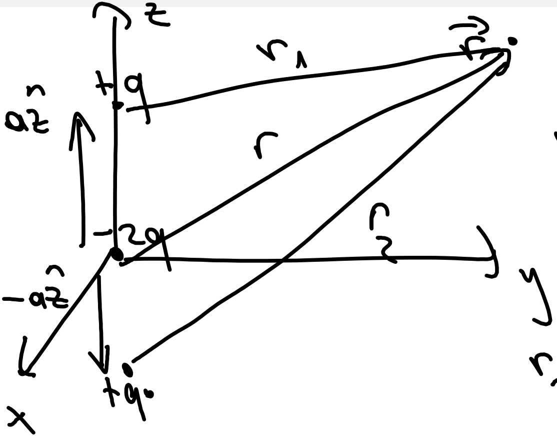
March 12, 2015



$$V(P) = ?$$

$$r_1 = \sqrt{(\vec{r} - a\hat{z})^2}$$

$$r_1 = \sqrt{r^2 - 2ar\cos\theta + a^2}$$

$$r_2 = \sqrt{(\vec{r} - (-a\hat{z}))^2}$$

$$= \sqrt{r^2 + a^2 + 2ar\cos\theta}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} + \frac{q}{r_2} - \frac{2q}{r}\right)$$

$$\frac{1}{r_1} = r_1^{-1} = \left(r^2 + a^2 - 2ar\cos\theta\right)^{-1/2}$$

$$= (r^2)^{-1/2}\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)^{-1/2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \underbrace{\frac{a^2}{r^2} - \frac{2a}{r}\cos\theta}_{\ll 1}\right)^{-1/2}$$

$$r \gg a$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$$

ignore anything smaller than $\frac{a^2}{r^2}$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)^{-1/2}$$

$$\simeq \frac{1}{r}\left(1 + \left(-\frac{1}{2}\right)\left(\frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(\frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)^2 + \cdots\right)$$

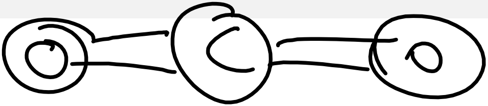
$$\simeq \frac{1}{r}\left[1 + \frac{a}{r}\cos\theta + \frac{a^2}{r^2}\left(-\frac{1}{4} + \frac{3}{\cancel{4}}\cancel{4}\cos^2\theta\right) + \mathcal{O}\left(\frac{a^3}{r^3}\right)\right]$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta + \frac{a^2}{2r^2}\left(-1 + 3\cos^2\theta\right)\right)$$

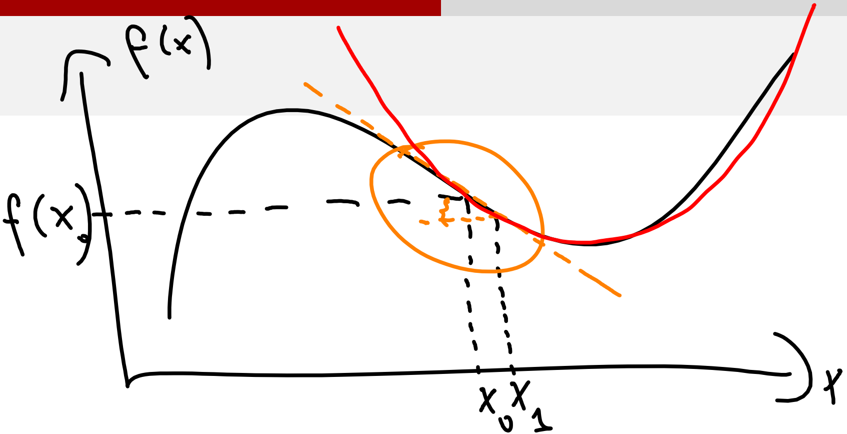
$$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta + \frac{a^2}{2r^2}\left(-1 + 3\cos^2\theta\right)\right)$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{2}{r}\right)$$

$$\boxed{V(\vec{r}) = \frac{qa^2}{4\pi\varepsilon_0}\frac{1}{2r^3}\left(-1 + 3\cos^2\theta\right)}$$



Taylor Expansion



$f(x_1) = ?$ if $x_0 \sim x_1$

$$f(x_1) = f(x_0) + f'(x_0)(x_1 - x_0) + \frac{1}{2} f''(x_0)(x_1 - x_0)^2$$

$$f(x_1) = f(x_0) + f'(x_0)(x_1 - x_0) + f''(x_0)\frac{(x_1-x_0)^2}{2!}$$

$$+ \cdots + f^{(n)}(x_0)\frac{(x_1-x_0)^n}{n!} + \cdots$$

$$f^{(n)}(x_0) \equiv \left.\frac{d^n f}{dx^n}\right|_{x=x_0}$$

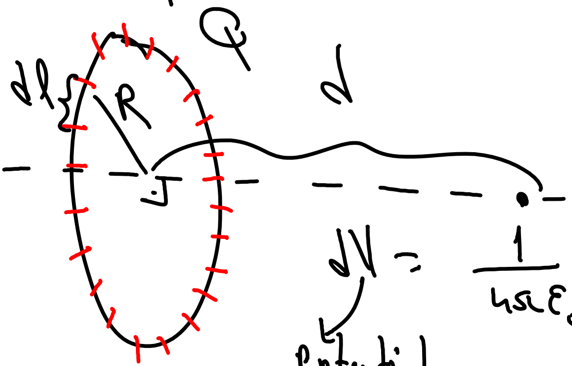
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

$$+ \frac{1}{4\pi\varepsilon_0}\frac{(\ )}{r^3}\left(-\frac{1}{4}+3\cos^2\theta\right)$$

$$+ \cdots$$

Example

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



$$dV = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2\pi R}dl\right)\frac{1}{\sqrt{d^2+R^2}}$$

potential

$$dq = \frac{Q}{2\pi R}dl$$

$$V = \sum dV = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\pi R}\frac{1}{\sqrt{d^2+R^2}}\sum dl$$

$$V = \sum dV = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\cancel{2\pi R}}\frac{1}{\sqrt{d^2+R^2}}\cancel{\sum dl}$$

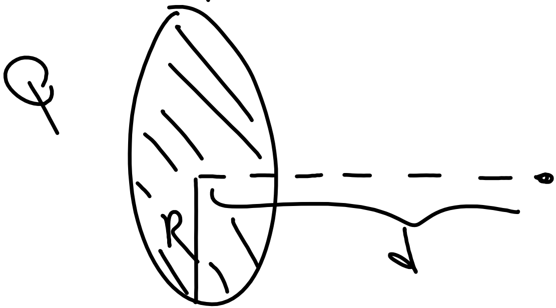
$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{d^2+R^2}}}$$

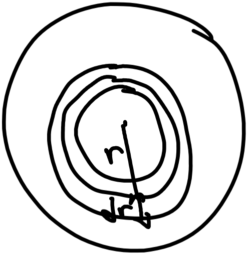
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{d}\left(1+\frac{R^2}{d^2}\right)^{-1/2} \qquad d \gg R$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{d}\left(1-\frac{1}{2}\frac{R^2}{d^2}+\cdots\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{d} - \frac{1}{8\pi\varepsilon_0}\frac{QR^2}{d^3}+\cdots$$

Example





$$dA = 2\pi r\,dr$$

$$dq = \frac{Q}{\pi R^2}\,dA$$

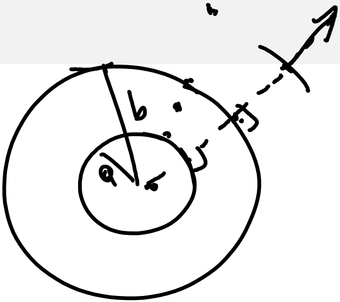
$$dV = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\pi R^2}2\pi r\,dr\right)\frac{1}{\sqrt{d^2+r^2}}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\,\frac{Q}{\pi R^2}\,2\pi\int_0^R \frac{r\,dr}{\sqrt{d^2+r^2}}}$$

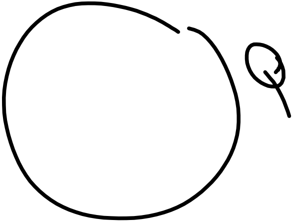
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}2\pi\int_0^R \frac{r\,dr}{\sqrt{d^2+r^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi R^2}2\pi\left(\sqrt{d^2+R^2}-d\right)$$

$$\boxed{\frac{r}{\sqrt{d^2+r^2}} = \frac{d}{dr}\left(\sqrt{d^2+r^2}\right)}$$

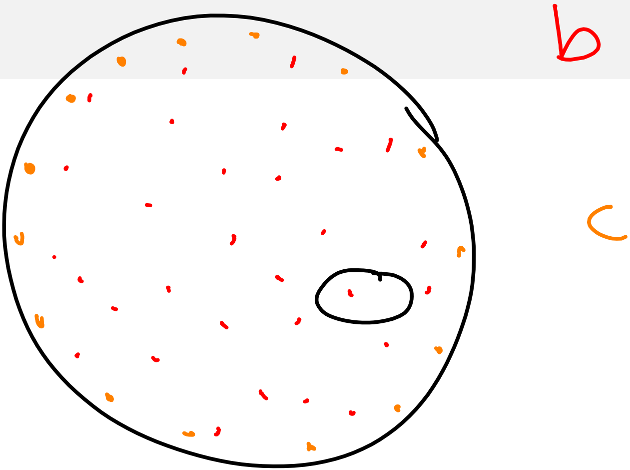


$\vec{E}(\vec{r})$?



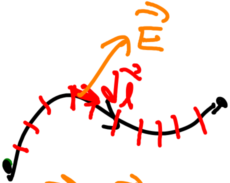
$$|\vec{E}| = \begin{cases} 0 & \text{inside} \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} & \text{outside} \end{cases}$$


+q
conductor


b
c

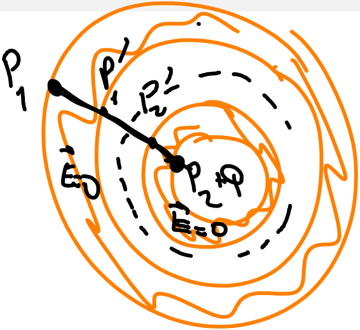
$$\oint_{S_1} \vec{E}\cdot d\vec{S} = 0$$

$$\oint_{S_2} \vec{E}\cdot d\vec{S} = +q/\varepsilon_0$$



$$\Delta V = -\int \vec{E}\cdot d\vec{l}$$

$$dV = -\vec{E}\cdot d\vec{l}$$



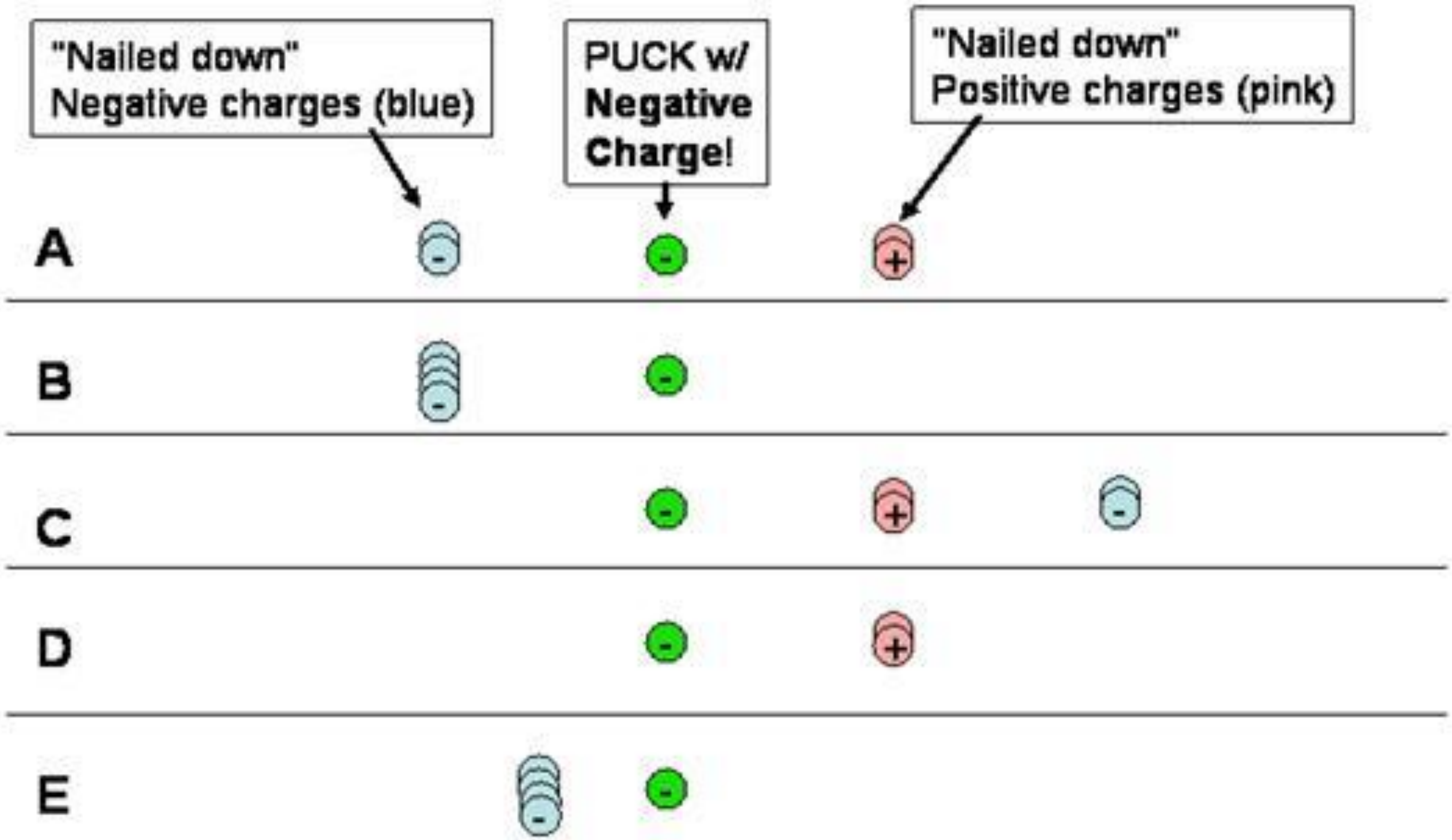
$$\Delta V = V(P_2) - V(P_1) = V(P_2') - V(P_1')$$

$$\vec{E} = E\hat{r}$$

$$\vec{E}\cdot d\vec{l} < 0$$

$$\int \vec{E}\cdot d\vec{l} < 0 \Rightarrow \Delta V > 0$$

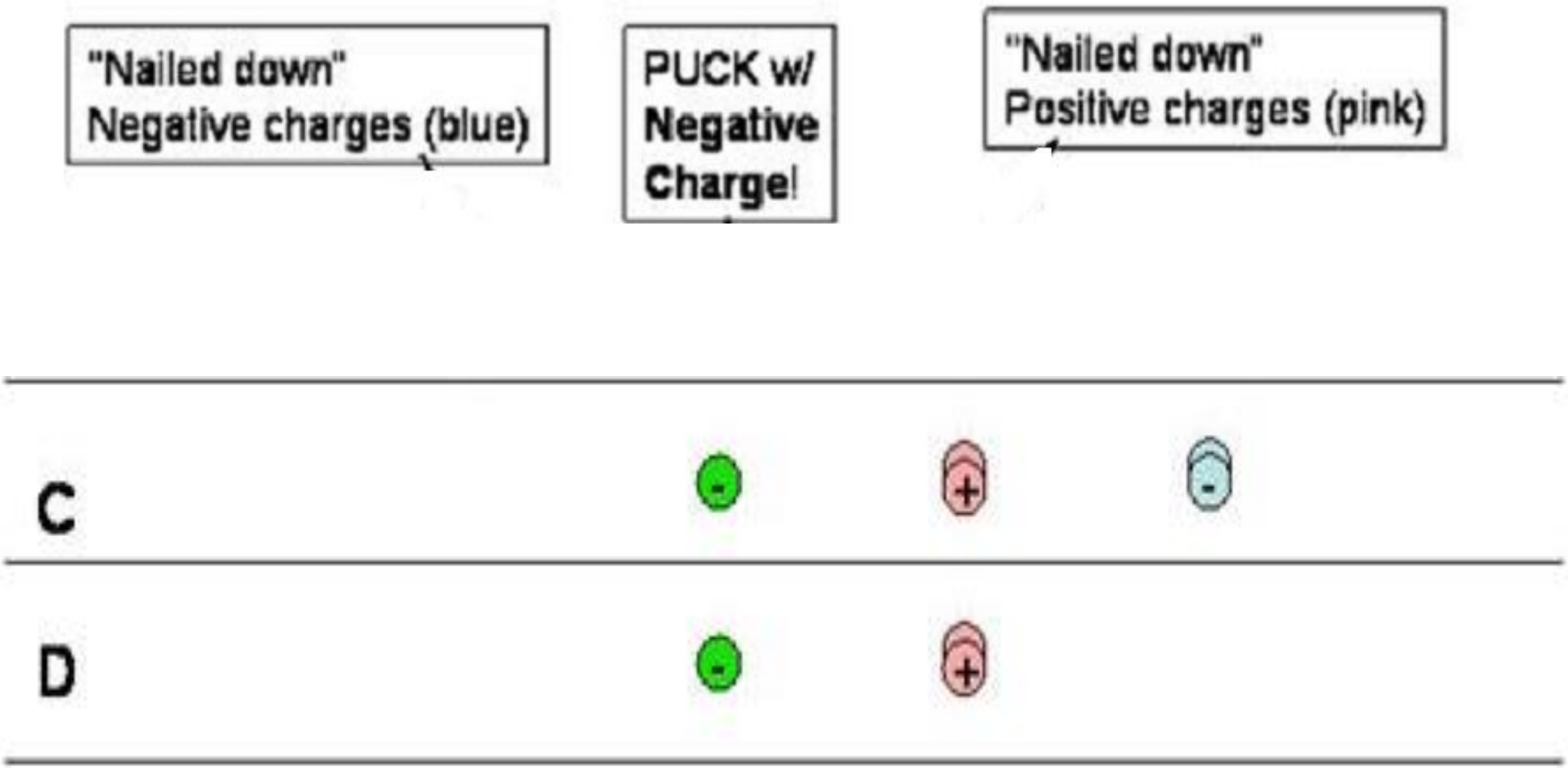
$$\oint \vec{E}\cdot d\vec{S} = 4\pi r^2 E = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$



All of the pucks feel a force to the right.

A. True B. False

Original question from Perkins University of Colorado

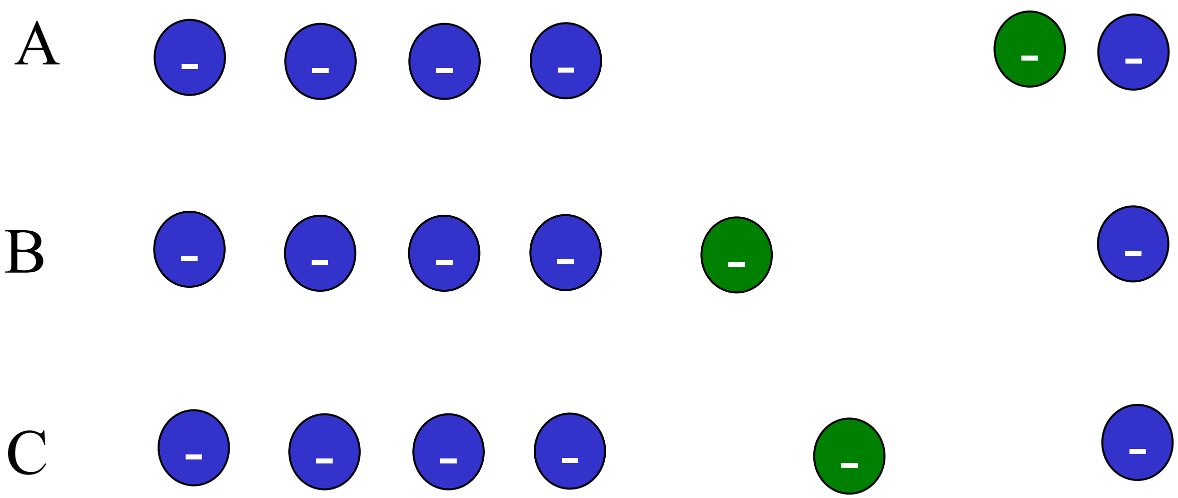


The puck  in C feels a greater force to the right than the puck in D.

A. True B. False

Original question from Perkins University of Colorado

For which of these choices is puck most likely not to move?



Original question from Perkins University of Colorado

If we put bunch of electrons in a box. They will

a. clump together.

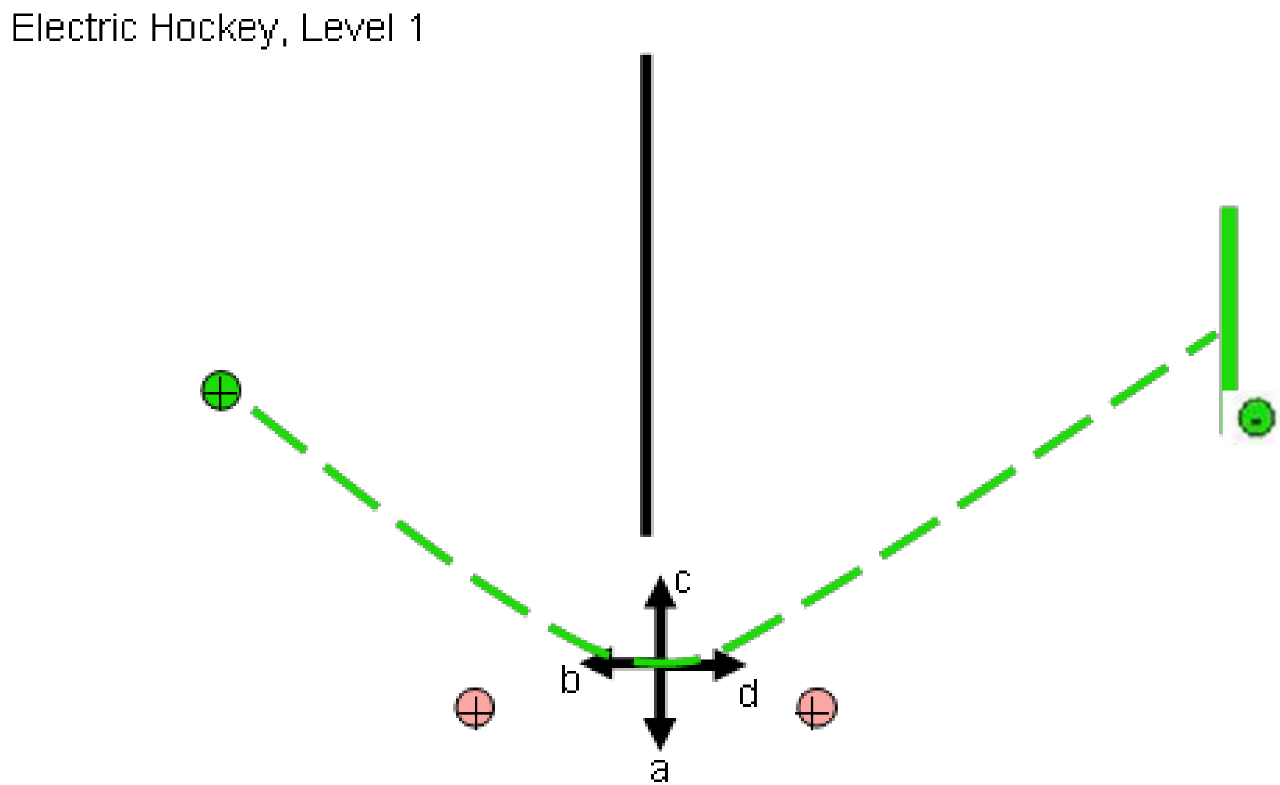
b. spread out uniformly across box.

c. make a layer on walls.

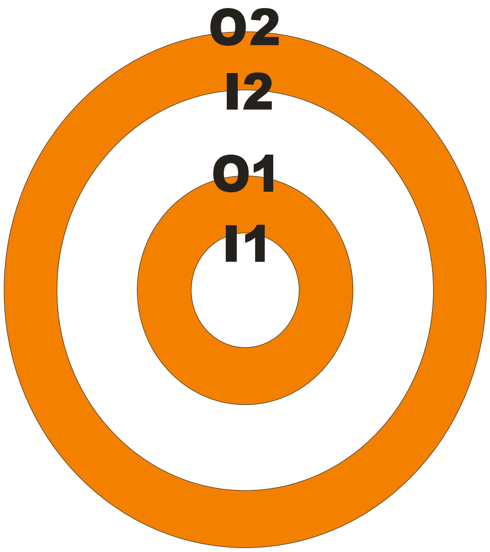
d. do something else.

Original question from Perkins University of Colorado

Which arrow best represents the direction of acceleration of the puck as it passes by the wall ?



Hollow Conductors



A point charge +Q is placed at the center of the conductors. The induced charges are:

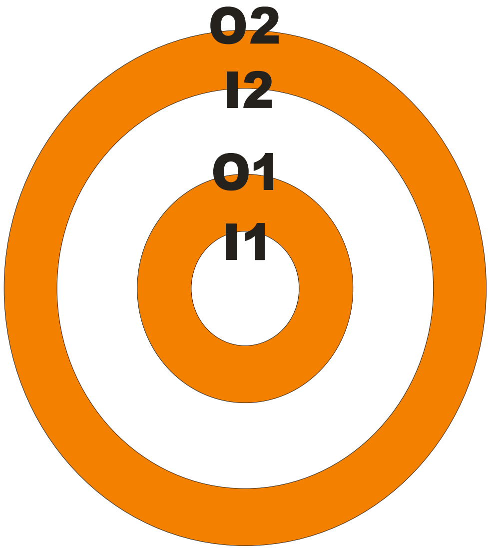
1. Q(I1) = Q(I2) = -Q;
   Q(O1) = Q(O2)= +Q

2. Q(I1) = Q(I2) = +Q;
   Q(O1) = Q(O2)= -Q

3. Q(I1) = -Q; Q(O1) = +Q
   Q(I2) = Q(O2)= 0

4. Q(I1) = -Q; Q(O2)= +Q
   Q(O1) = Q(I2)= 0

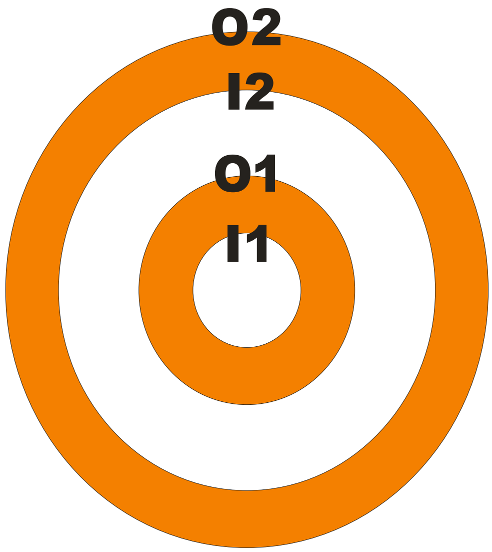
Hollow Conductors



A point charge +Q is placed at the center of the conductors. The potential at O1 is:

1. Higher than at I1
2. Lower than at I1
3. The same as at I1

Hollow Conductors



A point charge +Q is placed at the center of the conductors. The potential at O2 is:

1. Higher than at I1

2. Lower than at I1

3. The same as at I1