EE-362 ELECTROMECHANICAL ENERGY CONVERSION-II

Torque in Induction Motors

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Mechanical Power

Linear Motion

Power(W) = Force (N) x Speed (m/s)

Rotational Motion

Power(W) = Torque (Nm) x Rotational Speed (rad/s)

$$P = T\omega \quad \rightarrow T = \frac{P}{\omega}$$

Can you guess a few applications that require high start-up torque?

Electric Cars: BMW i3

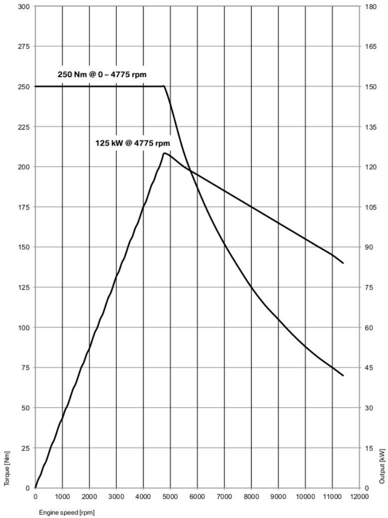
Start-up Torque

Electric motor		
Motor technology		BMW eDrive technology: hybrid synchronous electric motor with inte electronics, charging unit and generator func recuperation
Max output	kW (hp)	125 (170)
Rated output / at	kW (hp) / rpm	75 (102) / 4800
Torque / at	Nm / rpm	250 / 0
Recuperation output	kW	up to 50

High-voltage battery		
Voltage	V	353
Battery capacity	Ah	94
Power output (gross / net)	kWh	33.2 / 27.2
Storage technology		Lithium-ion

For curious students: BMW i3 Specs,

Importance of Start-up Torque: BMW i3 vs WV Golf GTI



For curious students: Tesla Induction Motor Info, Reverse engineering a Tesla drivetrain, BMW i3 Specs,

Torque-Power Relation

$$P_{mech} = 3I_2'^2 \frac{(1-s)}{s} r_2'$$

$$T = \frac{P}{\omega}$$

What is ω of the rotor?

$$\omega_r = (1-s)\omega_s$$

Torque-Power Relation

$$P_{mech} = T(1 - s)\omega_s = 3I_2'^2 \frac{(1 - s)}{s} r_2'$$

$$T\omega_s = 3I_2'^2 \frac{r_2'}{s}$$

Airgap power

Generated Torque

$$T = 3I_2'^2 \frac{r_2'}{s} \frac{1}{\omega_s}$$, which is equal to:

$$T = \frac{P_{ag}}{\omega_s}$$ or $$T = \frac{P_{mech}}{\omega_r}$$

ω_s is the mechanical synchronous speed! $$\omega_s = \frac{2\pi f_e}{(p/2)}$$

Generated Torque

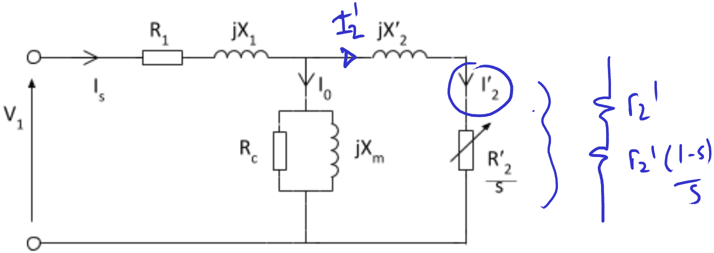
$$T = 3 I_2'^2 \frac{r_2'}{s} \frac{1}{\omega_s}$$

We know:

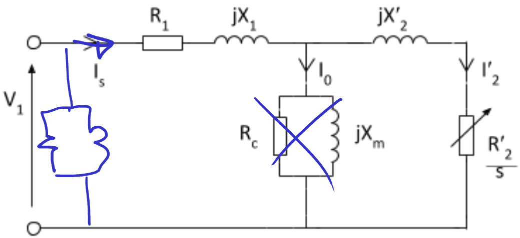
- ω_s, if we know f_e and number of poles
- s: if we know rotor speed
- r_2' from locked-rotor test

How can we calculate I_2'?

How can we calculate I'_2?

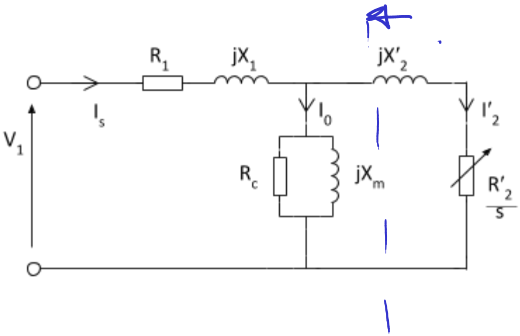


How can we calculate I_2'?



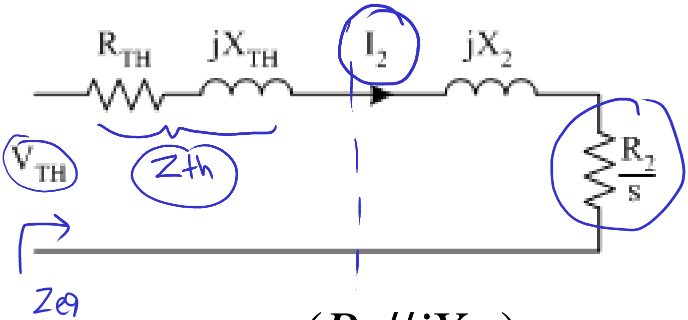
- Inaccurate but easy: Move parallel branch to source side

How can we calculate I_2'?



- Inaccurate but easy: Move parallel branch to source side
- More accurate: Calculate Thevenin equivalent as seen from the rotor side

Thevenin Equivalent Circuit



$$V_{th} = \frac{(R_c // jX_m)}{R_1 + jX_1 + (R_c // jX_m)} V_1$$

$$Z_{th} = (R_1 + jX_1) // (R_c // jX_m)$$

$$I_2' = \frac{V_{th}}{Z_{eq}} = \frac{V_{th}}{\sqrt{\left(R_{th} + \frac{R_2'}{s}\right) + \left(X_{th} + X_2'\right)^2}}$$

Torque

$$T_e = \frac{3V_{th}^2}{(R_{th} + \frac{r_2'}{s})^2 + (X_{th} + X_2')^2} \frac{r_2'}{s\omega_s}$$

Torque

$$T_e = \frac{3V_{th}^2}{(R_{th} + \frac{r_2'}{s})^2 + (X_{th} + X_2')^2} \frac{r_2'}{s\omega_s}$$

If you're in a hurry, move the parallel branch to motor terminals and replace:

$$V_{th} \to V_1 \quad R_{th} \to R_1 \quad X_{th} \to X_1$$

Torque

$$T_e = \frac{3V_1^2}{(R_1 + \frac{r_2'}{s})^2 + (X_1 + X_2')^2} \frac{r_2'}{s\omega_s}$$

$$V_{th} \rightarrow V_1 \quad R_{th} \rightarrow R_1 \quad X_{th} \rightarrow X_1$$

Torque Characteristics

Can you guess the waveform wrt rotor speed?

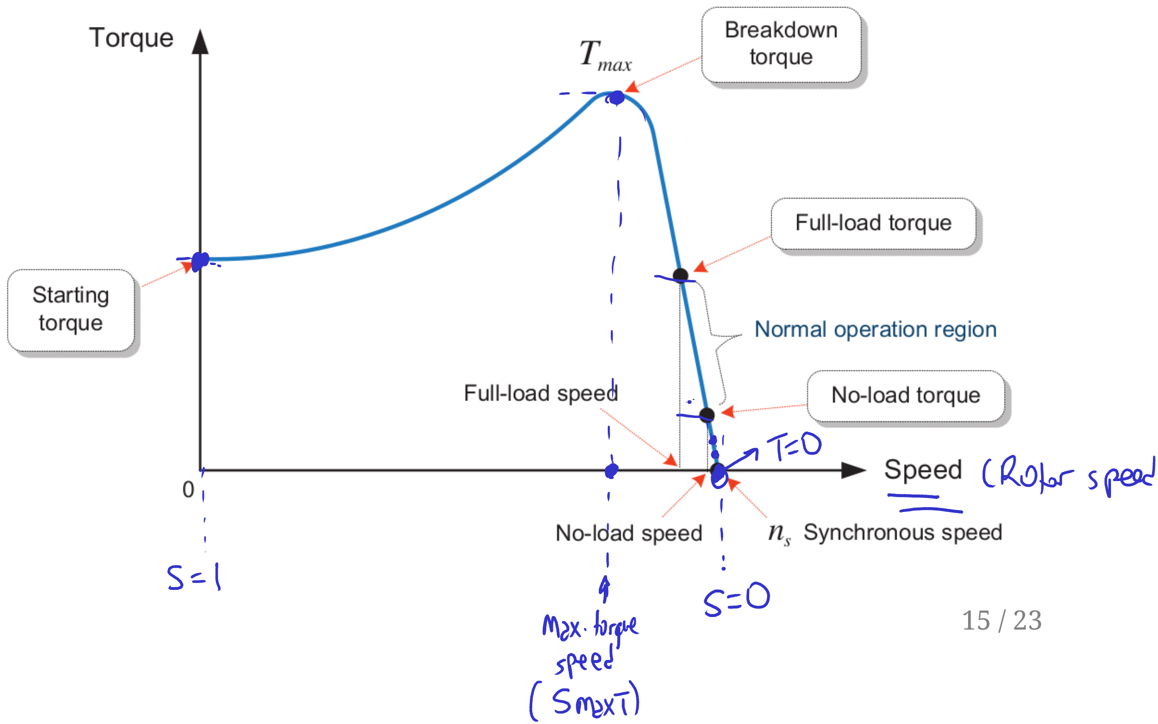
$$T_e = \frac{3V_{th}^2}{(R_{th} + \frac{r_2'}{s})^2 + (X_{th} + X_2')^2} \frac{r_2'}{s\omega_s}$$

[Torque Graphs]

$s = 1 \Rightarrow T_e > 0$

$s = 0 \quad T_e = 0$

Typical Torque Curve of an Induction Motor



Torque characteristics

For small values of slip: Torque is proportional to slip

For large values of slip: Torque is inversely proportional to slip

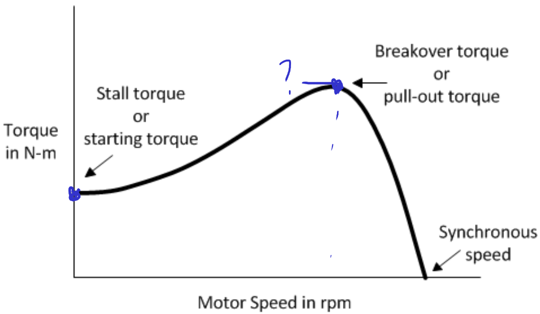
Rated slip is usually smaller than 0.05

Start-up Torque

Substitute s=1 in the torque equation

$$T_{start} = \frac{3V_{th}^2}{(R_{th} + r_2')^2 + (X_{th} + X_2')^2} \frac{r_2'}{\omega_s}$$

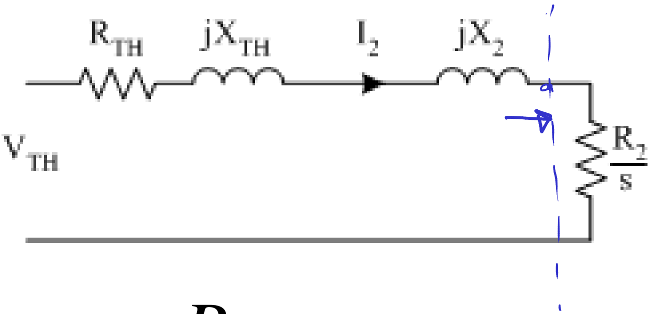
Maximum Torque Point



Speed-Torque Curve for a Three-Phase Induction Motor

$$T_e = \frac{3V_{th}^2}{(R_{th} + \frac{r_2'}{s})^2 + (X_{th} + X_2')^2} \frac{r_2'}{s\omega_s}$$

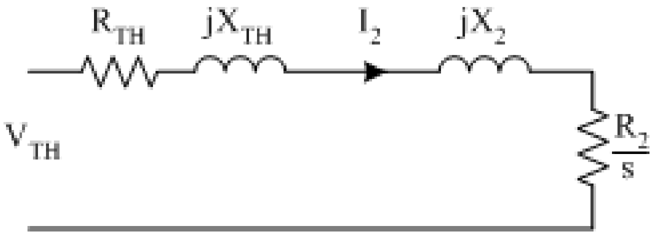
Maximum Torque Point



$$T_e = \frac{P_{ag}}{\omega_s}$$

Maximum torque point = Maximum airgap power point

Maximum Torque Point



What is the condition for maximium airgap Power?

Maximum Power Transfer Theorem:

$$\frac{r_2'}{s} = \sqrt{R_{th}^2 + (X_{th} + X_2')^2}$$

Slip for maximum torque

$$s_{maxT} = \frac{r_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}}$$

Maximum Torque (substitute s)

$$T_{max} = 3\frac{0.5V_{th}^2}{\omega_s}\frac{1}{(R_{th} + \sqrt{R_{th}^2 + (X_{th} + X_2')^2}}$$

Notice that s_{maxT} depends on r_2' but T_{max} doesn't.