EE-362Multiply-Excited Systems

# Dynamic Mechanical Systems 

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## Review:

## Rotational Systems:


or alternatively

$$
F=\frac{1}{2} I^{2} \frac{d L(x)}{d x}
$$

## Review:

## Rotational Systems:

$$
T=-\frac{1}{2} \Phi^{2} \frac{d R(\theta)}{d \theta}
$$

or alternatively

$$
T=\frac{1}{2} I^{2} \frac{d L(\theta)}{d \theta}
$$

What is the torque in the following systems?
a) If Coil\#1 is excited only,
b) If Coil $\# 2$ is excited only,

## What is the torque?

## Cylindrical Rotor, Cylindrical Stator



## What is the torque?

## Cylindrical Rotor, Salient Stator



## What is the torque?

Salient Rotor, Cylindrical Stator


## Multiply-Excited Systems

What happens if both of the coils are excited?


## Multiply-Excited Systems



Electrical Energy = Magnetic Energy + Mechanical Energy

## Multiply-Excited Systems



Electrical Energy = Magnetic Energy + Mechanical Energy

## Mutual Inductance



Write down the voltage equation of Inductor 2.

## What is the stored energy in coill?

$W_{\text {mag1 }}=\frac{1}{2} i^{2} L$

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$W_{\text {mag } 1}=\frac{1}{2} i^{2} L \quad \rightarrow$ Not Correct!
$d W_{m a g 1}=i_{1} d \lambda_{1}$
$d W_{m a g 1}=i_{1}\left(L_{11} d i_{1}+L_{12} d i_{2}\right)$

## Total stored energy (coil1+coil2)?

$d W_{\text {mag }}=i_{1} d \lambda_{1}+i_{2} d \lambda_{2}$

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## Total stored energy (coil1+coil2)?

$d W_{\text {mag }}=i_{1} d \lambda_{1}+i_{2} d \lambda_{2}$
Or it can be written as:
$W_{m a g}=\int d W_{m a g}$
$=\frac{1}{2} L_{11} i_{1}^{2}+\frac{1}{2} L_{22} i_{2}^{2}+M i_{1} i_{2}$

## Stored Energy in Matrix Form

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$$
W_{m a g}=\frac{1}{2}\left[\begin{array}{ll}
i_{1} & i_{2}
\end{array}\right]\left[\begin{array}{cc}
L_{11} & M \\
M & L_{22}
\end{array}\right]\left[\begin{array}{l}
i_{1} \\
i_{2}
\end{array}\right]
$$

## Stored Energy in Matrix Form

$W_{\text {mag }}=\frac{1}{2}\left[\begin{array}{ll}i_{1} & i_{2}\end{array}\right]\left[\begin{array}{cc}L_{11} & M \\ M & L_{22}\end{array}\right]\left[\begin{array}{l}i_{1} \\ i_{2}\end{array}\right]$
Generalized case
$W_{\text {mag }}=\frac{1}{2} \mathbf{I}_{t} \mathbf{L I}$
An application of multiply excited systems: Contactless Surgery. More information

## Torque in Multiply Excited Sytems

still depends on the derivative of $W_{\text {mag }}$

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still depends on the derivative of $W_{\text {mag }}$

$$
\begin{aligned}
T_{\text {mech }}= & \frac{1}{2} \frac{d L_{11}}{d \theta} i_{1}^{2}+\frac{1}{2} \frac{d L_{22}}{d \theta} i_{2}^{2} \\
& +\frac{d M}{d \theta} i_{1} i_{2}
\end{aligned}
$$

## Torque in Multiply Excited Sytems

still depends on the derivative of $W_{\text {mag }}$

$$
\begin{aligned}
T_{\text {mech }}= & \frac{1}{2} \frac{d L_{11}}{d \theta} i_{1}^{2}+\frac{1}{2} \frac{d L_{22}}{d \theta} i_{2}^{2} \\
& +\frac{d M}{d \theta} i_{1} i_{2} \\
T_{\text {mech }}= & \frac{1}{2} \mathbf{I}_{t} \frac{d \mathbf{L}}{d \theta} \mathbf{I}
\end{aligned}
$$

## Cylindrical Rotor, Cylindrical Stator


$T=i_{1} i_{2} \frac{\partial M}{\partial \theta}:\left(L_{11}, L_{22}\right.$ constant $)$

## Cylindrical Rotor, Salient Stator


$T=\frac{1}{2} i_{2} \frac{\partial L_{22}}{\partial \theta}+i_{1} i_{2} \frac{\partial M}{\partial \theta}:\left(L_{11}\right.$ constant $)$

## Salient Rotor, Cylindrical Stator


$T=\frac{1}{2} i_{1}^{2} \frac{\partial L_{11}}{\partial \theta}+i_{1} i_{2} \frac{\partial M}{\partial \theta}:\left(L_{22}\right.$ constant $)$

## Combination with Mechanical Systems:

Linear and Rotational Motion

| Linear | Rotationa |
| :---: | :---: |
| X: (m) | ( $\theta$ ): (radians) |
| v : Velocity (m/s) | ) $\omega$ : Angular Velocity ( $\theta / \mathrm{s}$ ) |
| F: Force (N) | T: Torque ( Nm ) |
| m : Mass (kg) | J:Inertia (kgm~2) |
| $\mathrm{F}=\mathrm{m} \mathrm{dv} / \mathrm{dt}$ | $\mathrm{T}=\mathrm{J} \mathrm{d} \omega / \mathrm{dt}$ |

## Dynamic Equations: Ideal Spring


$F=k\left(x-x_{0}\right)$ : No energy dissipation ( $\sim$ deal Inductor)

## Dynamic Equations: Damping


$F=B v=B \frac{d x}{d t}:$ Dissipates energy (~Resistance)
Overdamped, underdamped (similar to RLC circuits)

Dynamic Equations: Inertia

$$
\begin{aligned}
& F=m a \\
& \text { or } \\
& F=m \frac{d v}{d t}=m \frac{d^{2} x}{d^{2} t}
\end{aligned}
$$

## Dynamic Equations: Mechanical Side

$f_{\text {mech }}=M \frac{d^{2} x}{d^{2} t}+B \frac{d x}{d t}$

$$
+k\left(x-x_{0}\right)+f_{\text {external }}
$$

## Bose's Active Suspension System



Bose suspension system, Bose suspension will be mass produced

## Bose Ride



Bose ride, Truck Driver comments-1, Truck Driver comments-2

## Summary

. Multiply excited systems still tries to minimize total stored magnetic energy

- Derivative of self inductances and mutual inductance can work together or oppose each other.
. Magnetic forces interact with the mechanical systems and generate a system response

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