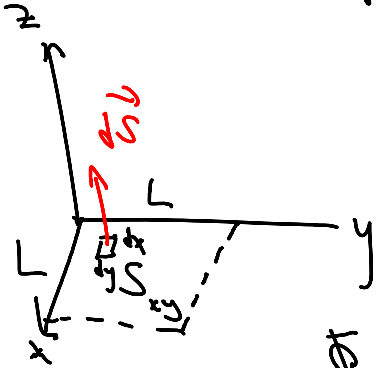
$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{S}$$



Examples

$$\vec{E} = E_0(\hat{x}+\hat{y})$$



$$\vec{E}\cdot\vec{E} = E_0^2(\hat{x}+\hat{y})\cdot(\hat{x}+\hat{y})$$

$$= E_0^2(1+0+0+1)$$

$$E = \sqrt{\vec{E}\cdot\vec{E}} = E_0\sqrt{2}$$

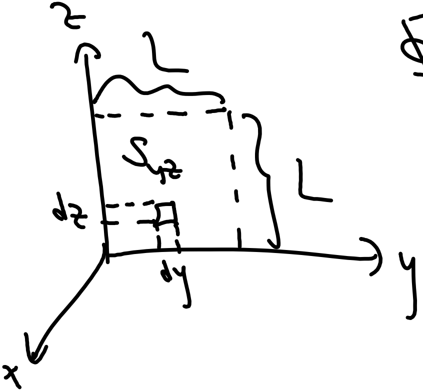
$$d\vec{S} = \hat{z}\, dx\, dy$$

$$\vec{E}\cdot d\vec{S} = 0 = E_0(\hat{x}+\hat{y})\cdot\hat{z}\, dx\, dy$$

$$\Phi_{xy} = \int \vec{E}\cdot d\vec{S} = 0$$

$$\vec{E}\cdot d\vec{S} = |\vec{E}||d\vec{S}|\cos\theta$$

$$= \sqrt{2}E_0\,dx\,dy\cos\frac{\pi}{2} = 0$$



$$\Phi_{yz} = \int \vec{E}\cdot d\vec{S}$$

$$= \int E_0(\hat{x}+\hat{y})\,dz\,dy\cdot\hat{x}$$

$$= E_0\int(1+0)\,dz\,dy$$

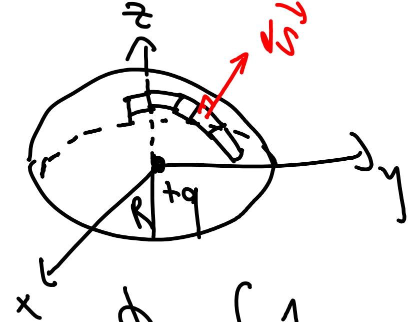
$$= E_0\int dz\,dy = E_0 L^2$$

$$\Phi_{yz} = \int \vec{E}\cdot d\vec{S} = \int |\vec{E}|\,|d\vec{S}|\cos\theta$$

$$= \cancel{\sqrt{2}}\, E_0 \int dy\,dz\, \underbrace{\cancel{\cos\frac{\pi}{4}}}_{\cancel{\frac{1}{\sqrt{2}}}}$$

$$\boxed{\Phi_{yz} = E_0 L^2}$$

Example



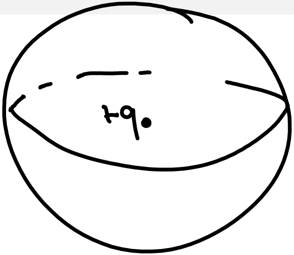
$$d\vec{S} = dS\,\hat{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{E}\cdot d\vec{S} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}dS$$

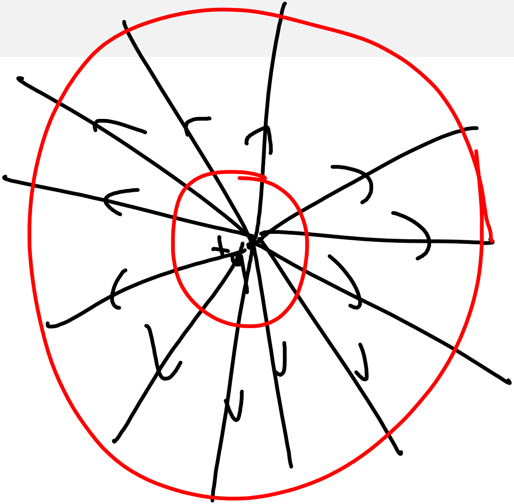
$$\phi = \int\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}dS = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\int dS$$

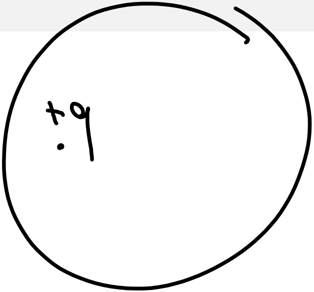
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}2\pi R^2 = \frac{q}{2\varepsilon_0}$$



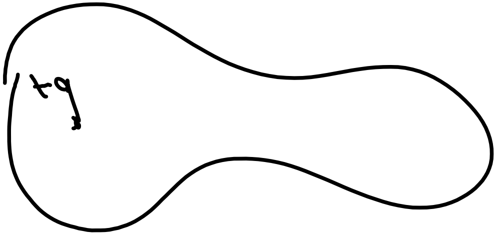
$$\int \vec{E} \cdot d\vec{S} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} 4\pi R^2$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

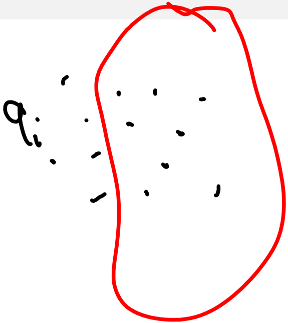

+q



$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$



$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$



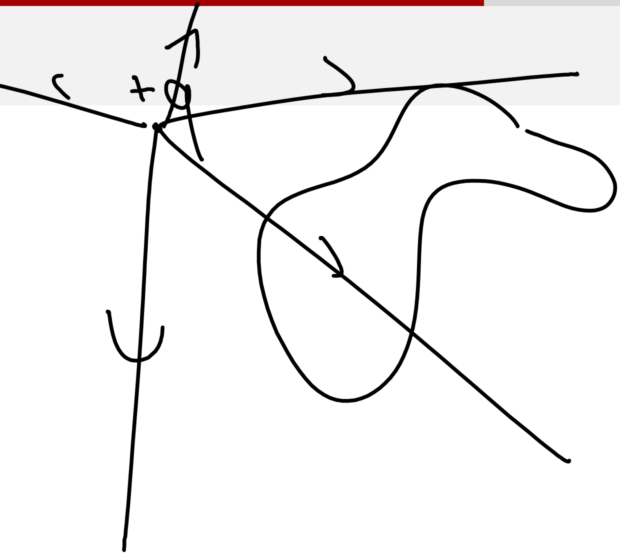
$$\vec{E} = \sum \vec{E}_i$$

$$\oint \vec{E}\cdot d\vec{S} = \sum \oint \vec{E}_i \cdot d\vec{S}$$

$$= \sum \frac{q_i}{\varepsilon_0}$$

sum over charges inside the surface

$$= \frac{Q_{enc(losed)}}{\varepsilon_0}$$



$$\oint \vec{E} \cdot d\vec{s} = 0$$

$$\boxed{\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}}$$

Gauss' Law

$$d\vec{S}_1 = -d\vec{S}_2$$

$$\Phi_1 = \oint \vec{E} \cdot d\vec{S}_1$$

$$\Phi_2 = \oint \vec{E} \cdot d\vec{S}_2 = -\Phi_1$$

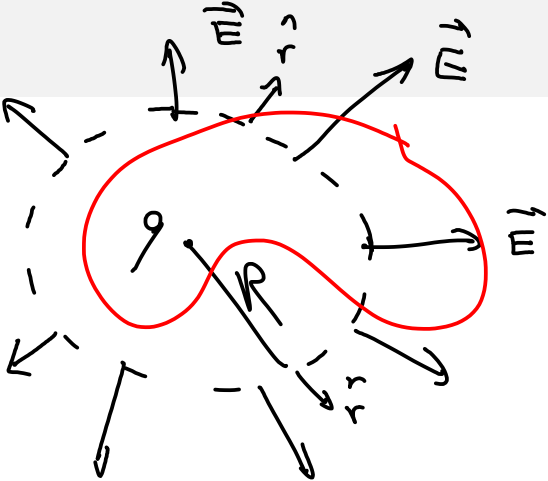
$$\Phi_1 = \frac{Q_2}{\varepsilon_0}$$

$$\Phi_2 = \frac{Q'_{enc}}{\varepsilon_0} = \frac{-Q_2}{\varepsilon_0}$$

Examples Gauss' Law

$\Rightarrow$ Coulomb's Law

$$\left|\vec{E}(\vec{r}_A)\right| = \left|\vec{E}(\vec{r}_B)\right|$$



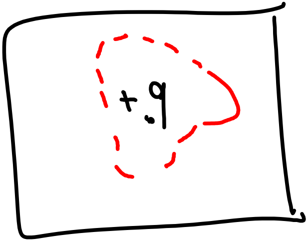
$$\int \vec{E} \cdot d\vec{S} = \oint E \, dS \cos\theta = E \oint dS = 4\pi R^2 E = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$$

$$\vec{E} = E\hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \hat{r}$$

Example

inside a conductor $\vec{E}=0$

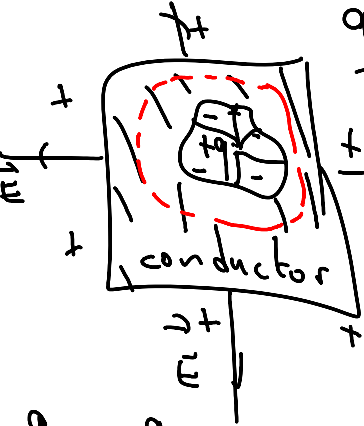


$$\frac{q}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{S} = 0$$

$$\Rightarrow q = 0$$

there can not be any charge inside a conductor!

Example



$$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{s} = 0$$

$$\boxed{q_{enc} = 0}$$

$$q_{enc} = (+q) + q_{ind} = 0$$

$$\Rightarrow \boxed{q_{ind} = -q}$$

q_{ind} : $q_{induced}$

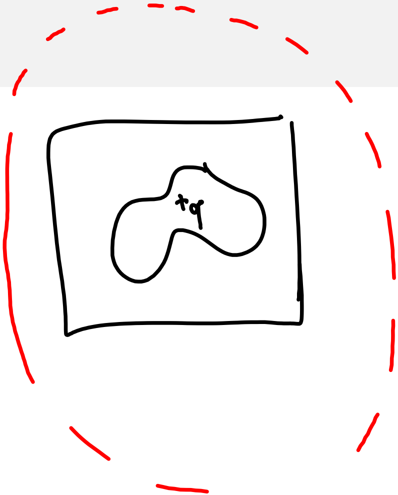
$$\frac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \cdot d\vec{S} = 0$$

$$q_{enc} = q + q_{induced}$$

$$\Rightarrow q_{ind} = -q$$

$$q_{outside} = +q$$

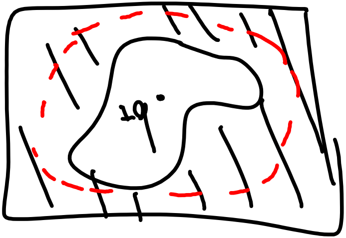
$$q_{cond} = q_{ind} + q_{outside} = 0$$



$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0}$$

Example

$Q_{cond} = Q_1$



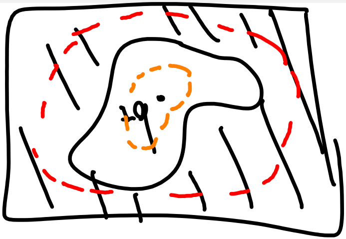
$q_1 \equiv q_{\text{inner surface}} = ? \quad +q \quad \frac{Q_1}{2}$

$q_2 \equiv q_{\text{outer surface}} = ? \quad 0 \quad \frac{Q_1}{2}$

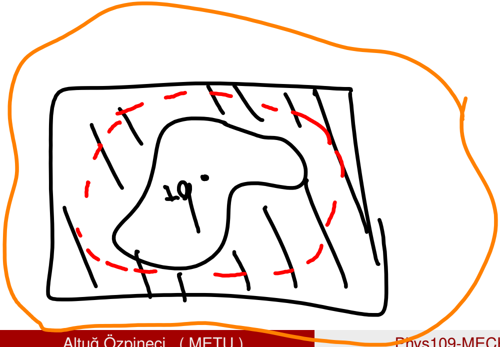
$q_{enc} = \varepsilon_0 \oint \vec{E} \cdot d\vec{S} = 0$

$q_{enc} = q + q_1 = 0 \Rightarrow q_1 = -q$

$q_1 + q_2 = Q_1 \Rightarrow q_2 = Q_1 + q$



$$\frac{q}{\varepsilon_0} = \int \vec{E} \cdot d\vec{S} \neq 0$$

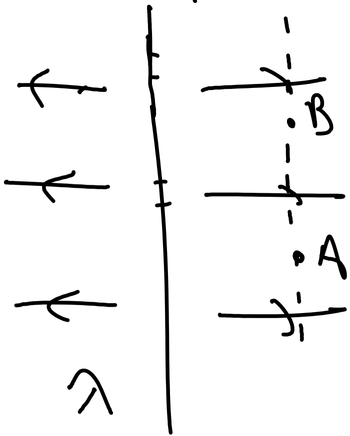


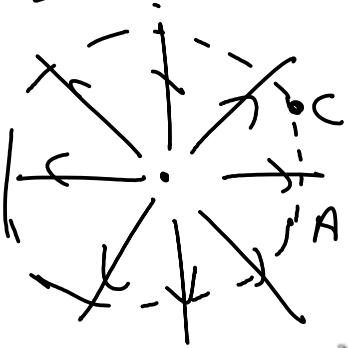
$$\oint \vec{E} \cdot d\vec{S} = \frac{q + Q_1}{\varepsilon_0}$$

Example

λ: charge per unit length.

$\vec{E} = ?$ $|\vec{E}_C| = |\vec{E}_A| = |\vec{E}_B|$

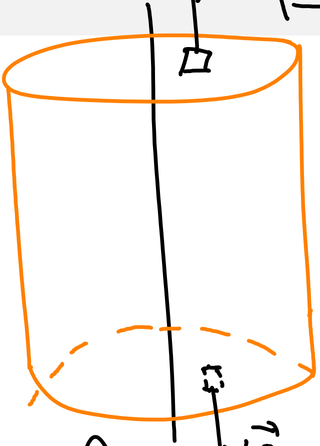




$d\vec{S}_{top} \perp \vec{E}_{top}$

$$\oint \vec{E}\cdot d\vec{S} = 0$$

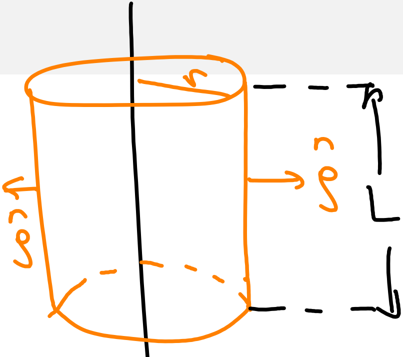
$$= \int_{side} \vec{E}\cdot d\vec{S} + \cancel{\int_{top} \vec{E}\cdot d\vec{S}}^{\,0} + \cancel{\int_{bottom} \vec{E}\cdot d\vec{S}}^{\,0}$$

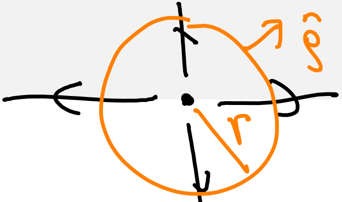


λ

$d\vec{S}_{botton} \perp \vec{E}_{botton}$

$$0 = \vec{E}\cdot d\vec{S}\Big|_{top} = \vec{E}\cdot d\vec{S}\Big|_{bottom}$$





$$\vec{E} = E(r)\,\hat{\rho}$$

$$d\vec{S} = dS\,\hat{\rho}$$

$$\vec{E}\cdot d\vec{S} = E(r)\,dS$$

$$\int_{\text{side}} \vec{E}\cdot d\vec{S} = \int_{\text{side}} E(r)\,dS = E(r)\int dS = E(r)\,2\pi r L$$

$$\oint \vec{E}\cdot d\vec{S} = E(r)\,2\pi r L$$

$$= \frac{\lambda L}{\varepsilon_0}$$

$$\boxed{E(r) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}}$$

$$\boxed{\vec{E}(r) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{\rho}}$$