$$\frac{d^2x}{dt^2} + w^2 x = 0 \qquad w^2 = \frac{k}{m}$$

Let us try the solution

$$x(t) = A\cos(wt+\delta)$$

$$\frac{dx}{dt} = -Aw\sin(wt+\delta)$$

$$\frac{d^2x}{dt^2} = -Aw^2\cos(wt+\delta) = -w^2 x$$

$$\therefore \frac{d^2x}{dt^2} + w^2 x = 0$$

i.e. $x(t) = A\cos(wt+\delta)$ is the solution

$$x(t) = A\cos(\omega t + \delta) = A\cos\delta\cos\omega t - A\sin\delta\sin\omega t$$

$$x_0 \equiv x(0) = A\cos\delta$$

$$v_0 \equiv \left.\frac{dx}{dt}\right|_{t=0} = -A\omega\sin\delta$$

$$\Rightarrow x(t) = x_0\cos\omega t + \frac{v_0}{\omega}\sin\omega t$$

is the solution of the simple harmonic oscillator (SHO)

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# Energy of a SHO

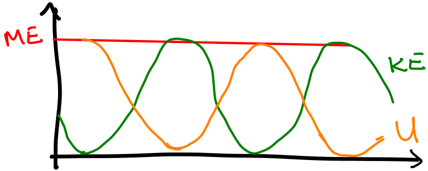
$$x(t) = A\cos(\omega t+\delta) \Rightarrow v(t) = -A\omega\sin(\omega t+\delta)$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\delta)$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t+\delta)$$

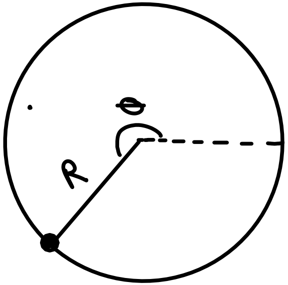
$$= \frac{1}{2}(m\omega^2)A^2\cos^2(\omega t+\delta)$$

$$ME = KE+U = \frac{1}{2}m\omega^2A^2$$




Energy oscillates between kinetic and potential energies

# Relation with Uniform Circular Motion




$\theta = wt + \theta_0$

$\vec{r}(t) = x(t)\,\hat{x} + y(t)\,\hat{y}$

$x(t) = R\cos\theta = R\cos(wt+\theta_0)$

$y(t) = R\sin\theta = R\cos(wt+\theta_0 - \frac{\pi}{2})$

Both x and y coordinates make SHO

# Damped Oscillator

Assume that there is a drag force also

$$F_D = -bv = -b\frac{dx}{dt}$$

then

$$m\frac{d^2x}{dt^2} = -kx - bv$$

$$\Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

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# Damped Oscillator

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

The soln. is $x(t) = Ae^{-\gamma t}\cos(\omega t + \delta)$

where

$$\gamma = \frac{b}{2m} \qquad \omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

<u>Note:</u> Frequency is smaller

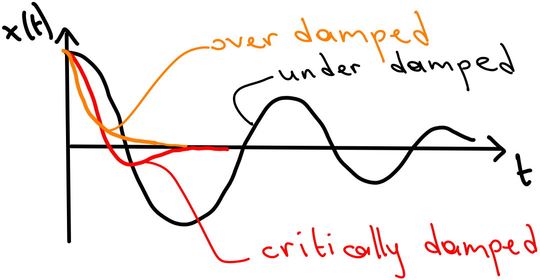
$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

**Cases:**

$b < \sqrt{4km}$ : under damped

$b = \sqrt{4km}$ : critically damped

$b > \sqrt{4km}$ : over damped



# Energy of the Damped Oscillator

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 e^{-2\gamma t}\cos^2(\omega t+\delta)$$

$$KE = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}mA^2 e^{-2\gamma t}\left[\gamma\cos(\omega t+\delta)+\omega\sin(\omega t+\delta)\right]^2$$

If damping is small ($\gamma \ll \omega$)

$$KE \simeq \frac{1}{2}mA^2 e^{-2\gamma t}\omega^2\sin^2(\omega t+\delta)$$

$$m\omega^2 \simeq k$$

$$\boxed{ME \simeq \frac{1}{2}kA^2 e^{-2\gamma t}}$$

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