

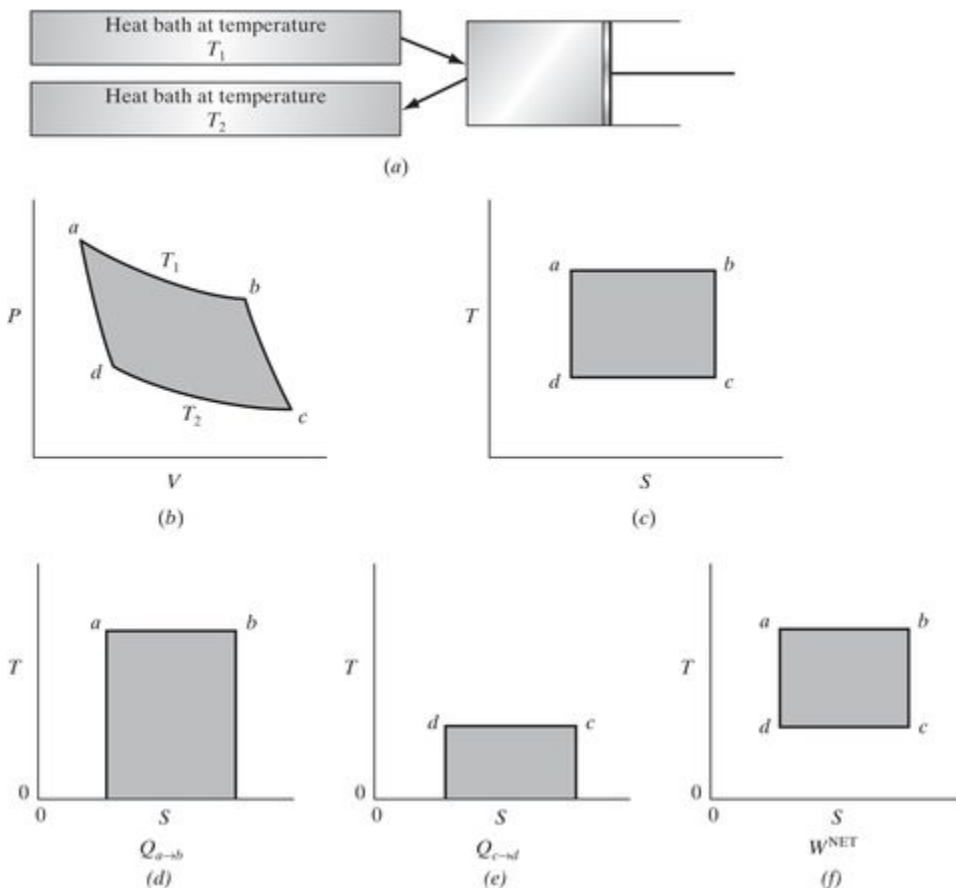
THE CARNOT CYCLE SELF STUDY MODULE

Objective

The objective of this module is to derive the heat and work interactions of a hypothetical cycle known as the Carnot cycle. The cycle runs around a gas confined in the piston and cylinder arrangement undergoing the following process steps:

- I. Reversible isothermal expansion
- II. Reversible adiabatic expansion
- III. Reversible isothermal compression
- IV. Reversible adiabatic compression

This cycle steps can be shown on PV and TS diagrams as follows (The figure is from Sandler)



The tables below summarize the states, and the changes during the processes between these states.

State	T	P	V	S
1	T_H	P_1	V_1	S_2
2	T_H	P_2	V_2	S_1
3	T_C	P_3	V_3	S_1
4	T_C	P_4	V_4	S_2

$$\Delta S = n C_v \ln \left(\frac{T_2}{T_1} \right) + n R \ln \left(\frac{V_2}{V_1} \right)$$

Path	ΔU	Q	W	ΔS
1	0	$nRT_H \ln(V_2/V_1)$	$-nRT_H \ln(V_2/V_1)$	$nR \ln(V_2/V_1)$
2	$nC_v(T_C - T_H)$	0	$nC_v(T_C - T_H)$	0
3	0	$nRT_C \ln(V_4/V_3)$	$-nRT_C \ln(V_4/V_3)$	$nR \ln(V_4/V_3)$
4	$nC_v(T_H - T_C)$	0	$nC_v(T_H - T_C)$	0
Overall cycle	0	$nR(T_H - T_C) \ln(V_2/V_1)$	$-nR(T_H - T_C) \ln(V_2/V_1)$	0

For the overall cycle $\Delta S=0= nR \ln(V_2/V_1) + nR \ln(V_4/V_3)=0$

$$Q_{\text{overall}} = nRT_H \ln(V_2/V_1) + nRT_C \ln(V_4/V_3)$$

$$\ln(V_4/V_3) = -\ln(V_2/V_1); \quad Q_{\text{overall}} = nR(T_H - T_C) \ln(V_2/V_1);$$

$W_{\text{overall}} = -Q_{\text{overall}}$ such that the substitution yields

$$\eta = -W_{\text{net}}/Q_H = \frac{T_H - T_C}{T_H}$$

This expression is known as the Carnot cycle efficiency and is a very important identifier about the power cycle performances.

Reading assignment

Read Chapter 5 of Sandler's thermodynamics, 5th edition. Also read about other power cycles, such as the Steam engine, the Otto engine, the Diesel engine etc.

Derive

Derive the efficiency of a hypothetical power cycle that operates between T_H and T_C to generate work. Observe the similarity between the efficiency terms of this cycle and the Carnot cycle.

Evaluate yourself

What are the muddiest points, what are the points that you understood well? What should you do to clarify the parts that are not at all clear.