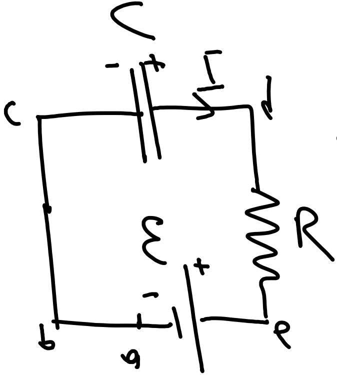
April 7, 2015

$$\oint \vec{E}\cdot d\vec{l} = 0 \Rightarrow \Delta V = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{l}$$



$$\oint \vec{E}\cdot d\vec{l} = 0$$

a → b → c → d → e → a

⇒ sum of all pot drops.

$$0 + 0 - \frac{Q}{C} + IR + \mathcal{E} = 0$$

$$I = -\frac{dQ}{dt}$$

$$-\frac{Q}{C}-\frac{dQ}{dt}R+\mathcal{E}=0$$

$$\frac{dQ}{dt}+\frac{Q}{RC}=\frac{\mathcal{E}}{R}$$

$$\boxed{\frac{dQ}{dt}=-\frac{Q}{RC}+\frac{\mathcal{E}}{R}}$$

$$Q=Ae^{\alpha t}+B$$

$$\frac{dQ}{dt}=\alpha Ae^{\alpha t}=\alpha(Q-B)=\alpha Q-\alpha B$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}e^x = e^x$$

$$\left[(\sin x)^{\cos x}\right]^{e^x}$$

$$e^{ix} \equiv \cos x + i\sin x$$

Euler Formula

$$Q = Ae^{\alpha t} + B$$

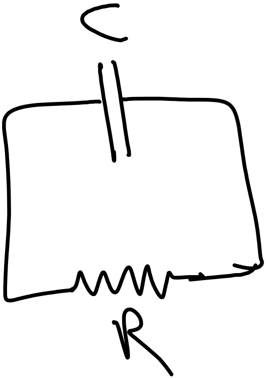
$$\frac{dQ}{dt} = \alpha A e^{\alpha t} = \alpha (Q - B) = \alpha Q - \alpha B$$

$$\frac{dQ}{dt} = -\frac{Q}{RC} + \frac{\mathcal{E}}{R}$$

$$\Rightarrow \alpha = -\frac{1}{RC} \qquad -\alpha B = \frac{\mathcal{E}}{R} \Rightarrow B = -\frac{\mathcal{E}}{\alpha R} = \mathcal{E} C$$

$$\boxed{Q(t) = A e^{-t/RC} + \mathcal{E} C}$$

Example



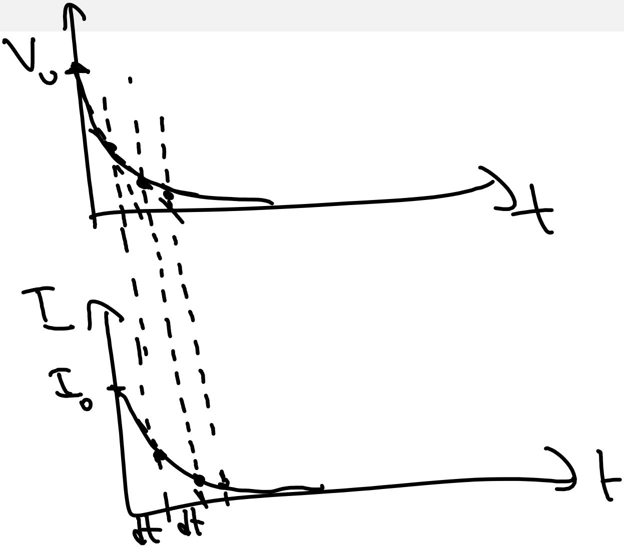
$$\mathcal{E} = 0$$

$$\boxed{Q(t) = A e^{-t/RC}}$$

$$Q(t=0) = Q_0 = A$$

$$\boxed{Q(t) = Q_0 e^{-t/RC}}$$

V



$$Q(t) = \mathcal{E} C \left(1 - e^{-t/RC}\right)$$

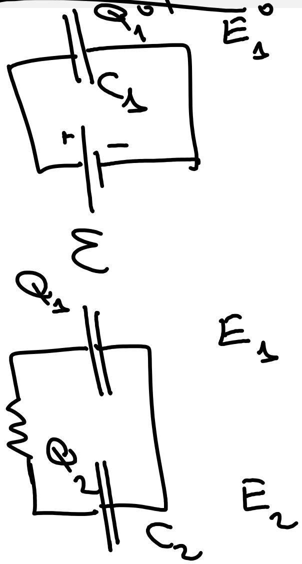
if $Q(t=0) = 0$

$$t = 5\tau = 5RC$$

$$e^{-5} \simeq \frac{1}{2.7^5} \simeq \frac{1}{3^5} \approx \frac{1}{250} \simeq 0.004$$

$$Q(5\tau) = Q_{max}\, 0.996$$

Example

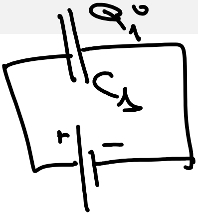


$$E_1^0 \neq E_1 + E_2$$

$$Q_1^0 = C_1 \mathcal{E}$$

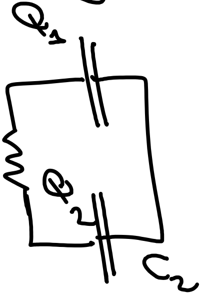
$$E_1^0 = \frac{1}{2} C V^2 = \frac{1}{2} C_1 \mathcal{E}^2$$

$$\boxed{E_1^0 = \frac{1}{2} C_1 \mathcal{E}^2}$$



E_1^0

$\mathcal{E}$



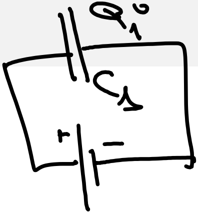
E_1

E_2

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_2 = \frac{C_2}{C_1} Q_1$$

$$Q = Q_1 + Q_2 = \frac{C_1 + C_2}{C_1} Q_1$$

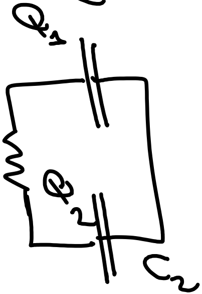
$$\boxed{\begin{aligned} Q_1 &= \frac{C_1}{C_1 + C_2} Q \\ Q_2 &= \frac{C_2}{C_1 + C_2} Q \end{aligned}}$$



E_1^0

$$E_1 = \frac{Q_1^2}{2C_1} = \frac{1}{2C_1}\frac{C_1^2}{(C_1+C_2)^2}Q^2$$

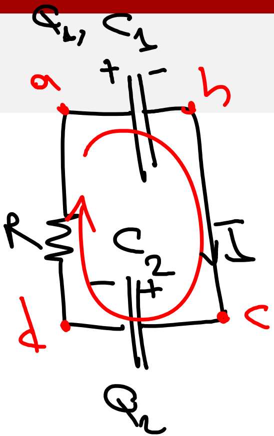
$$E_1 = \frac{C_1}{2(C_1+C_2)^2}Q^2$$



E_1

$$E_2 = \frac{C_2}{2(C_1+C_2)^2}Q^2$$

$$E_2 \quad E_1 + E_2 = \frac{1}{2}\frac{Q^2}{(C_1+C_2)} \neq \frac{1}{2}\frac{Q^2}{C_1}$$



$$\frac{Q_1}{C_1} + 0 + \frac{Q_2}{C_2} + IR = 0$$

$$|Q_1| + |Q_2| = Q$$

$$Q_1 - Q_2 = Q$$

$$I = +\frac{dQ_1}{dt} = +\frac{dQ_2}{dt}$$

$$dQ_1 = dQ_2$$

$$Q_2 = Q_1 - Q$$

$$\frac{Q_1}{C_1} + 0 + \frac{Q_2}{C_2} + IR = 0$$

$$\frac{Q_1}{C_1} + \frac{Q_1 - Q}{C_2} + IR = 0$$

$$Q_1\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{dQ_1}{dt}R - \frac{Q}{C_2} = 0$$

$$\frac{dQ_1}{dt} + \frac{Q_1}{RC_{eq}} - \frac{Q}{RC_2} \Rightarrow \boxed{Q_1(t) = Ae^{-\frac{t}{RC_{eq}}} + Q\frac{C_{eq}}{C_2}}$$

$$Q_1(t) = Ae^{-\frac{t}{RC_{eq}}} + Q\frac{C_{eq}}{C_2}$$

$$Q_1(t=0) = A + Q\frac{C_{eq}}{C_2} = Q$$

$$\Rightarrow A = Q\left(1 - \frac{C_{eq}}{C_2}\right)$$

$$\Rightarrow Q_1(t) = Q\left[\left(1 - \frac{C_{eq}}{C_2}\right)e^{-\frac{t}{RC_{eq}}} + \frac{C_{eq}}{C_2}\right]$$

$$Q_1(t) = Qe^{-t/RC_{eq}} + Q\frac{C_{eq}}{C_2}\left(1 - e^{-t/RC_{eq}}\right)$$

$$Q_1(t) = Q e^{-t/RC_{eq}} + Q \frac{C_{eq}}{C_2}\left(1 - e^{-t/RC_{eq}}\right)$$

$$I = \frac{Q}{RC_{eq}}\left(-e^{-t/RC_{eq}}\right) + Q \frac{\cancel{C_{eq}}}{C_2} \frac{1}{R\cancel{C_{eq}}} e^{-\frac{t}{RC_{eq}}}$$

$$I = -\frac{Q}{R} e^{-\frac{t}{RC_{eq}}}\left(\frac{1}{C_{eq}} - \frac{1}{C_2}\right)$$

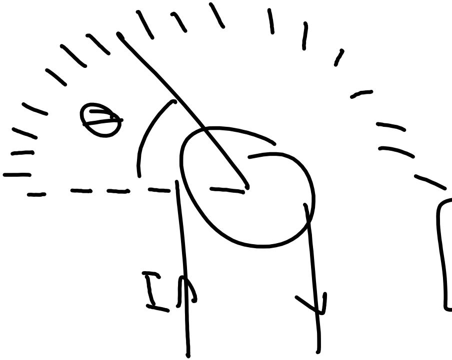
$$I = -\frac{Q}{R} e^{-\frac{t}{RC_{eq}}} \left(\frac{1}{C_{eq}} - \frac{1}{C_2} \right).$$

$$\boxed{I = -\frac{Q}{RC_1} e^{-\frac{t}{RC_{eq}}}}$$

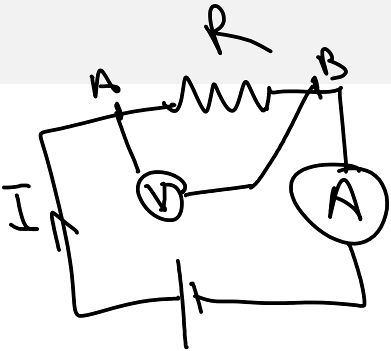
$$P = I^2 R = \frac{Q^2}{C_1^2 R} e^{-\frac{2t}{RC_{eq}}}$$

$$W = \int_0^\infty P\,dt = \frac{Q^2}{\cancel{R} C_1^2} \frac{\cancel{R} C_{eq}}{2} = \frac{Q^2 C_2}{2C_1(C_1+C_2)}$$

Ammeter, Voltmeters, Galvanometer, ...

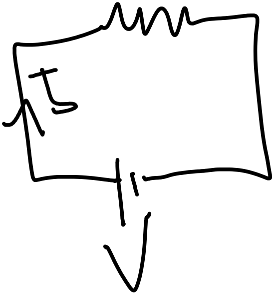


$$\boxed{\theta \propto I}$$



$R_{ammeter} \ll R$

$R_{voltmeter} \gg R$



$$V = IR$$

$$I = \frac{V}{R}$$