Prof. Dr. Deniz Uner
Chemical Engineering Middle East Technical University Ankara Turkey

SELF STUDY MODULE

Unsteady state material and energy balances

Objective

Material and energy balances under unsteady state conditions. General conservation laws will be derived and applied.

Vocabulary

System is defined as the part of the universe that we are interested in. We define the rest of the universe as the **surroundings**. System and the surroundings are connected through a real or a hypothetical boundary. The boundary connecting (or separating) the system and the surroundings is called the **control volume** or sometimes **control surface**.

The system can be open, closed or isolated. The open and closed systems definitions is towards mass exchange. If a system allows mass exchange it is called an **open** system. If a system does not allow mass exchange, the system is **closed**.

Derive

First we will derive a general balance equation for the conserved quantities. Within the context of the course, we define the conserved quantities as mass and energy.

The open system general balance equation

$$\begin{bmatrix} time\ rate\ of\ change \\ of\ the\ conserved \\ quantity\ within\ the \\ system\ boundaries \end{bmatrix} = \begin{bmatrix} rate\ at\ which \\ conserved\ quantity \\ enters\ through\ the \\ system\ boundaries \end{bmatrix} - \begin{bmatrix} rate\ at\ which \\ conserved\ quantity \\ leaves\ through\ the \\ system\ boundaries \end{bmatrix}$$

The first term defines the unsteady state changes that can take place within the system boundaries. For example, we can be filling a bucket with water, and the amount of water in the bucket is changing with respect to time. Or conversely, we could be emptying a tank and the mass in the tank could change with respect to time.

The second term defines the rate at which the conserved quantity enters through our system such as the flow rate of water with which we are filling the bucket. The final term is a term for the rate at which the conserved quantity leaves our system boundaries.

As a result, the general mass conservation law can be written as

$$\frac{dm}{dt} = \sum_{in} m_i - \sum_{out} m_j$$

Similarly in its simplified form, the general conservation of energy can be written as

$$\frac{dU}{dt} = \sum_{in} m_i h_i - \sum_{out} m_j h_j + \dot{Q} + \dot{W} - P\frac{dv}{dt}$$

Calculate

1. A 25 L LPG tank is being filled through a line delivering gas at 80 bar and 300 K. The flow rate of the gas into the tank is constant at 1 kg/s. The filling is completed after the flow stops under natural forces.
   a. What is the temperature and the pressure of the gas in the tank initially? How many number of moles? Why do we call the tank empty?

   b. Answer the following for the period immediately after the mechanical equilibrium:
      i. What is the pressure of the gas in the tank?
      ii. What is the temperature of the gas in the tank?
      iii. How many moles of gas are there in the tank?

   c. Answer the following for the period after the thermal equilibrium:
      i. What is the temperature of the gas in the tank?
      ii. What is the pressure of the gas in the tank?

2. A water tank is draining under natural forces through an orifice at the bottom of a tank. The water level in the tank is measured as a function of time and the data is provided below.

Time (s)	0	1.5	3.3	5.2	7.2	9.4	12.0	15.1	19.0	21.3
Level in the tank (cm)	45	40	35	30	25	20	15	10	5	3

   a. Determine the relationship between the mass flow rate of the fluid and the height of the tank if we assume that there is a constitutive relationship between the mass and the tank height as follows $\dot{m} = ah^b$
   b. Using these constants, generate a plot of height as a function of time that has the original data as well as the prediction of the equation you have derived.
   c. If you are interested, you can also generate your own data using a disposable plastic water bottle. Compare your results with the given data. Comment on the differences.

3. 1 L of water initially at room temperature is in a teapot is on the stove. At time t=0, the stove is turned on and heat is being transferred at a constant rate of 100 W. Using your skills in material and energy balances answer the following questions:
   a. Derive an expression for the amount of water in the teapot as a function of time until water starts to boil. State all of your assumptions clearly.
   b. Derive an expression for the temperature in the teapot as a function of time until water starts to boil. State all of your assumptions clearly.
   c. How does the temperature of water change as a function of time after boiling starts?
   d. How does the amount of water change as a function of time after boiling starts?

4. Write down the unsteady state material balance around the ventricle and atrium of the heart both in the right and in the left. Stepwise include the blood circulation system in your analysis as well as the lungs.

5. How much time is needed to evaporate water in a tea glass (approximately 100 ml capacity) at room temperature, if the rate of evaporation is considered as constant at 1 ml/hour? List all the assumptions you make.

6. How much energy is required to evaporate all of the water in the tea glass at room temperature?

Bibliography

S. Sandler Chemical Biochemical and Engineering thermodynamics, 4th edition, Wiley

M. Koretsky, Engineering and Chemical Thermodynamics, 2nd edition, Wiley, 2013, NY.

M.J. Moran, H. N. Shapiro, D.D. Boettner, M.B. Bailey, Principles of Engineering Thermodynamics, 7th edition, John Wiley and Sons, 2012, NY.