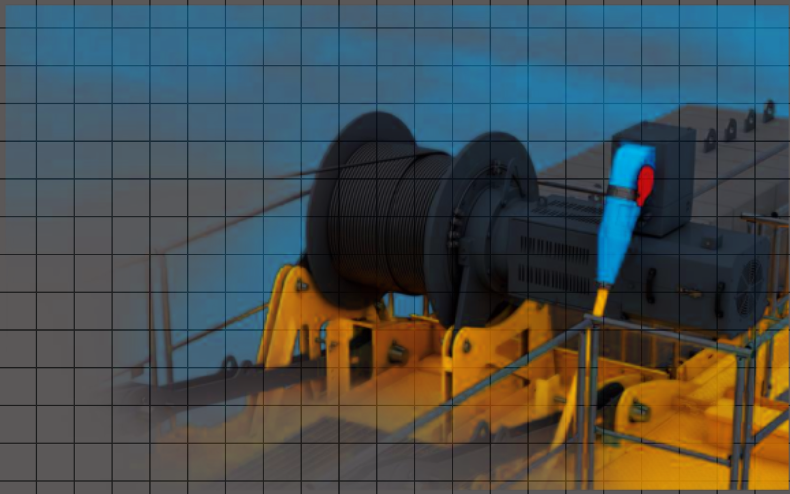
Induction motors are commonly used in tower cranes with variable voltage and frequency drives as shown in the figures below.

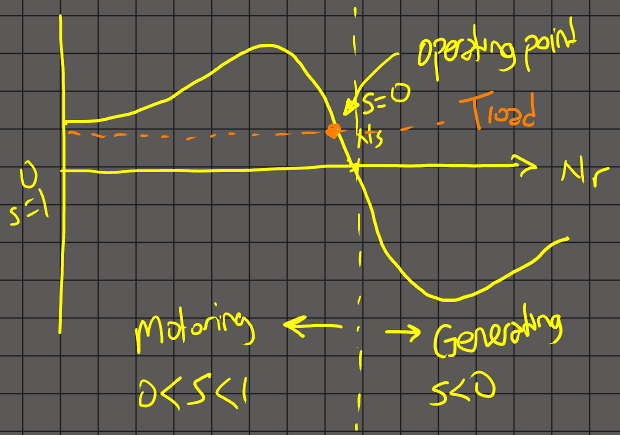


In this problem, ignore the parallel branch of the induction motor, the friction and windage losses.

**a) (4pts)** Assume that, the crane operator is lifting (moving up) a mass at constant speed. (Constant-torque load)

i) Please sketch the torque-speed characteristics of a typical induction machine between s=1 and s=-1. (Label the critical points and clearly show operating mode regions).

ii) In the same graph, draw a load torque line and label the operating point. In what mode does the machine is operating in this case? Describe the direction of the power flow.




Machine is operating as a motor

power flows from electrical system to mechanical system

**b) (6pts)** Torque characteristic of induction machines can be approximated using a linear equation ($T_e$=ks), if the rotor speed is close to the synchronous speed, where k is a factor that depends on the machine and supply characteristics.

Starting from the electromechanical torque expression given below, derive the value of k. Please state any assumptions you made, for full credit.

$$\tau = \frac{3}{\omega_s} \cdot \frac{V_1^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s}$$

$T = ks \Rightarrow$ only valid if s is small

$s \approx 0$

$\frac{r_2'}{s} \gg r_1, X_1, X_2'$

$(r_1 + r_2'/s)^2 + (X_1 + X_2')^2 \approx \left(\frac{r_2'}{s}\right)^2$

$\rightarrow T = \frac{3V_1^2}{\omega_s \left(\frac{r_2'}{s}\right)^2} \cdot \frac{r_2'}{s} = \frac{3V_1^2 \cdot s}{\omega_s r_2'} \rightarrow = k$

**c) (5pts)** Assume you have a 400V (l-l) wye-connected, 16kW, 3-phase, 6-pole squirrel-cage induction machine connected to a variable voltage variable frequency drive. The referred rotor resistance ($r_2'$) is 0.5 Ω.

The crane operator lifts a mass which exerts 91 Nm of torque on the shaft of the induction machine. Calculate the rotor speed in rpm, if the induction machine is supplied with 50 Hz, 400 V (l-l) voltage.

$V_{ll} = 400V \Rightarrow V_{ph} = \frac{400}{\sqrt{3}} = 230V$

6 pole $\Rightarrow N_s = 1000$ rpm

$\omega_s = \frac{2\pi \cdot 50}{(6/2)} = \frac{100\pi}{3}$ rad/s

synchronous mech. speed

$T = \frac{3V_1^2}{\omega_s \cdot r_2'} \cdot s$

$91 = \frac{3 \cdot (230)^2}{\frac{100\pi}{3} \cdot 0{,}5} \cdot s$

$s = 0{,}03 \Rightarrow N_r = (1-s) \cdot N_s$

$= 0{,}97 \cdot 1000$

$= 970$ rpm

**d) (5pts)** Now, the operator would like to increase the speed of the load using the motor controller, which responds by suddenly changing the applied frequency to 60 Hz under **constant V/f mode of operation**, calculate the speed of the rotor once the system reaches the steady-state under 60 Hz excitation.

Constant flux (V/f) operation $\Rightarrow$ f↑ V↑

50Hz ($\omega_{s1}$), 230V ($V_1$) $\Rightarrow$ 60Hz ($\omega_{s2}$), $V_2$

$V_2 = \frac{60}{50} \cdot 230 = 276V$ (where $\frac{60}{50} = 1{,}2$)

$91 = \frac{3(1{,}2 \cdot V_1)^2}{(1{,}2\omega_{s1})\, r_2'} \cdot s_2 \Rightarrow s_2 = \frac{s_1}{1{,}2} = \frac{5}{6} \cdot s_1 \Rightarrow s_2 = 0{,}025$

$91 = \frac{3 \cdot (1{,}2 \cdot 230)^2}{(1{,}2 \times \frac{100\pi}{3}) \cdot 0{,}5} \cdot s_2 \Rightarrow s_2 = 0{,}025$

$N_s = 1200$ rpm $\Rightarrow N_r = (1-s) \cdot N_s = (1-0{,}025) \cdot 1200$

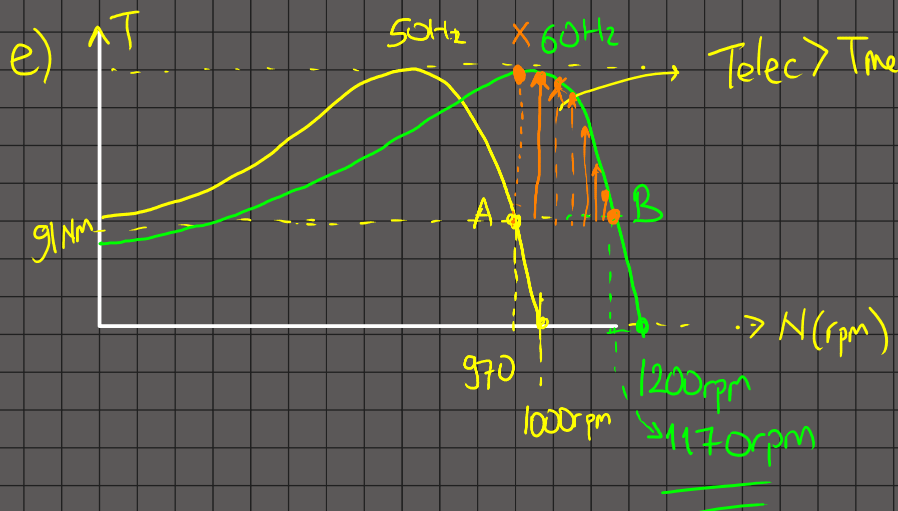
$= 1170$ rpm

**e) (5pts)** In a graph, sketch the the torque characteristics of the machine (torque vs rotor speed in rpm) in part (c) and label the operating point with label **A**. Then label the steady state operating point of part (d) with label **B**. Describe in detail how the machine moves from the state in point A to the state in point B.

**f) (6 pts)** For the transient period from point A to point B, sketch the following.

i) Rotational speed vs time
ii) Electromagnetic torque vs time
iii) Gross mechanical power vs time.

e)

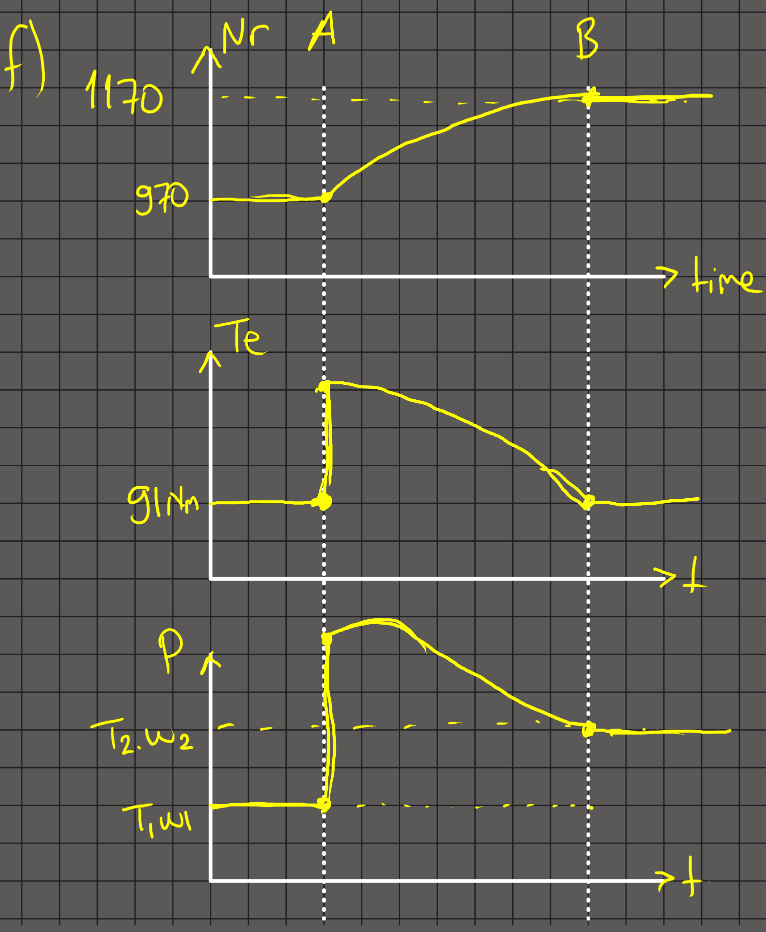



$T_{elec} > T_{mech} \Rightarrow J\frac{d\omega}{dt} > 0$

↳ crane accelerates

When the freq. changes, the speed cannot change instantaneously, but torque characteristics change (as in green line), so the operating point jumps to point X. Extra torque ($T_{elec} - T_{load}$) accelerates the motor until it reaches point B. Acceleration is higher at first then reduces, as it reaches 1170rpm

f)




$T_e - T_\ell = J\frac{d\omega}{dt}$

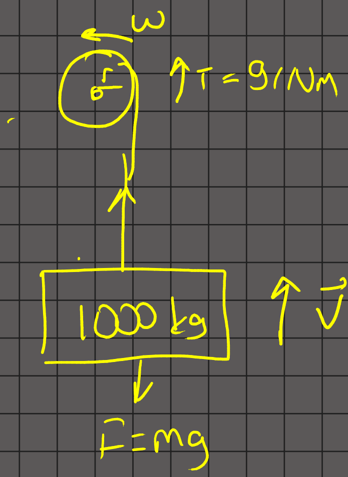
$P = T.\omega$

$T_1 = T_2$

$\omega_2 > \omega_1$

**g) (4 pts)** If the crane is lifting a 1000 kg load at constant speed, and the motor is delivering rated power of 16 kW. What is the linear speed of the load in m/s?

$P = T \cdot \omega$




$P = T\omega = F \cdot V$ $\quad F = mg$

$16000 = 1000 \cdot 10 \cdot V$

$V = 1,6\ m/s$