EE-362

Multiply-Excited Systems

Dynamic Mechanical Systems

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Review:

Linear

~~Rotational~~ Systems:

$$F = -\frac{1}{2}\Phi^2 \frac{dR(x)}{dx}$$

or alternatively

$$F = \frac{1}{2} I^2 \frac{dL(x)}{dx}$$

Review:

Rotational Systems:

$$T = -\frac{1}{2}\Phi^2 \frac{dR(\theta)}{d\theta}$$

or alternatively

$$T = \frac{1}{2}I^2 \frac{dL(\theta)}{d\theta}$$

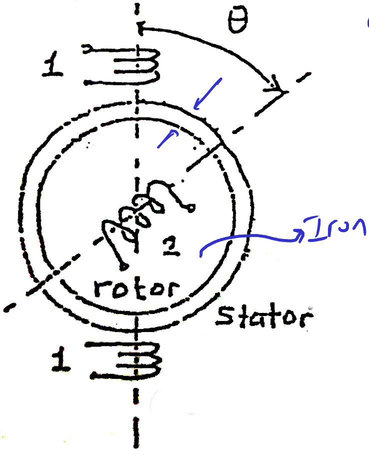
What is the torque in the following systems?

a) If Coil#1 is excited only,

b) If Coil#2 is excited only,

What is the torque?

Cylindrical Rotor, Cylindrical Stator



$R(\theta) \Rightarrow \text{constant}$

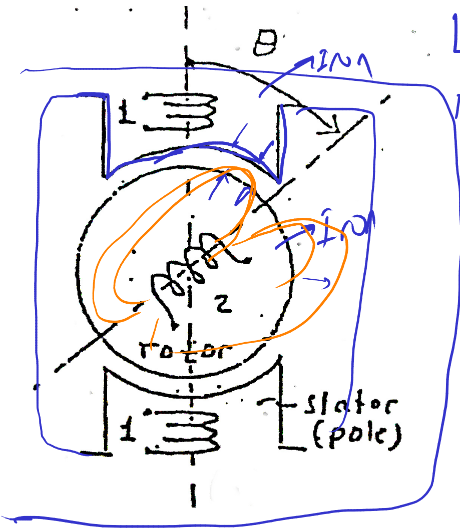
$L(\theta) \Rightarrow \text{constant}$

$\hookrightarrow \frac{\partial R(\theta)}{\partial \theta} = 0$

$\frac{\partial L(\theta)}{\partial \theta} = 0$

What is the torque?

Cylindrical Rotor, Salient Stator



coil #1

$L_{11}(\theta) \Rightarrow$ constant

No torque if only coil #1 is excited.

coil #2

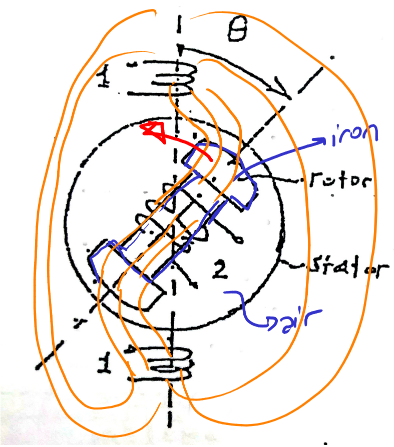
$L_{22}(\theta) \neq$ constant

$R_2(\theta)$ is minimum when rotor is vertical ($\theta=0$)

it will rotate C.C.W.

What is the torque?

Salient Rotor, Cylindrical Stator



coil #1 is excited

$R_1(\theta)$ is minimum, when the rotor is vertically aligned ($\theta = 0$)
it will rotate C.C.W

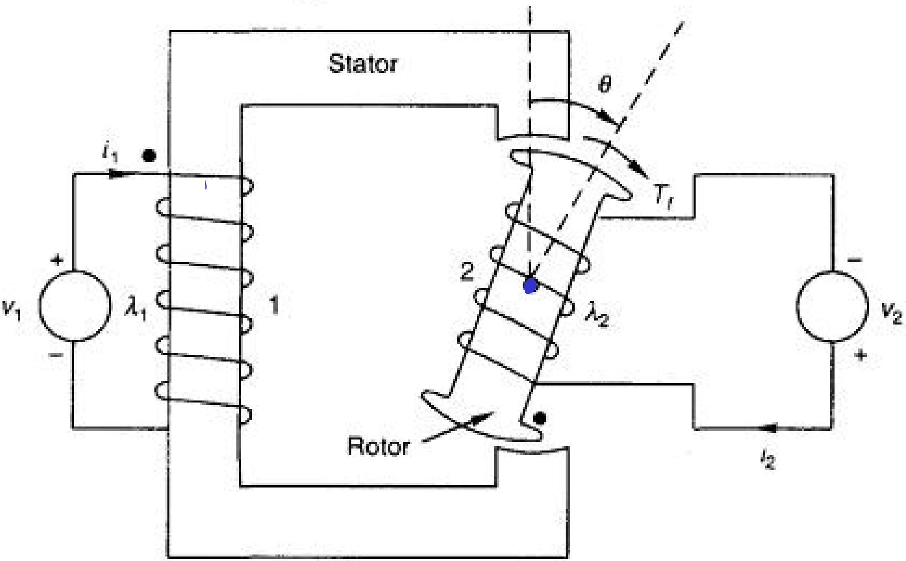
coil #2 is excited

$R_2(\theta) \Rightarrow$ is constant

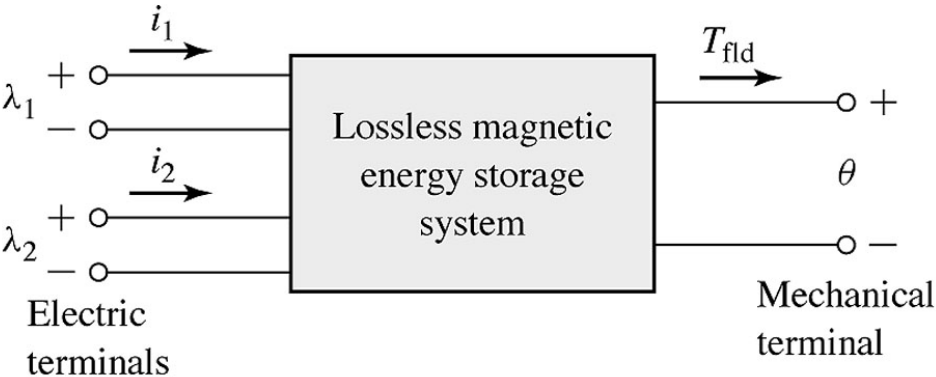
No torque

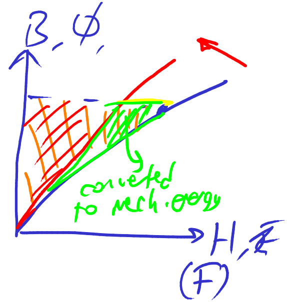
Multiply-Excited Systems

What happens if both of the coils are excited?



Multiply-Excited Systems

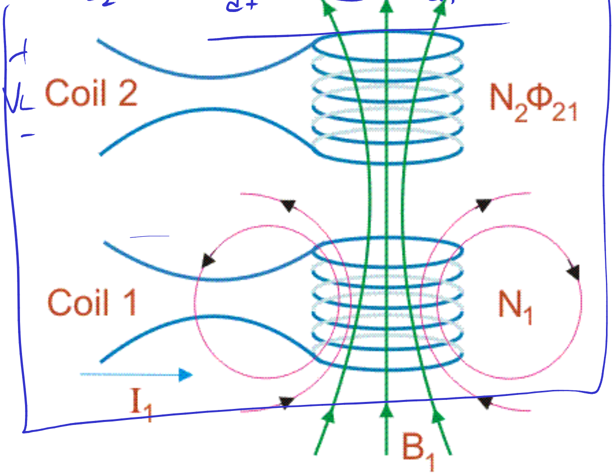




Electrical Energy = Magnetic Energy + Mechanical Energy

Mutual Inductance

$V_{L2} = L_2 \frac{di_2}{dt} + L_{12} \cdot \frac{di_1}{dt}$, with $L_{12} \to M$



$V_L = L \cdot \frac{di}{dt}$

$L = \lambda / I$

Write down the voltage equation of Inductor 2.

What is the stored energy in coil1?

$$W_{mag1} = \frac{1}{2} i^2 L \quad \rightarrow \text{Not Correct!}$$

$$dW_{mag1} = i_1 d\lambda_1$$

mutual inductance

$$dW_{mag1} = i_1 (L_{11} di_1 + L_{12} di_2)$$

Total stored energy (coil1+coil2)?

$$dW_{mag} = i_1 d\lambda_1 + i_2 d\lambda_2$$

Or it can be written as:

$$W_{mag} = \int_0^{i_2}\int_0^{i_1} dW_{mag}$$

$$= \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + Mi_1i_2$$

coil #1

$$W_{mag} = \int_0^t V i \, dt$$

$$= \int_0^t \left(L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}\right) \cdot i_1 \, dt$$

$$\frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{12} \cdot i_1 \cdot i_2$$

second coil

$$\frac{1}{2}L_{22}i_2^2 + \frac{1}{2}L_{21} \cdot i_1 \cdot i_2$$

Add

$$L_{12} = L_{21} = M$$

Stored Energy in Matrix Form

inductance matrix

$$W_{mag} = \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix}\begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Stored Energy in Matrix Form

$$W_{mag} = \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix}\begin{bmatrix} L_{11} & M \\ M & L_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Generalized case

$$W_{mag} = \frac{1}{2}\mathbf{I}_t\mathbf{L}\mathbf{I}$$

An application of multiply excited systems: Contactless Surgery More information

Torque in Multiply Excited Sytems

still depends on the derivative of W_{mag}

$$T_{mech} = \frac{1}{2}\frac{dL_{11}}{d\theta}i_1^2 + \frac{1}{2}\frac{dL_{22}}{d\theta}i_2^2$$

$$+\frac{dM}{d\theta}i_1 i_2$$

can be (+) or (−)

Torque in Multiply Excited Sytems

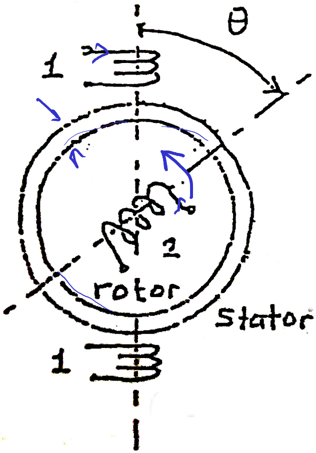
still depends on the derivative of W_{mag}

$$T_{mech} = \frac{1}{2}\frac{dL_{11}}{d\theta}i_1^2 + \frac{1}{2}\frac{dL_{22}}{d\theta}i_2^2$$

$$+\frac{dM}{d\theta}i_1 i_2$$

$$T_{mech} = \frac{1}{2}\mathbf{I}_t\frac{d\mathbf{L}}{d\theta}\mathbf{I}$$

Cylindrical Rotor, Cylindrical Stator



$L_{11}(\theta) = \text{constant}$

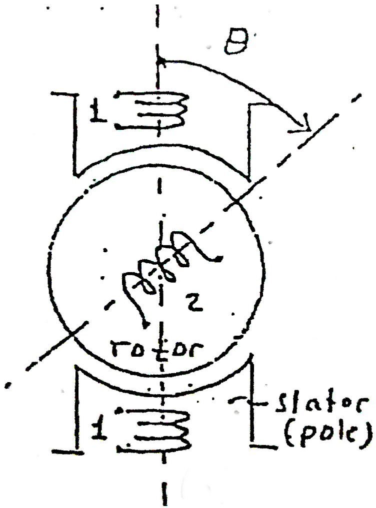
$L_{22}(\theta) = \text{constant}$

$$T = i_1 i_2 \frac{\partial M}{\partial \theta} : (L_{11}, L_{22} \text{ constant})$$

will be reverse

reverse

Cylindrical Rotor, Salient Stator



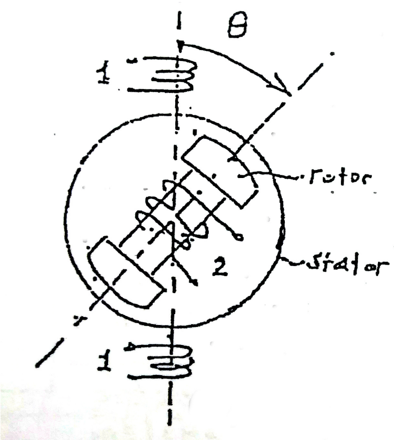
$L_{11}(\theta) = \text{constant}$

$L_{22}(\theta) \neq \text{constant}$

↳ $L_{22}(\theta)$ generates torque.

$$T = \frac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} + i_1 i_2 \frac{\partial M}{\partial \theta} : (L_{11} \text{ constant})$$

Salient Rotor, Cylindrical Stator



$L_{22}(\theta) \Rightarrow$ constant

$$\frac{\partial L_{22}(\theta)}{\partial \theta} = 0$$

$$T = \frac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + i_1 i_2 \frac{\partial M}{\partial \theta} : (L_{22} \text{ constant})$$

Combination with Mechanical Systems:

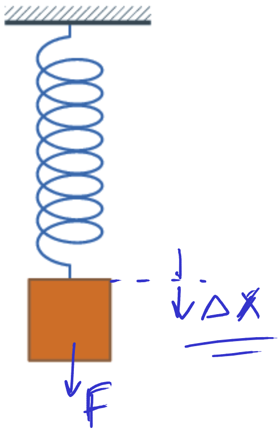
Linear and Rotational Motion

Linear	Rotational
X: (m)	(θ): (radians) → displacement
v: Velocity (m/s)	ω: Angular Velocity (θ/s) → speed
F: Force (N)	T: Torque (Nm)
m: Mass (kg)	J: Inertia (kgm^2)
F=m dv/dt	T=J dω/dt

$\frac{1}{2} m v^2 \Rightarrow \frac{1}{2} J \omega^2 \rightarrow F = ma \Rightarrow T = J \cdot \frac{d\omega}{dt}$

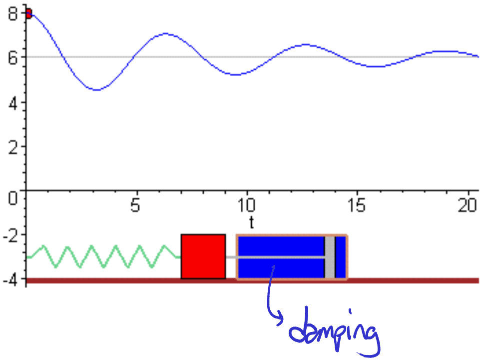
$P = F \cdot V \Rightarrow P = T \cdot \omega$

Dynamic Equations: Ideal Spring



$F = k(x - x_0)$: No energy dissipation (~Ideal Inductor)

Dynamic Equations: Damping



$$F = Bv = B\frac{dx}{dt}$$: Dissipates energy (~Resistance)

Overdamped, underdamped (similar to RLC circuits)

Dynamic Equations: Inertia

$$F = ma$$

or

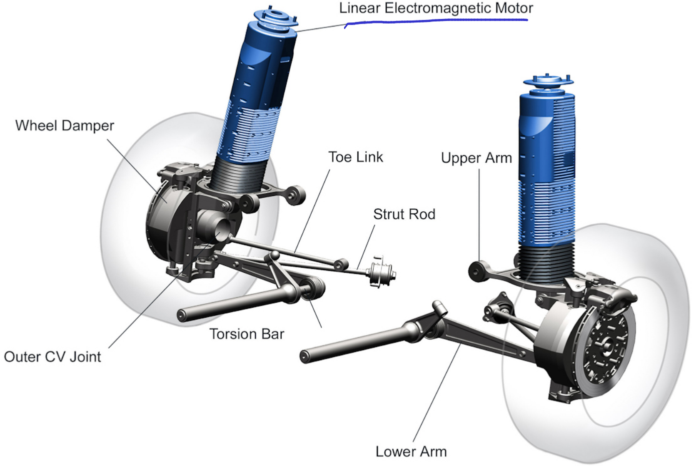
$$F = m\frac{dv}{dt} = m\frac{d^2x}{d^2t}$$

Dynamic Equations: Mechanical Side

$$f_{mech} = M\frac{d^2x}{d^2t} + B\frac{dx}{dt}$$

$$+k(x - x_0) + f_{external}$$

Bose's Active Suspension System



[Bose suspension system](), [Bose suspension will be mass produced]()

Bose Ride

[Bose ride](), [Truck Driver comments-1](), [Truck Driver comments-2]()

Summary

- Multiply excited systems still tries to minimize total stored magnetic energy
- Derivative of self inductances and mutual inductance can work together or oppose each other.
- Magnetic forces interact with the mechanical systems and generate a system response