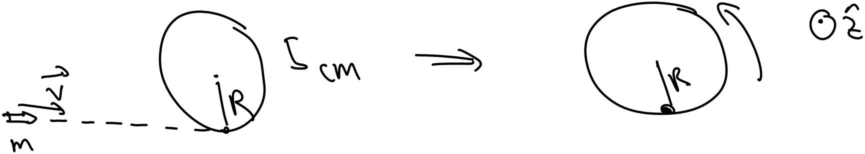
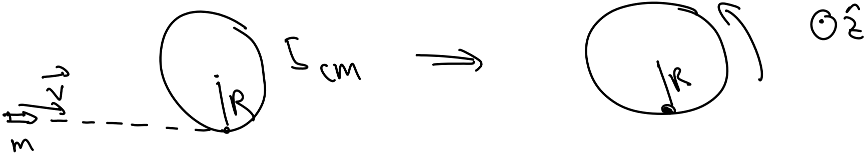
Example



Energy Conservation $K_i = \frac{1}{2} m v^2$

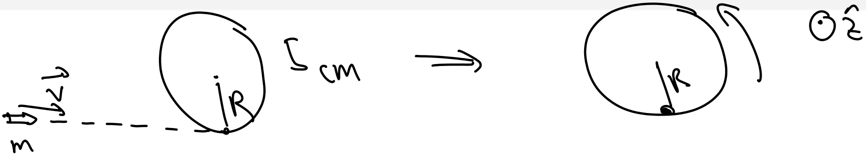
$$K_f = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} m (\omega R)^2$$

if $K_i = K_f \Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} \left(I_{cm} + m R^2 \right) \omega^2$



Conservation of momentum.

$$\frac{d\vec{P}_{cm}}{dt} = \vec{F}^{ext}$$



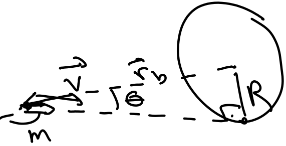
Conservation of angular mom:

$$\frac{d\vec{L}}{dt} = \vec{\tau}^{\,ext} \qquad \vec{\tau}^{\,ext} = \sum \vec{r} \times \vec{F}^{ext}$$

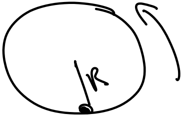
$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$ origin on the axis

$\vec{\tau} = 0 \Rightarrow \vec{L}$ is conserved

$$I = \sum m_i R_i^2$$



I_{cm} $\Longrightarrow$



$\odot \hat{z}$

$\vec{\omega} = \omega \hat{z}$

$$\vec{L}_i = \sum \vec{r}_i \times \vec{p}_i$$
$$= \vec{r}_b \times (m\vec{v})$$
$$= r_b m v \sin(\pi - \theta) \hat{z}$$
$$\vec{L}_i = m v r_b \sin\theta \, \hat{z}$$
$$= m v R \, \hat{z}$$

$$\vec{L}_f = \sum \vec{r}_i \times \vec{p}_i$$
$$= I_{cm} \vec{\omega} + mR^2 \vec{\omega}$$
$$= (I_{cm} + mR^2) \, \omega \hat{z}$$
$$= m v R \, \hat{z}$$
$$\Rightarrow \omega = \frac{m v R}{I_{cm} + mR^2}$$

$$K_i = \frac{1}{2} m v^2$$

$$\Rightarrow \omega = \frac{m v R}{I_{cm} + mR^2}$$

$$K_f = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} m (\omega R)^2$$

$$= \frac{1}{2}\cancel{(I_{cm} + mR^2)} \frac{(m v R)^2}{(I_{cm} + mR^2)^{\cancel{2}}} = \frac{1}{2} m v^2 \left[\frac{mR^2}{I_{cm} + mR^2}\right]$$

$$K_f - K_i = \frac{1}{2} m v^2 \left[\frac{mR^2}{I_{cm} + mR^2} - 1\right]$$

$$= \frac{1}{2} m v^2 \left(\frac{-I_{cm}}{I_{cm} + mR^2}\right)$$

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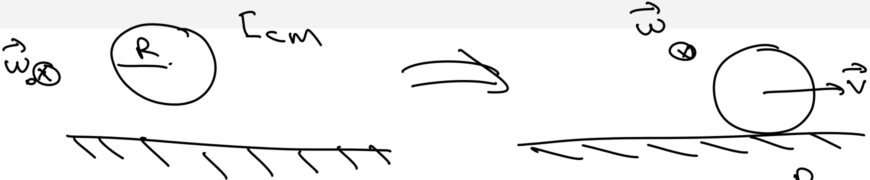


μ_k

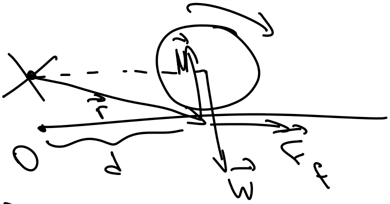
$v = \omega R$

$\omega = ?$

Energy ✗

Linear momentum ✗

Angular momentum ?



$\vec{\tau}^{ext} = ?$

$\vec{\tau} = \vec{r} \times \vec{F}_f \quad \odot$

$\neq 0$

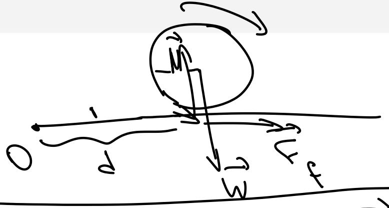
$|\vec{\tau}| = r\, F \sin\theta$

$\vec{\tau}_0^{F_f} = 0$

$\left.\begin{aligned} \vec{\tau}_0^{N} &= N d \;\hat{z} \\ \vec{\tau}_0^{W} &= W d\,(-\hat{z}) \end{aligned}\right\} N = W$

$+$

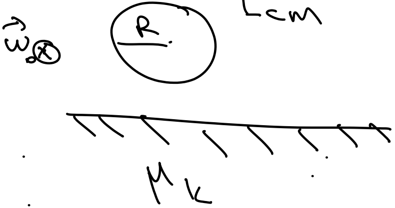
$\vec{\tau}_0 = 0 \Rightarrow \dfrac{d\vec{L}}{dt} = 0 \rightarrow \vec{L}$ is conserved



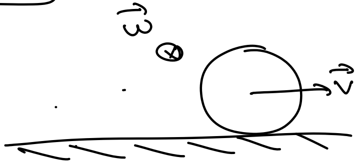
$$\frac{d\vec{p}}{dt} = \vec{F}^{ext} = \vec{F}_f \neq 0$$

$$\boxed{\vec{L} = \vec{r}_{cm} \times \vec{p}_{cm} + \vec{L}_{cm}}$$

I_{cm}

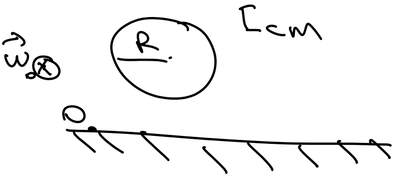


$\Longrightarrow$

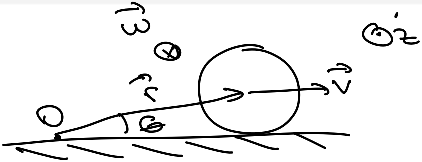


$v = \omega R$

$\omega = ?$



$\Longrightarrow$



$v = \omega R$

$\omega = ?$

$$\vec{L} = \vec{r}_{cm} \times \vec{p}_{cm} + \vec{L}_{cm}$$

$$\vec{L}_i = I_{cm} \vec{\omega}_0$$

$$\vec{\omega}_0 = \omega_0 (-\hat{z})$$

$$\vec{L}_i = -I_{cm} \omega_0 \hat{z}$$

$\equiv$

$$\vec{L}_f = Rmv(-\hat{z}) + I_{cm} \vec{\omega}$$

$$\vec{\omega} = \omega(-\hat{z})$$

$$\vec{L}_f = (-m\omega R^2 - I_{cm}\omega)\hat{z}$$

$$\Longrightarrow \boxed{\omega = \frac{I_{cm}\omega_0}{I_{cm} + mR^2}} < \omega_0$$

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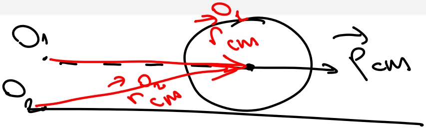
$$\vec{L}_i^{O_1} = I\vec{\omega}_0$$

$$\vec{L}_i^{O_2} = I\vec{\omega}_0$$

$$\vec{L} = \vec{r}_{cm} \times \vec{p}_{cm} + \vec{L}_{cm}$$



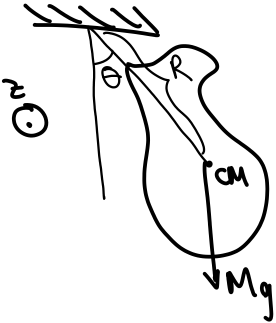
$$\vec{L}_f^{O_1} = I\vec{\omega}$$

$$\vec{L}_f^{O_2} = \vec{r}_{cm} \times \vec{p}_{cm} + I\vec{\omega}$$

$$\vec{r}_{cm}^{\,O_1} \times \vec{p}_{cm} = 0$$

$$\vec{r}_{cm}^{\,O_2} \times \vec{p}_{cm} \neq 0$$

Oscillations



Let I_{CM} be the moment of inertia around CM.

$$I = I_{CM} + MR^2$$

$$\vec{\tau} = -MgR\sin\Theta\,\hat{z}$$

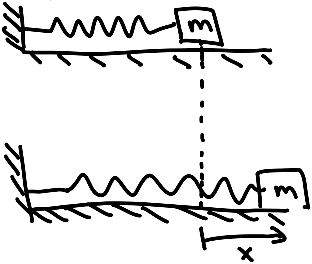
$$\vec{\alpha} = \frac{d^2\Theta}{dt^2}\hat{z}$$

$$-MgR\sin\Theta = I\frac{d^2\Theta}{dt^2}$$

For $\Theta \ll 1$, $\sin\Theta \simeq \Theta$

$$\boxed{\frac{d^2\Theta}{dt^2} + \left(\frac{MgR}{I}\right)\Theta = 0}$$

Equilibrium:



$$F = -kx$$

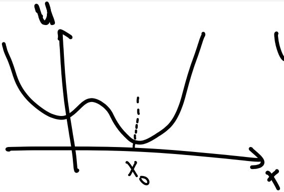
$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Compare with:

$$\frac{d^2\theta}{dt^2} + \left(\frac{MgR}{I}\right)\theta = 0$$

Motion Close to a Minimum of Potential



$$U(x) = U(x_0) + \left.\frac{dU}{dx}\right|_{x=x_0}(x-x_0) + \left.\frac{d^2U}{dx^2}\right|_{x=x_0}\frac{(x-x_0)^2}{2} + \dots$$

If x_0 is a minimum

$$\left.\frac{dU}{dx}\right|_{x=x_0} = 0 \quad \& \quad \left.\frac{d^2U}{dx^2}\right|_{x=x_0} > 0$$

Then $U(x) \simeq U(x_0) + \frac{1}{2}\left.\frac{d^2U}{dx^2}\right|_{x=x_0}(x-x_0)^2$

$$F(x) \simeq -\frac{dU}{dx} = -k(x-x_0)$$

where $k = \frac{d^2U}{dx^2}$. This is nothing but Hooke's Law

In physics, close to minimum energy everything behaves like a spring!