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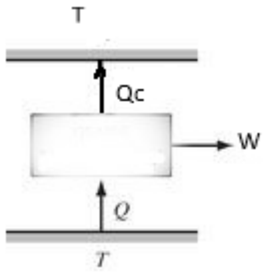
# *THE POWER CYCLES SELF STUDY MODULE*

## *Objective*

The objective of this module is to guide you in analyzing the power cycles. You will be able to differentiate the cycle efficiency from the process unit efficiency, you will learn about the energy conversion processes in general.

## *Compare*

So far we have learned two perspectives of a power cycle. Let us first look at the general perspective of a power cycle. The general characteristics are:



1. The operation is cyclic, i.e. the process ends at the same state as it has started.
2. The cycle absorbs heat, $Q_H$, from the sink kept at constant $T_H$,
3. The cycle generates work, W
4. The cycle releases heat $Q_C$ to the sink kept at constant temperature $T_C$. The cycle repeats itself.

Figure 1. The power cycle operating between a heat source at $T_H$ and a heat sink operating at $T_C$

In this cycle the first law analysis reveals that

$\Delta U=Q_H-Q_C-W=0$ Because the operation is cyclic. Thus,

$W=Q_H-Q_C$ or, $Q_C= Q_H-W$

$\Delta S=0 =\frac{Q_H}{T_H}-\frac{Q_C}{T_C}+S_{generated}$ (Self Test: Why is $\Delta S=0$?)

Substituting $Q_C$ in terms of $Q_H$ and W in the entropy balance yields

$0= \frac{Q_H}{T_H}-\frac{Q_H-W}{T_C}+S_{generated}$

If the cycle is reversible, i.e., $S_{gen}=0$, we obtain a relationship between the amount of work that we can obtain from this cycle in terms of the amount of the heat input and the difference between the source and the sink temperatures:

$$\eta = -\frac{W_H}{Q_H} = \frac{T_H - T_C}{T_H}$$

This general characteristics of a power cycle is reflected in the hypothetical Carnot cycle, shown below:

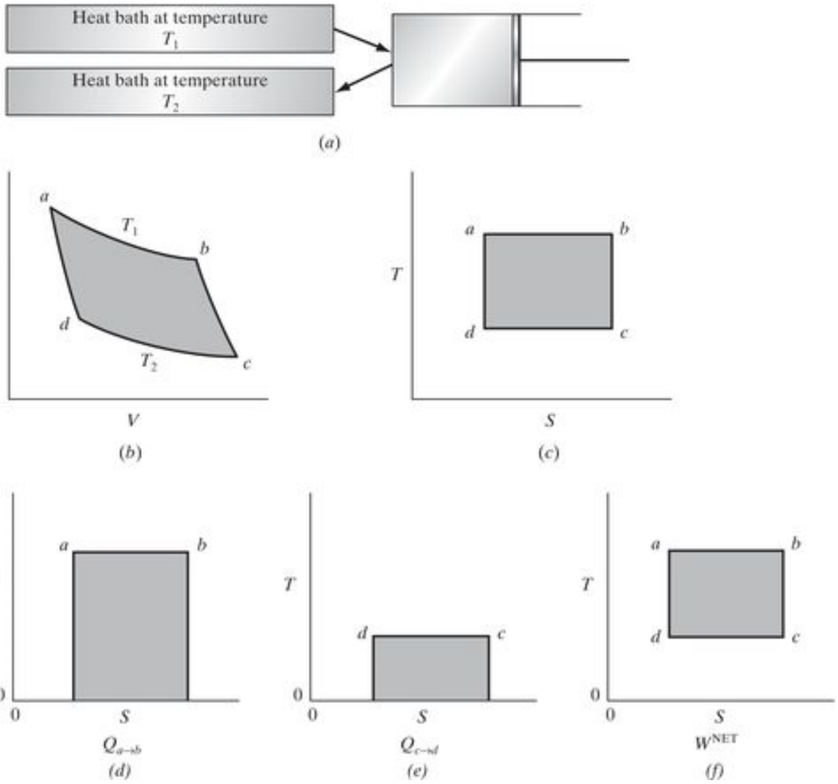


Figure 2. Sandler Figure 4.3-2 The Carnot Cycle and the heat and work interactions shown on a PV and TS diagrams.

## *The Rankine Cycle, a.k.a. the steam engine*

The general perspectives of the power cycle will prevail through all of the power cycles. We will have a high temperature source, a low temperature sink and while heat flows from hot to cold, we can convert some of it to work. Now let us remember the thermodynamic landscape, we have four different independent variables, as seen in the sketch on the right side, P,V, T,S. In the Carnot cycle we have navigated in the T, S domain of the processes. The Carnot cycle followed a route with constant T, constant S, constant T, constant S, in other words, TSTS sequence. Now we will analyze a PSPS cycle, the steam engine. There are different modifications of the steam engine. The ones currently used in thermal plants, including the Nuclear Power Generation plants, are based on the Rankine cycle. The Rankine Cycle has the following features:

I. Reversible adiabatic, i.e. isentropic expansion, S
II. Constant pressure condensation, P
III. Reversible adiabatic, i.e. isentropic compression, S
IV. Constant pressure evaporation, P

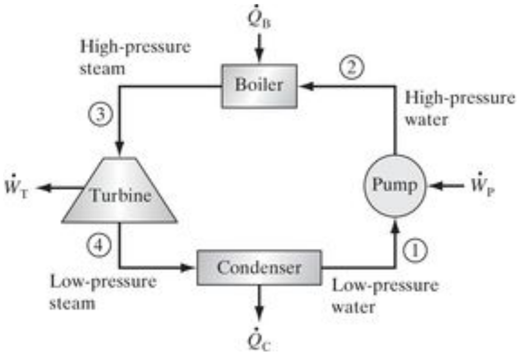


Figure 3. (this is FIGURE 5.2-1 in Sandler) rankine power cycle

In the figure shown above the flow chart of a Rankine power cycle is summarized. Now we will analyze they cycle step by step. The figure below shows the Rankine power cycle on the TS diagram (now, does the coordinates make sense?). The figure on the left, labeled C has point 4 in the two phase domain, while figure on the right, labeled d has the same point on exactly the two phase dome.

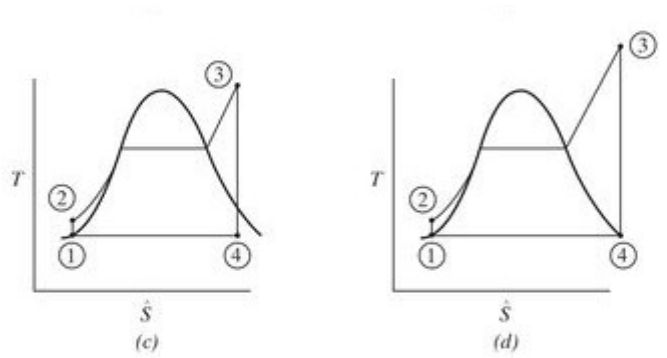


Figure 4. parts c and d of Figure 5.2.-2 in Sandler, rankine power cycle shown on TS diagram of steam. Points 1,2,3 and 4 are on one-to-one correspondence with those in Figure 1.

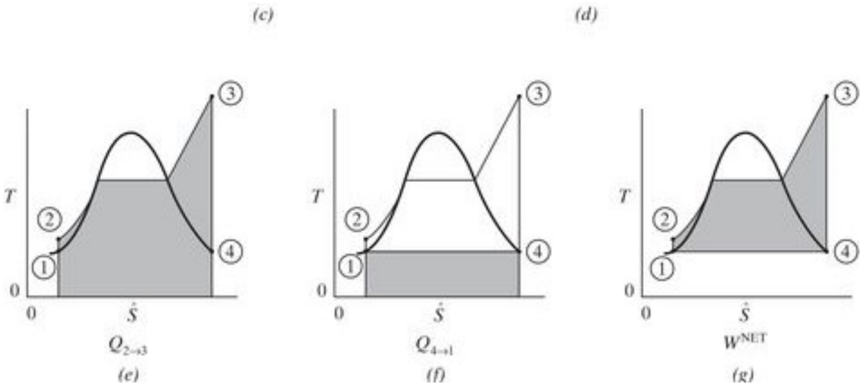


Figure 5. parts e, f and g of Figure 5.2.-2 in Sandler, continued. Rankine power cycle shown on TS diagram of steam, indicating the heat and work domains. Points 1,2,3 and 4 are on one-to-one correspondence with those in Figure 3.

Now we will design our own power cycle by selecting our own temperatures and pressures for this process. You are going to fill the table below, by reading from the TS diagram or by reading from the steam tables for the relevant information.

First the balance equations about the process units. Each of the process units is considered as an open system operating at steady state.

System selected around the boiler

The tables below summarize the states, and the changes during the processes between these states.

| State | T | P | V | S |
|---|---|---|---|---|
| 1 | $T_H$ | $P_1$ | $V_1$ | $S_2$ |
| 2 | $T_H$ | $P_2$ | $V_2$ | $S_1$ |
| 3 | $T_C$ | $P_3$ | $V_3$ | $S_1$ |
| 4 | $T_C$ | $P_4$ | $V_4$ | $S_2$ |

$$\Delta S = n\, C_v \ln\left(\frac{T_2}{T_1}\right) + n\, R \ln\left(\frac{V_2}{V_1}\right)$$

| Path | $\Delta U$ | Q | W | $\Delta S$ |
|---|---|---|---|---|
| 1 | 0 | $nRT_H\ln(V_2/V_1)$ | $-nRT_H\ln(V_2/V_1)$ | $nR\ln(V_2/V_1)$ |
| 2 | $nC_v(T_c-T_H)$ | 0 | $nC_v(T_c-T_H)$ | 0 |
| 3 | 0 | $nRT_c\ln(V_4/V_3)$ | $-nRT_c\ln(V_4/V_3)$ | $nR\ln(V_4/V_3)$ |

| | | | | |
|---|---|---|---|---|
| 4 | $nC_v(T_H\text{-}T_c)$ | 0 | $nC_v(T_H\text{-}T_c)$ | 0 |
| Overall cycle | 0 | $nR(T_H\text{-}T_c)\ln(V_2/V_1)$ | $-nR(T_H\text{-}T_c)\ln(V_2/V_1)$ | 0 |

For the overall cycle $\Delta S=0= nR\ln(V_2/V_1)+ nR\ln(V_4/V_3)=0$

$Q_{overall}= nRT_H\ln(V_2/V_1)+ nRT_c\ln(V_4/V_3)$

$\ln(V_4/V_3)=-\ln(V_2/V_1)$; $Q_{overall}=nR(T_H-T_c)\ln(V_2/V_1)$;

$W_{overall}=-Q_{overall}$ such that the substitution yields

$\eta=-W_{net}/Q_H=\frac{T_H-T_C}{T_H}$

This expression is known as the Carnot cycle efficiency and is a very important identifier about the power cycle performances.

## *Reading assignment*

Read Chapter 5 of Sandler's thermodynamics, 5th edition. Also read about other power cycles, such as the Steam engine, the Otto engine, the Diesel engine etc.

## *Derive*

Derive the efficiency of a hypothetical power cycle that operates between $T_H$ and $T_C$ to generate work. Observe the similarity between the efficiency terms of this cycle and the Carnot cycle.

## *Evaluate yourself*

What are the muddiest points, what are the points that you understood well? What should you do to clarify the parts that are not at all clear.

Why do we use TS diagrams for analyzing power cycles?