**Name and Surname:**
**Student Id:**
**Department:**
**Signature:**

## MIDTERM 1-INSTRUCTIONS

Read each question carefully. You should show and explain each one of your steps. A correct answer without any intermediate steps will not earn you any points. Any wrong units, any misplaced vector signs, etc. will cost you 2 points. You can use the formulas given on this page. If you want to use any other formulas, you have to derive them first. The questions might contain unnecessary information or insufficient information. If the question contains insufficient information, make any necessary assumptions. You will lose points if you make unnecessary assumptions. Unless required otherwise, express your answers in terms of the parameters given in the question.

| Question: | 1(20) | 2(20) | 3(25) | 4(25) | 5(30) | Total(100+20) |
|---|---|---|---|---|---|---|
| Grades: | | | | | | |

Formula Sheet

| | | | |
|---|---|---|---|
| $\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$ | $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$ | $\vec{v} = \frac{d\vec{r}(t)}{dt}$ | $\vec{a} = \frac{d\vec{v}}{dt}$ |
| $\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ | $\vec{v}(t) = \vec{v}_0 + \vec{a}t$ | $\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t} dt' \vec{v}(t')$ | $\vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t} dt' \vec{a}(t')$ |
| $a_c = \frac{v^2}{r}$ | $\vec{F} = m\vec{a}$ | $\lvert\vec{F}_{fr}\rvert = \mu_k \lvert\vec{N}\rvert$ if $v > 0$ | $\lvert F_{fr}\rvert < \mu_s \lvert\vec{N}\rvert$ if $v = 0$ |
| $\vec{F}_{12} = G_N \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$; $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ | | $\vec{F}_D = -b\vec{v}$ | $W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{\ell}$ |
| $W = \vec{F} \cdot \vec{r}$ | $\vec{A} \cdot \vec{B} = AB\cos\theta$ | $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ | |
| $i\frac{\partial \psi}{\partial t} = H\psi$ | $H = \frac{\vec{p}^2}{2m} + V(\vec{r})$ | $W = \Delta T$ | $T = \frac{1}{2}mv^2$ |
| $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$ | $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ | $\vec{\nabla} \cdot \vec{B} = 0$ | $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ |
| $G_N = 6.67 \times 10^{-11} N(m/kg)^2$ | | $\vec{\tau} = \vec{r} \times \vec{F}$ | $\tau = I\alpha$ |
| $I = \sum_i m_r R_i^2$ | $\vec{v} = \vec{\omega} \times \vec{r}$ | $\vec{a}_{tan} = \vec{\alpha} \times \vec{r}$ | $\lvert\vec{A} \times \vec{B}\rvert = AB\sin\theta$ |
| $\oint \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0}$ | $\epsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2$ | | $\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$ |
| $u_E = \frac{1}{2}\epsilon_0 \vec{E}^2$ | $C = K\epsilon_0 \frac{A}{d}$ | $Q = CV$ | $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$ |
| $P = VI$ | $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$ | $V = IR$ | $\lvert\vec{\mu}\rvert = IA$ |
| $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$ | | $d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \hat{r}}{r^2}$ | $d\vec{F} = Id\vec{\ell} \times \vec{B}$ |
| $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ | $\oint \vec{B} \cdot d\vec{S} = 0$ | $\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$ | $u_B = \frac{1}{2}\frac{\vec{B}^2}{\mu_0}$ |
| $\mu_0 = 4\pi \times 10^{-7}\ Tm/A$ | $\vec{L} = \vec{r} \times \vec{p}$ | $\vec{p} = m\vec{v}$ | $\frac{d^2x}{dt^2} + \omega^2 x = 0$, $T = \frac{2\pi}{\omega}$ |
| $\sin x \simeq x$, $x \ll 1$ | $\cos x \simeq 1 - \frac{x^2}{2}$, $x \ll 1$ | $(1+x)^n \simeq 1 + nx$, $x \ll 1$ | |
| $PV = Nk_bT$ | $PV^\gamma = const$; $\gamma = \frac{5}{3}$ | $U = -G_N \frac{m_1 m_2}{r}$ | $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$ |
| $e = 1.6 \times 10^{-19}\ C$ | | | |

## QUESTIONS

1. Mark the following questions as true or false. (2 points each, 20 points total)

i) F The potential of a given charge distribution is the work that needs to be done to bring the charges into their final configuration.

ii) T The force acting on an electric dipole in a uniform electric field is zero.

iii) F For a given charge distribution, the electric field at a point can change depending on whether some of the charges are in a conductor or not.

iv) T The force acting on an electric dipole in a non uniform electric field might be non-zero.

v) T The electric field is defined as the electric force felt by a unit test charge.

vi) F In electrostatics, the potential difference between two different points in a dielectric is always zero.

vii) T The electric field lines are perpendicular to equipotential surfaces.

viii) F If $\oint \vec{E} \cdot d\vec{S} = 0$ for a spherical Gauss' surface, it is necessary that $\vec{E} = 0$.

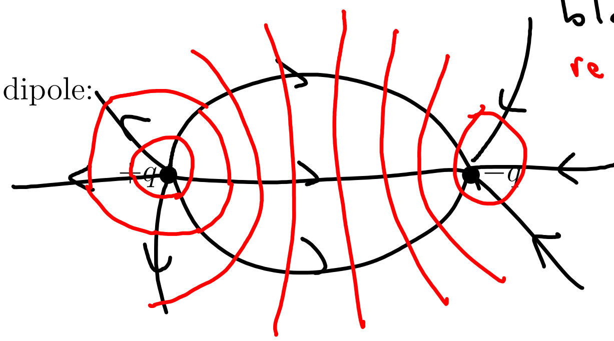
ix) F Consider a point charge. Around the point there is a spherical shell. Outside this spherical shell, there is a second spherical shell. Both of the shells are made of a conducting material, and they are not touching each other. The potential difference between the outer surface of the outer shell and the inner surface of the inner shell is zero.

x) T The electric field lines that start from a point charge inside the cavity in a conductor, terminate on the surface of the cavity.

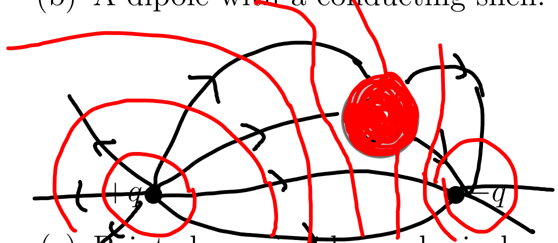
2. Sketch the electric field lines and equipotential surfaces for the following charge distributions (5 points each). (black dots represent point charges, gray regions represent conductors)

black lines: electric field lines
red lines: equipotential surfaces

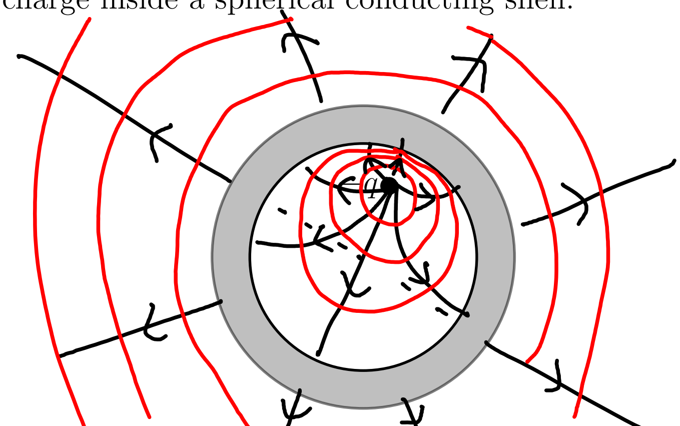
(a) A dipole:



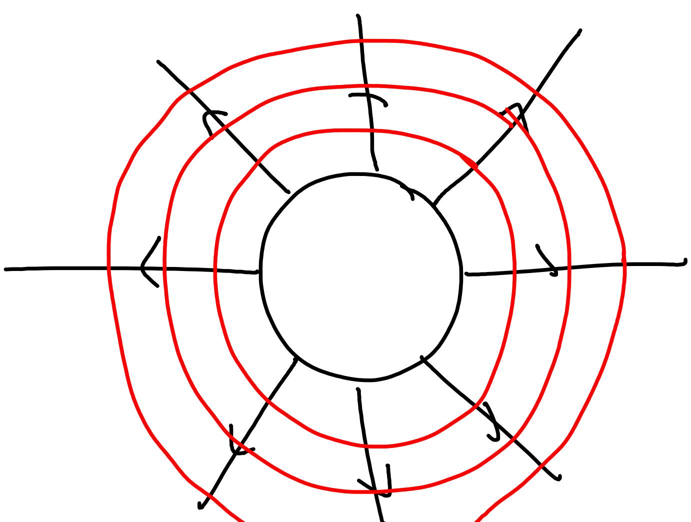

(b) A dipole with a conducting shell:

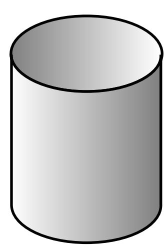



(c) Point charge inside a spherical conducting shell:

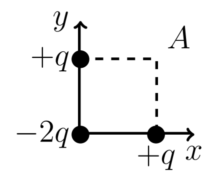



(d) A uniformly charged (with a positive charge) infinite cylindrical shell (sketch from an angle that sees only the cross-section of the cylinder):





3. Consider the three point charges shown in the figure. The charges are at $\vec{r}_1 = a\hat{x}$, $\vec{r}_2 = 0$ and $\vec{r}_3 = a\hat{y}$. (25 points total)




(a) What is the electric field at the point $A$? (5 points)

Using superposition principle and the electric field of a point charge

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{a^2}\hat{x} + \frac{q}{a^2}\hat{y} - \frac{2q}{2a^2}\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right]$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left[\hat{x}\left(1-\frac{1}{\sqrt{2}}\right) + \hat{y}\left(1-\frac{1}{\sqrt{2}}\right)\right] = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\frac{(\hat{x}+\hat{y})}{\sqrt{2}}(\sqrt{2}-1)$$

(b) What is the potential at the point $A$? (5 points)

Using superposition principle and the electric pot. of a point charge:

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} + \frac{q}{a} + \frac{(-2q)}{a\sqrt{2}}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}\left(2-\sqrt{2}\right)$$

(c) What is the electrostatic energy stored in this system? (5 points)

Bringing the charges one by one, the work done and hence the energy stored is

$$U = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q(-2q)}{a} + \frac{1}{4\pi\varepsilon_0}\left[\frac{qq}{a\sqrt{2}} + \frac{q(-2q)}{a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-4 + \frac{1}{\sqrt{2}}\right]$$

(d) How much work should be done to bring a charge $q$ to the point $A$ from infinity? (5 points)

The work can be calculated using

$$W = qV_A$$

Using the result of part (b)

$$W = q\frac{1}{4\pi\varepsilon_0}\frac{q}{a}(2-\sqrt{2}) = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}(2-\sqrt{2})$$

(e) Consider a spherical surface whose centre is at the position of charge $-2q$ and its surface contains the point $A$. What is $\oint \vec{E} \cdot d\vec{S}$ integrated over this surface?

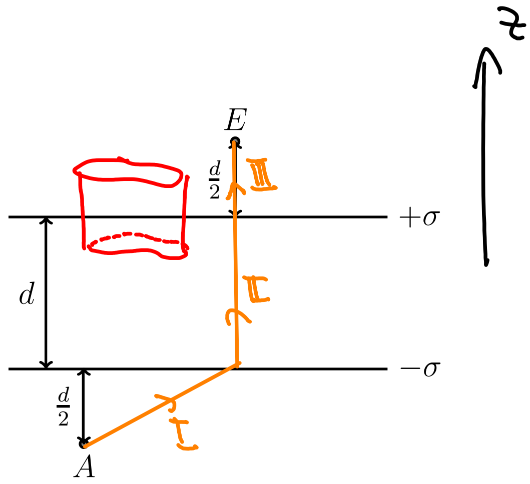
Using Gauss' Law $\oint \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$.

The described surface encloses all three charges. Hence $Q_{enc} = q + q + (-2q) = 0$.

Therefore

$$\oint \vec{E} \cdot d\vec{S} = 0$$

4. Consider two infinite parallel plates as shown in the figure. The plates are separated by a distance $d$. The upper plate is positively charged and the lower plate is negatively charged. The potential at the lower plate is set to be zero. There are also two points, $A$ and $E$, shown in the figure (there are not point charges at the points $A$ and $E$.). (25 points)




(a) Using Gauss' law, calculate the electric field everywhere. (you should also explain why you choose a particular Gauss' surface) (10 points)

Each plate will create an electric field in the vertical direction. Since the field lines will be parallel, they will be uniform, and hence the electric field will have constant magnitude. In the regions where the points A and E lies, the electric field created by the $+\sigma$ and $-\sigma$ plates will be of equal magnitude and opposite direction, hence they will cancel each other. In the region between the plates, there will be a uniform non-zero electric field. Let's choose a Gauss' surface of the shape of a right cylinder with the base having an arbitrary shape as shown in the above figure. There will only be a contribution to the Gauss' integral from the below surface:
Denoting $\vec{E} = E_0 \hat{z}$, and for the below surface $d\vec{S} = dS(-\hat{z})$

$$\oint \vec{E} \cdot d\vec{S} = \int_{\substack{\text{below} \\ \text{surface}}} -E_0 \, dS = -E_0 A = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow E_0 = -\frac{\sigma}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{z}}$$

(b) Using the definition of the potential difference $V_{AE} \equiv V_E - V_A \equiv -\int \vec{E} \cdot d\vec{\ell}$, calculate the potential difference between the points $A$ and $E$ (make sure that you specify also the path along which you are calculating the integral) (5 points)

Consider the orange path shown. Along part I and part III, $\vec{E} = 0$. Hence

$$V_{AE} = -\int_{\vec{E}} \vec{E} \cdot d\vec{l} = -\int_{II} \vec{E} \cdot d\vec{l}$$

along path II, $\vec{E} = -\frac{\sigma}{\varepsilon_0}\hat{z}$. Let $d\vec{l} = dl\,\hat{z}$ Hence

$$V_{AE} = +\int \frac{\sigma}{\varepsilon_0} dl = \frac{\sigma}{\varepsilon_0} d$$

(c) What is the potential difference between the plates? (5 points)

Since, out of the plates, $\vec{E} = 0$

The potential difference between the plates is equal to $V_{AE}$:

$$V = V_{AE} = \frac{\sigma}{\varepsilon_0} d$$

(d) If the space between the plates is filled by a dielectric with dielectric constant $K$, what would be $V_{AE}$? (5 points)

The dielectric reduces the electric field by a factor $K$, hence the potential difference is also reduced by a factor $K$:

$$V \longrightarrow \frac{V}{K} = \frac{\sigma}{K\varepsilon_0} d$$

5. In this question, you will analyse a possible difference between the charges of the electron and proton. Let the charge of the electron be $q_e = -e$ and the charge of the proton $q_p = e(1+\delta)$. Experimentally the constant $\delta$ is known to be $|\delta| < 10^{-21}$. Consider three spherical masses, two of which is of C, and the other one of Fe. The atomic masses of C and Fe are 12.0107 amu and 55.845 amu (1 amu= $1.660 \times 10^{-27}$ kg). (30 points)

   (a) How many protons are in each one of the masses? (5 points)

The number of C atoms is $\dfrac{1\,kg}{12.0107 \times 1.66 \times 10^{-27}\,kg} = 5.01 \times 10^{25}$

Each C atom contains 6 protons. Hence number of protons in the C spheres are

$n^p_c = 3.0 \times 10^{26}$

Similarly the number of protons in the Fe sphere are

$n^p_{Fe} = 26 \dfrac{1kg}{55.845 \times 1.66 \times 10^{-27}\,kg} = 2.8 \times 10^{26}$

   (b) Assuming that the number of electrons in each mass is equal to the number of protons in each mass, what is the net charge of each mass? (5 points)

The charge of the C sphere:

$Q_c = n^p_c e(1+\delta) + n^p_c(-e) = n^p_c e\delta = 4.8 \times 10^7 \delta$ C

Similarly for the Fe sphere:

$Q_{Fe} = n^p_{Fe} e(1+\delta) + n^p_{Fe}(-e) = n^p_{Fe} e\delta = 4.5 \times 10^7 \delta$ C

   (c) Consider the two masses made of C. If their centres are at a distance of $d = 1$ $m$, what is the gravitational force that they exert? what is the electrostatic force they exert on each other? (5 points)

Gravitational force $\vec{F}_G = -G_N \dfrac{(1kg)^2}{(1m)^2} \hat{r} = (-6.67\, 10^{-11}\, N)\hat{r}$

Electrostatic force $\vec{F}_E = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q_c^2}{(1m)^2}\hat{r} = (2.1\, 10^{25}\, \delta^2\, N)\hat{r}$

(d) Repeat the previous part with one mass made of C and the other one made of Fe. (5 points)

Since the masses are the same, gravitational force does not change

$$\vec{F}_G = -G_N \frac{(1kg)^2}{(1m)^2}\hat{r} = \left(-6.67\ 10^{-11}\ N\right)\hat{r}$$

Electrostatic force $\vec{F}_E = \frac{1}{4\pi\varepsilon_0}\frac{Q_C Q_{Fe}}{(1m)^2}\hat{r} = \left(1.9\ 10^{25}\ \delta^2\ N\right)\hat{r}$

(e) What is the difference of the magnitudes of the net forces exerted on the spheres in each case? (5 points)

The difference is only due to the electrostatic force and hence is given by

$$\boxed{0.2\ 10^{25}\ \delta^2\ N}$$

(f) Assuming that this difference is less than 1% of the average force in both cases, what is an upper limit on $\delta$? (5 points)

Since the electrostatic force would be negligible compared to the gravitational force, the average of the forces is equal to the gravitational force. Taking

$$\frac{0.2\times10^{25}\ \delta^2\ N}{6.67\times10^{-11}\ N} = 3.0\times10^{34}\ \delta^2 < 10^{-2}$$

$$\Rightarrow \boxed{\delta < 5.8\times10^{-19}}$$