# THE POWER CYCLES SELF STUDY MODULE-Rankine Cycle

# **Objective**

The objective of this module is to guide you in analyzing the power cycles. You will be able to differentiate the cycle efficiency from the process unit efficiency, you will learn about the energy conversion processes in general.

### Reading assignment

Read Chapter 5 of Sandler's thermodynamics, 5<sup>th</sup> edition. Also read about other power cycles, such as the Steam engine, the Otto engine, the Diesel engine etc.

### Remember

So far we have learned two perspectives of a power cycle. Let us first look at the general

perspective of a power cycle. The general characteristics are:



1. The operation is cyclic, i.e. the process ends at the same state

as it has started.

- 2. The cycle absorbs heat,  $Q_{H},$  from the sink kept at constant  $T_{H},$
- 3. The cycle generates work, W
- 4. The cycle releases heat  $Q_C$  to the sink kept at constant

temperature T<sub>c</sub>. The cycle repeats itself.

Figure 1. The power cycle operating between a heat source at  $T_H$  and a heat sink operating at  $T_C$ In this cycle the first law analysis reveals that

 $\Delta U=Q_{H}-Q_{C}-W=0$  Because the operation is cyclic. Thus,

 $W=Q_H-Q_C$  or,  $Q_C=Q_H-W$ 

$$\Delta S=0 = \frac{Q_H}{T_H} - \frac{Q_C}{T_C} + S_{generated} \text{ (Self Test: Why is } \Delta S=0\text{?)}$$

Substituting  $Q_C$  in terms of  $Q_H$  and W in the entropy balance yields

$$0 = \frac{Q_H}{T_H} - \frac{Q_H - W}{T_C} + S_{generated}$$

If the cycle is reversible, i.e., S<sub>gen</sub>=0, we obtain a relationship between the amount of work that we can obtain from this cycle in terms of the amount of the heat input and the difference between the source and the sink temperatures:

$$\eta = \frac{W_H}{Q_H} = \frac{T_H - T_C}{T_H}$$

Note that the sign convention of this derivation is different from the derivation in Sandler. Sandler takes all the energy flows as positive, while we chose to introduce the signs of the outgoing energy components as negative, hence the negative sign in front of the W/Q term in the efficiency definition is no longer needed.

This general characteristics of a power cycle is reflected in the hypothetical Carnot cycle, shown below:



Figure 2. Sandler Figure 4.3-2 The Carnot Cycle and the heat and work interactions shown on a PV and TS diagrams.

## The Rankine Cycle, a.k.a. the steam engine

The general perspectives of the power cycle will prevail through all of the power cycles. We will have a high temperature source, a low temperature sink and while heat flows from hot to cold, we can convert some of it to work. Now let us remember the thermodynamic landscape, we have four different independent variables, as seen in the sketch on the right side, P,V, T,S. In the Carnot cycle we have navigated in the T, S domain of the processes. The Carnot cycle followed a route with constant T, constant S, constant T, constant S, in other words, TSTS sequence. Now we will analyze a PSPS cycle, the steam engine. There are different modifications of the steam engine. The ones currently used in thermal plants, including the Nuclear Power Generation plants, are based on the Rankine cycle. The Rankine Cycle has the following features:

- I. Reversible adiabatic, i.e. isentropic expansion, S
- II. Constant pressure condensation, P
- III. Reversible adiabatic, i.e. isentropic compression, S
- IV. Constant pressure evaporation, P



Figure 3. (this is FIGURE 5.2-1 in Sandler) flow chart of the Rankine power cycle

In the figure shown above the flow chart of a Rankine power cycle is summarized. Now we will analyze they cycle step by step. The figure below shows the Rankine power cycle on the TS diagram (now, does the coordinates make sense?). The figure on the left, labeled C has point 4 in the two phase domain, while figure on the right, labeled d has the same point on exactly the two phase dome.



Figure 4. parts c and d of Figure 5.2.-2 in Sandler, rankine power cycle shown on TS diagram of steam. Points 1,2,3 and 4 are on one-to-one correspondence with those in Figure 1.



Figure 5. parts e, f and g of Figure 5.2.-2 in Sandler, continued. Rankine power cycle shown on TS diagram of steam, indicating the heat and work domains. Points 1,2,3 and 4 are on oneto-one correspondence with those in Figure 3.

Now we will design our own power cycle by selecting our own temperatures and pressures for this process. You are going to fill the table below, by reading from the TS diagram or by reading from the steam tables for the relevant information.

First the balance equations about the process units. Each of the process units is considered as an open system operating at steady state. The mass flow rate of the cycle fluid, water in this case, is constant throughout the cycle, due to the steady state conditions. We will desgnate that as  $\dot{m}$  from this point onwards.

I. System is selected around the pump, i.e. between points 1 and 2. The pump operates adiabatically and reversibly (Self test: Why do we make such approximations?). The energy balance around this system simplifies to

$$0 = \dot{m} \big( \hat{H}_{in} - \hat{H}_{out} \big) + \dot{W}_{shaft}$$

Or,

$$\dot{W}_{pump} = \dot{m} \big( \hat{H}_2 - \hat{H}_1 \big)$$

II. System is selected around the boiler

$$0 = \dot{m} \big( \hat{H}_{in} - \hat{H}_{out} \big) + \dot{Q}_{boiler}$$

Or,

$$\dot{Q}_{boiler} = \dot{m} \big( \hat{H}_3 - \hat{H}_2 \big)$$

III. System is selected around the turbine, i.e. between points 3 and 4. The turbine

operates adiabatically and reversibly (Self test: Why do we make such

approximations?)

$$0 = \dot{m} \big( \widehat{H}_{in} - \widehat{H}_{out} \big) + \dot{W}_{shaft}$$

Or,

$$\dot{W}_{turbine} = \dot{m} \big( \hat{H}_4 - \hat{H}_3 \big)$$

IV. The system is selected around the condenser.

$$0 = \dot{m} \big( \hat{H}_{in} - \hat{H}_{out} \big) + \dot{Q}_{boiler}$$

Or,

$$\dot{Q}_{condenser} = \dot{m} \big( \hat{H}_1 - \hat{H}_4 \big)$$

In other words, if we can determine the states of water at the given process points of 1,2,3, and 4 we can determine the quantitative amounts of heat, net work and the efficiency of the cycle.

### Design your Rankine Cycle

During this exercise you will design your own Rankine cycle. You will select your boiler pressure, condenser pressure, boiler effluent temperature T<sub>3</sub> and whether you wish to have a saturated steam or wet steam at the exit of your turbine. Your freedom has limitations which we will discuss during our online time. You can start working on your design!

Use the table below for your design process. Fill the table with the information you know or

and remember that for pure components you need only two state variables to know the state

of the matter.

State	Т	Р	Ĥ	Ŝ
1				
2				
3				
4				

Now fill this table with the information you collected and using the energy and entropy

balances.

Process unit	ΔH	Q	W	ΔS
The Pump				
Boiler				
Turbine				
Condenser				
Overall cycle				

After filling the table with the relevant information and performing the necessary

calculations, determine the cycle efficiency

 $\eta$ =-W<sub>net</sub>/Q<sub>H</sub>

Compare this efficiency with that of the carnot cycle efficiency. Then we will discuss our results.

$$\eta = \frac{T_H - T_C}{T_H}$$

# Derive

Derive the efficiency of a hypothetical power cycle that operates between  $T_H$  and  $T_c$  to generate work. Observe the similarity between the efficiency terms of this cycle and the Carnot cycle.

# Evaluate yourself

What are the muddlest points, what are the points that you understood well? What should you do to clarify the parts that are not at all clear.

Why do we use TS diagrams for analyzing power cycles?

Why do we assume reversible operation?

What is the reason behind the adiabatic operation of the turbine and the pump?

If the turbine is not reversible, How can we deal with it?