EE-464 STATIC POWER CONVERSION-II

Controller Design in Power Electronics

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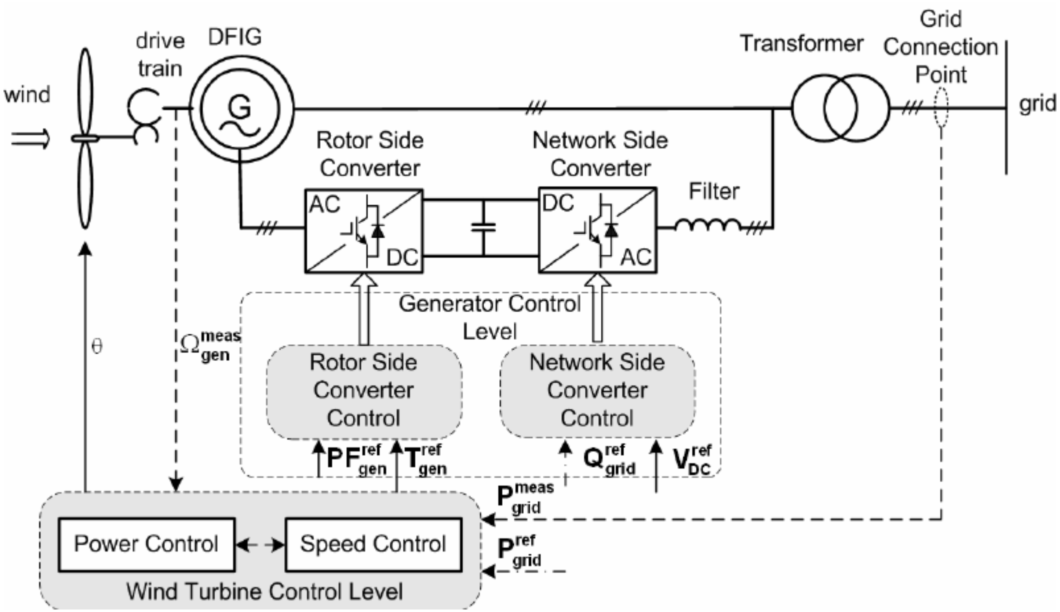
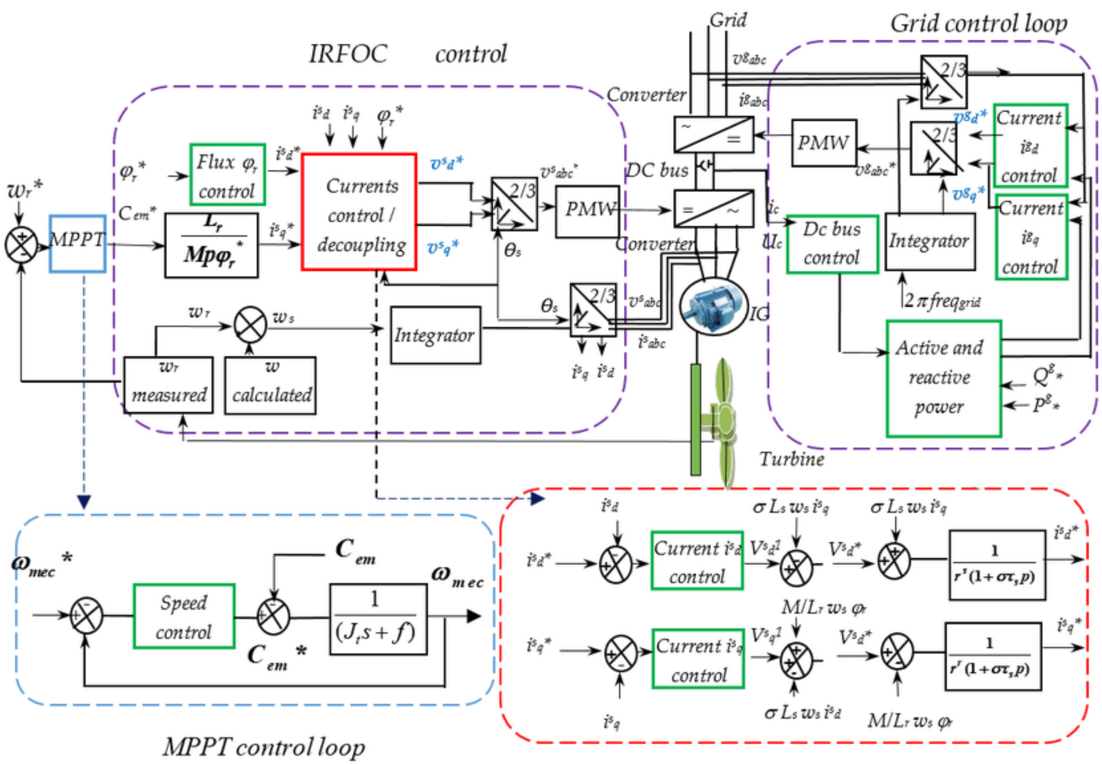
Control in Power Electronics

Control of a Wind Turbine



Detailed Control of a Wind Turbine

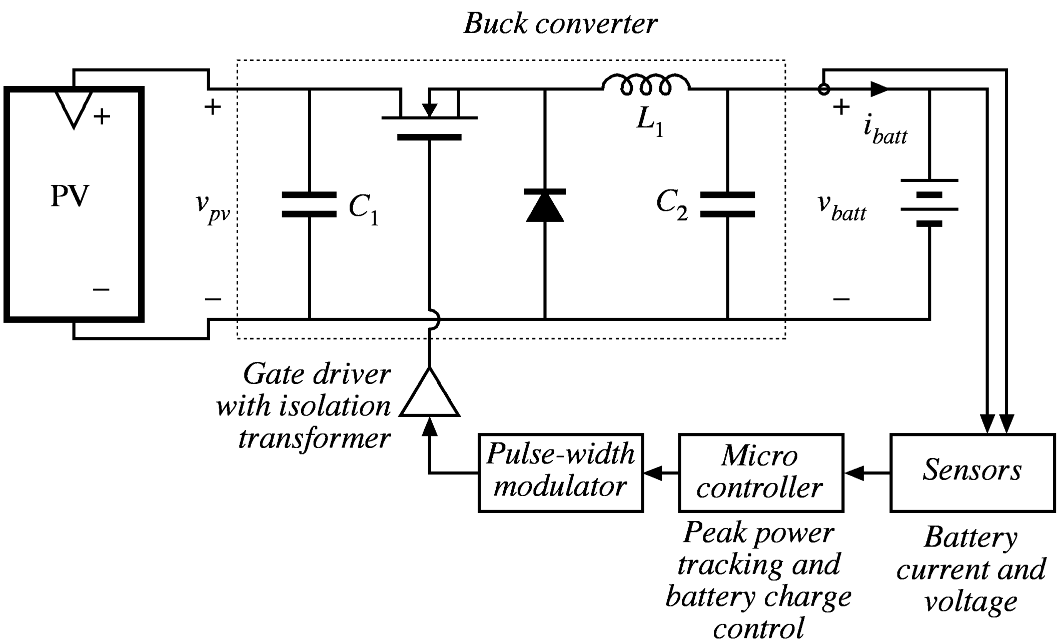


Control in Power Electronics

Most DC/DC converters controlled by analog controllers:

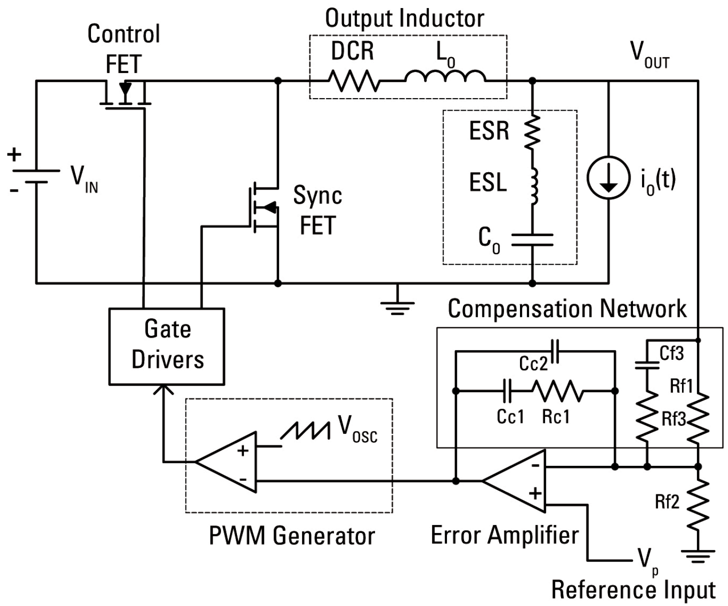
- Micro-controllers are not fast enough (both for computing and sampling) at high switching frequencies
- Cheap (just an IC and a few passive elements)
- Could be integrated to with drive circuit (LM1771)

Control in Power Electronics



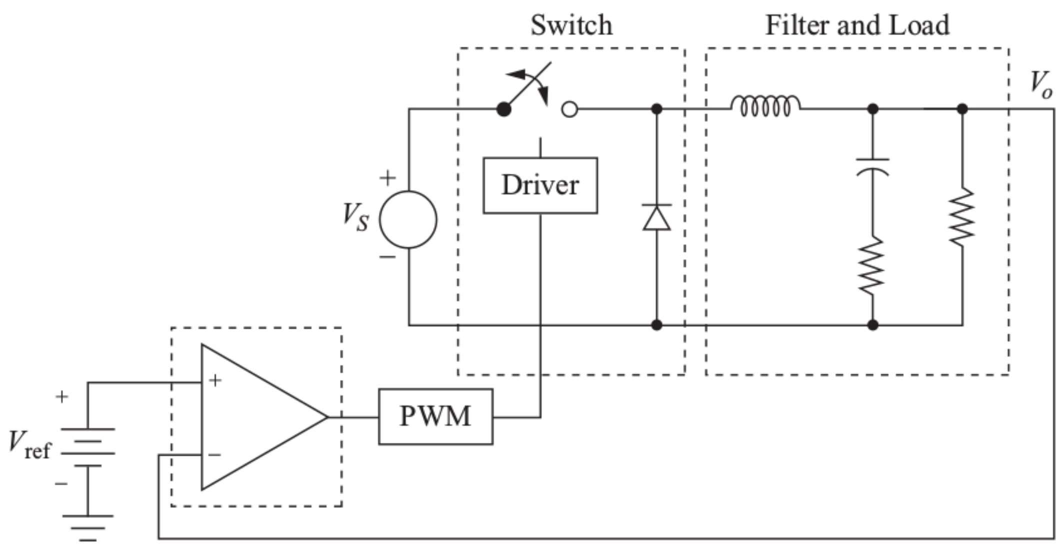
Control with a microcontroller

Control in Power Electronics

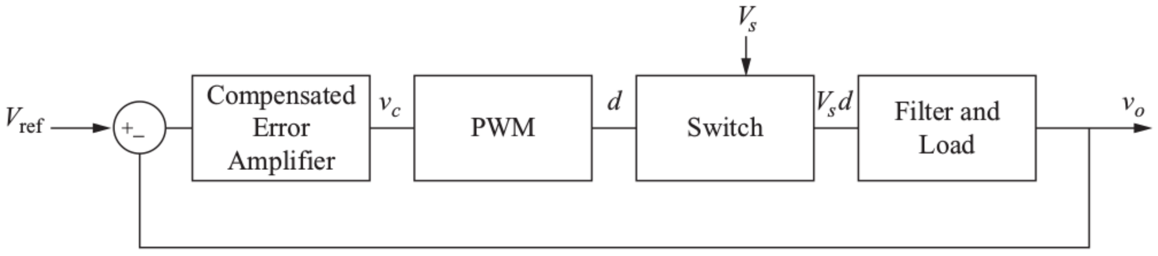


Control with an error amplifier

Buck Converter Controller



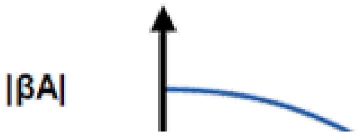
Buck Converter Controller



Control Loop Stability

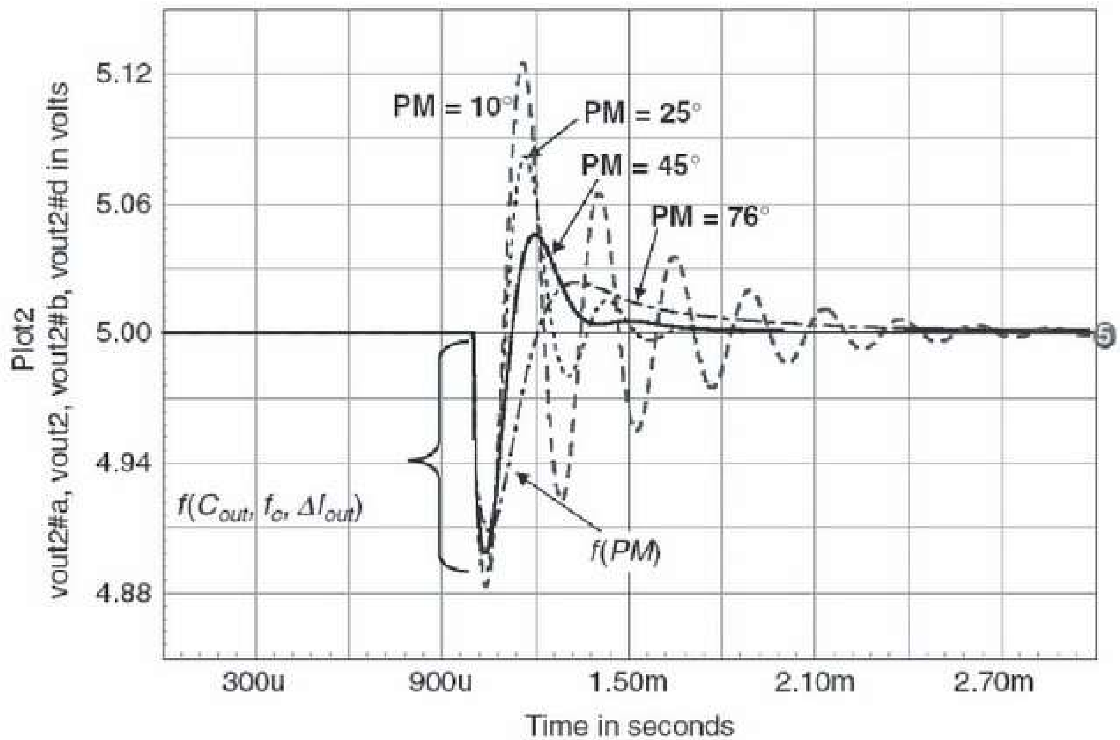
- Small steady-state error (i.e. gain at low frequencies should be large)
- No resonance: (i.e. gain at switching frequency should be small)
- Enough phase-margin: (usually at least 45 degree phase margin is aimed for stability)

Phase Margin



Difference to -180 degrees when the gain is unity (0dB)

Phase Margin



Small Signal Analysis

Don't worry, will be revisited!

Small Signal Analysis

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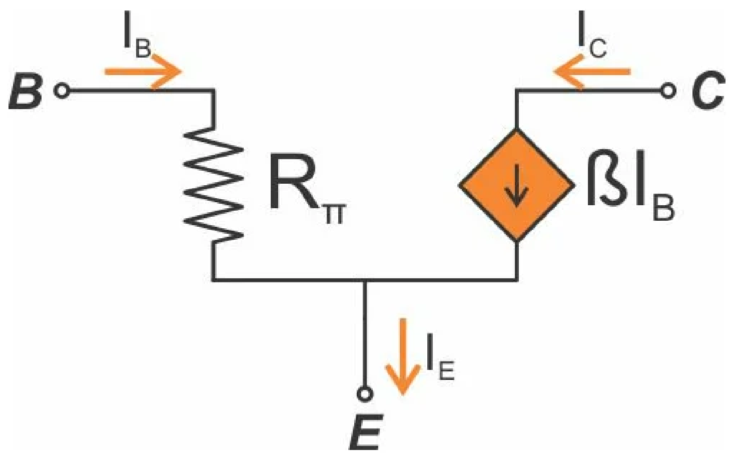
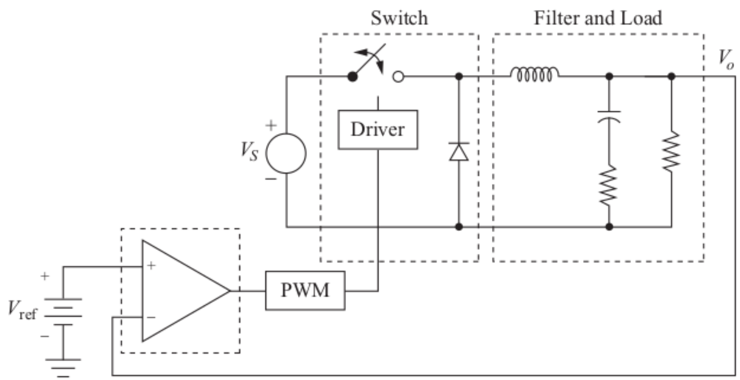
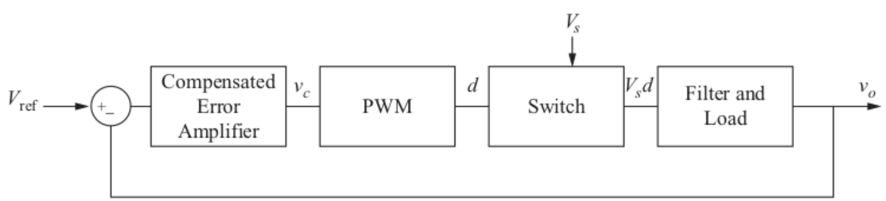
Small Signal Model of a Transistor (EE311)



Small Signal Analysis for the Buck Converter



(a)



(b)

Small Signal Analysis

Small Signal Analysis

For a parameter, x:

Small Signal Analysis

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- x: total quantity

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- $\tilde{x}$: AC term (small-signal variation)

Small Signal Analysis

For a parameter, x:

- x: total quantity
- X: steady-state (DC) component
- $\tilde{x}$: AC term (small-signal variation)

$$x = X + \tilde{x}$$

Small Signal Analysis

Small Signal Analysis

For the buck converter

$$v_o = V_o + \tilde{v}_o$$

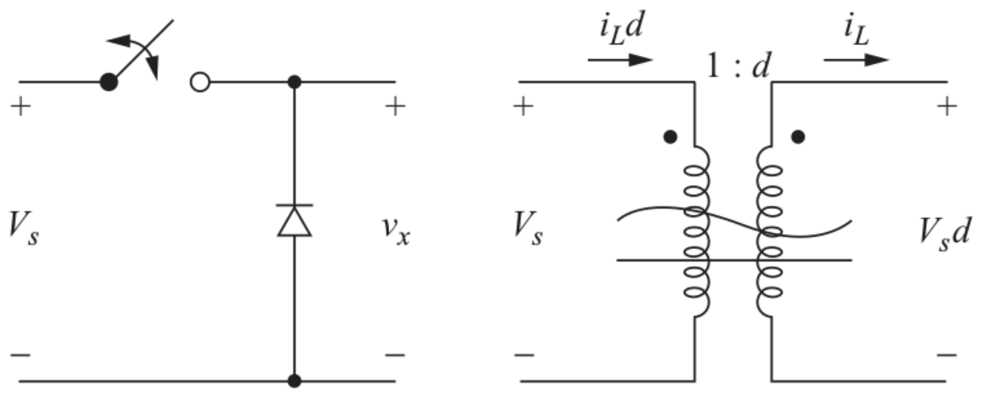
$$d = D + \tilde{d}$$

$$i_L = I_L + \tilde{i}_L$$

$$v_s = V_s + \tilde{v}_s$$

Small Signal Analysis

Average Model of the buck converter



Small Signal Analysis for the Buck Converter

Let's derive the small signal model for voltage

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Small Signal Analysis for the Buck Converter

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ignoring the last term

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ignoring the last term

$$v_x \approx V_s D + \tilde{v}_s D + V_s \tilde{d} = v_s D + V_s \tilde{d}$$

Small Signal Analysis for the Buck Converter

Let's repeat for current

$$i_s = i_L d = (I_L + \tilde{i}_L)(D + \tilde{d})$$

$$\approx i_L D + I_L \tilde{d}$$

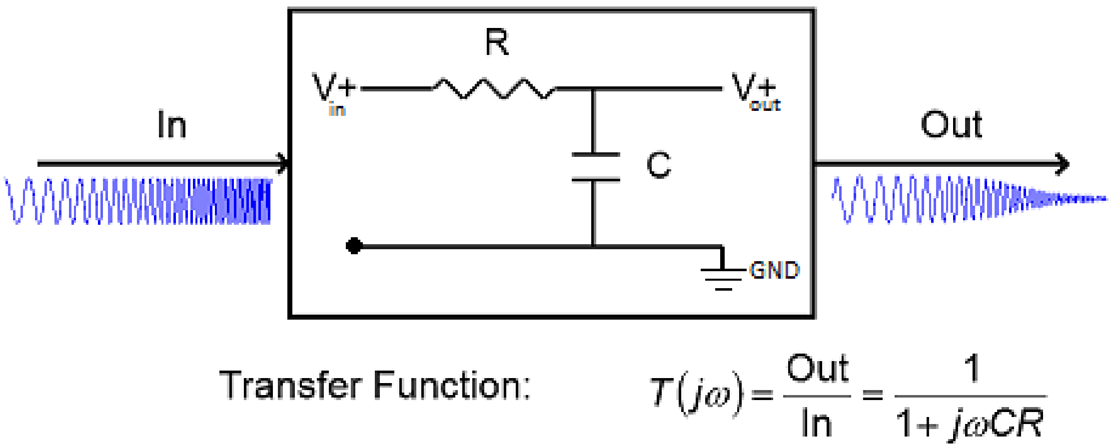
Exercise Assignment:

Power Electronics, Hart, Section 7.13

Buck Converter Small Signal Model

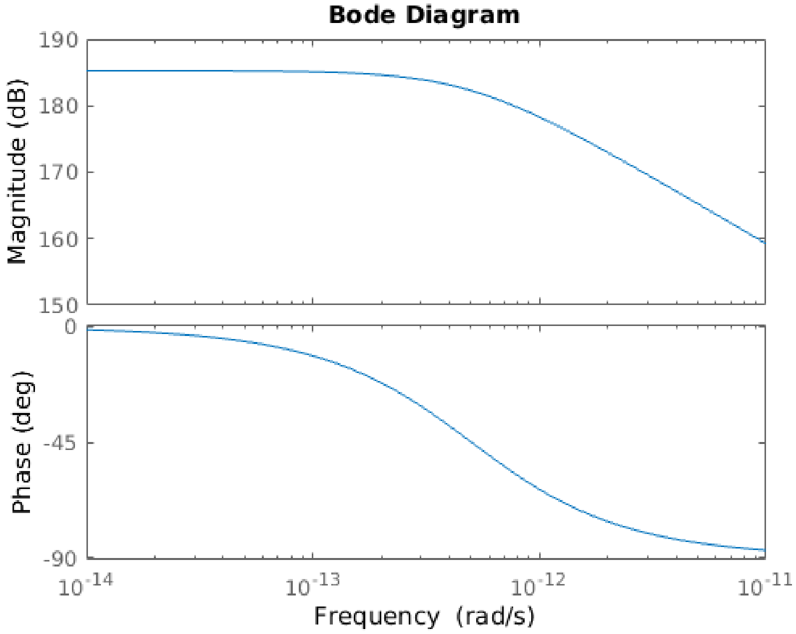
Transfer Functions

RC Filter



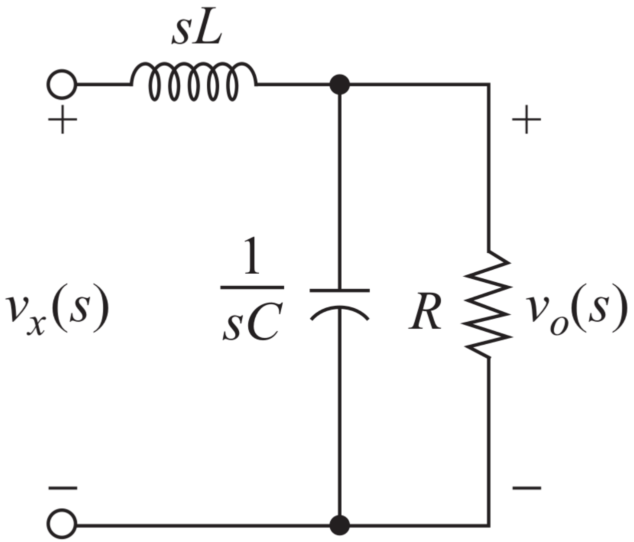
Transfer Functions

RC Filter Bode Plot



Transfer Functions

Let's do for the LCR part of the converter



Representation in the s-domain

Transfer Functions

Let's do for the LCR part of the converter

$$\frac{v_o(s)}{v_x(s)} = \frac{1}{LC(s^2 + (1/RC)s + 1/LC)}$$

$$v_x(s) = V_s d(s)$$

Transfer function in terms of d(s)

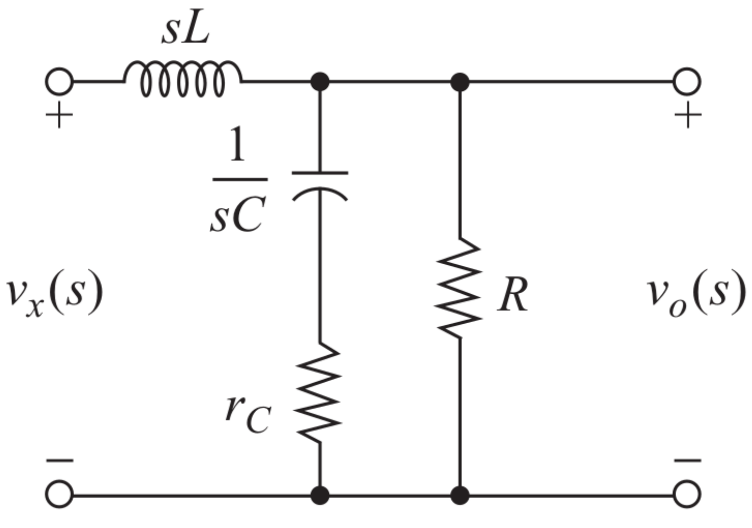
$$\frac{v_o(s)}{d(s)} = \frac{V_s}{LC(s^2 + (1/RC)s + 1/LC)}$$

Realistic RLC

Non-ideal elements can effect stability

- Resistance of the inductor
- ESR of capacitor (series resistance)

Let's repeat the case with non-ideal capacitor



Capacitor with series resistance

Let's repeat the case with non-ideal capacitor

$$\frac{v_o(s)}{d(s)} = \frac{V_s}{LC} \frac{1 + s r_C R}{s^2(1 + r_C/R) + s(1/RC + r_C/L) + 1/LC)}$$

Let's repeat the case with non-ideal capacitor

$$\frac{v_o(s)}{d(s)} = \frac{V_s}{LC} \frac{1 + s r_C R}{s^2(1 + r_C/R) + s(1/RC + r_C/L) + 1/LC)}$$

Can be simplified by assuming $r_C << R$

$$\frac{V_s}{LC} \frac{1 + s r_C R}{s^2 + s(1/RC + r_C/L) + 1/LC)}$$

Notice the extra zero introduced by ESR!

PWM Block Transfer Function

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$$d = \frac{v_c}{V_p}$$

for a saw-tooth PWM generator with Vp peak voltage

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Transfer function

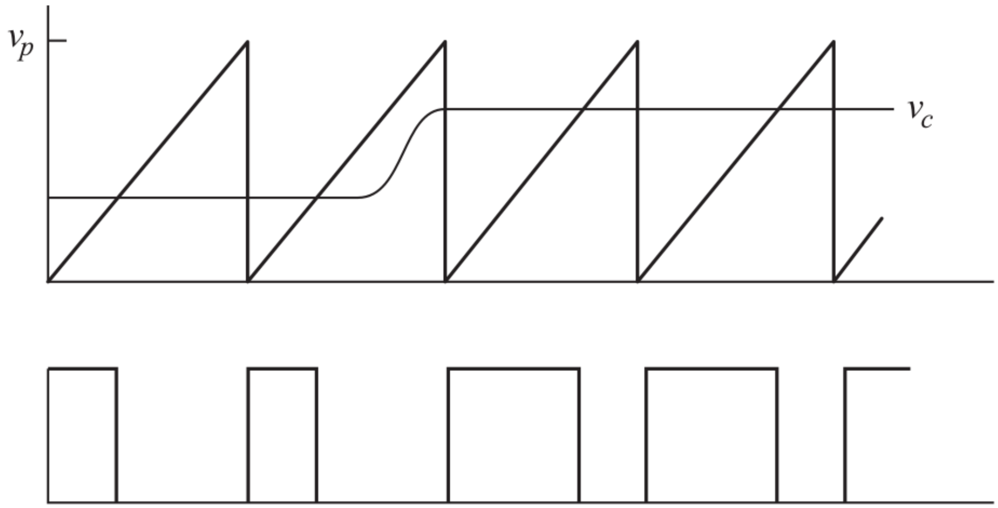
$$\frac{d(s)}{v_c(s)} = \frac{1}{V_p}$$

PWM Block Transfer Function

Be careful with high-frequency control bandwidth

PWM Block Transfer Function

Be careful with high-frequency control bandwidth



What about in switching components?

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- Multi-mode systems (topology changes with switching)

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- Different transfer function for on-off states

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- Multi-mode systems (topology changes with switching)
- Different transfer function for on-off states
- Can use non-linear controller, or multiple linear controllers (but difficult to implement)

What about in switching components?

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Solution:

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- Convert multi-mode to single-mode system

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- Convert multi-mode to single-mode system
- Linearizing the system with averaging wrt duty cycle

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- Use a linear controller with required characteristics

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Solution:

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Details in the textbook (Mohan)

Case Study (Mohan 10-1)

Find the transfer function of forward converter

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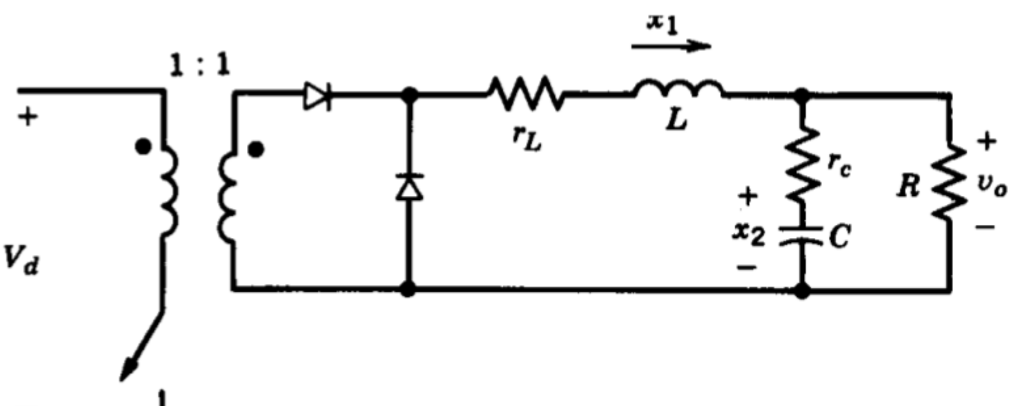
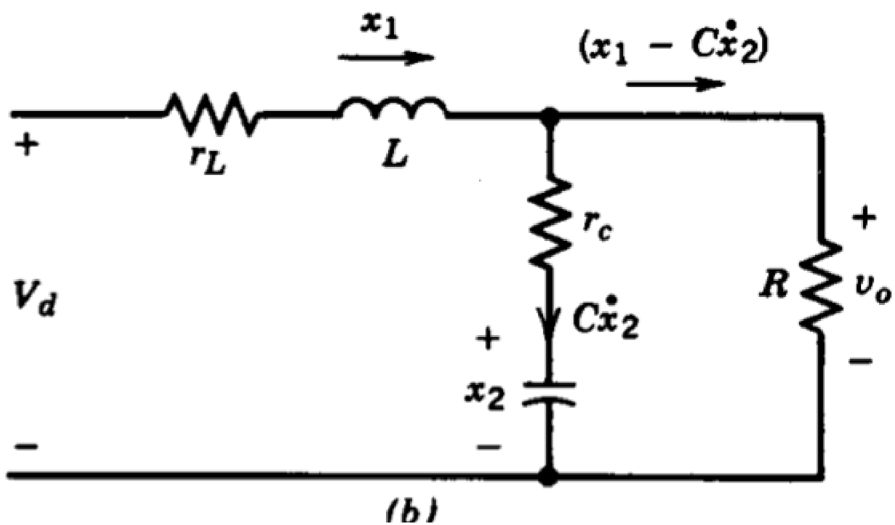


(a)

Note the state variables

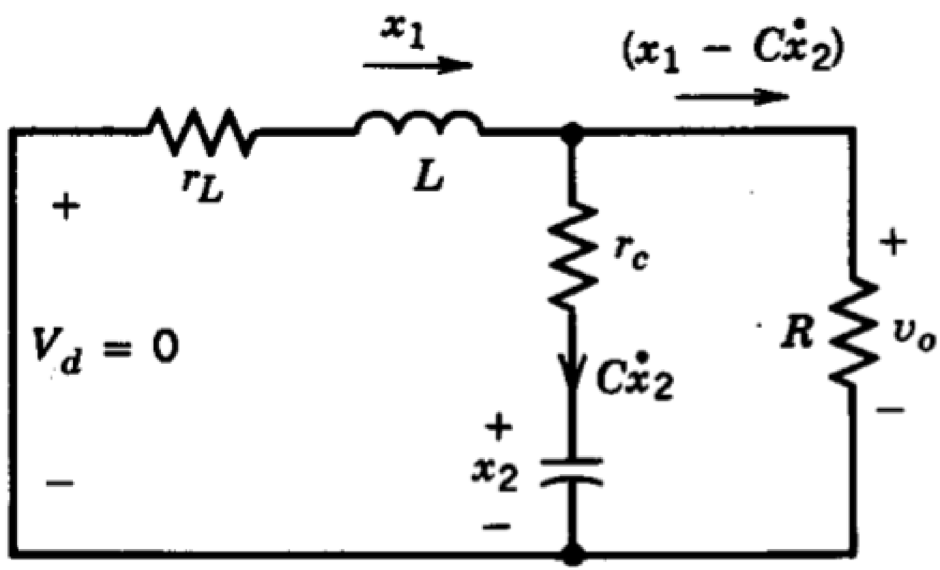
Case Study (Mohan 10-1)

Switch ON



Case Study (Mohan 10-1)

Switch OFF



Case Study (Mohan 10-1)

Steady State Transfer Function

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$$\frac{V_o}{V_d} = -CA^{-1}B$$

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Steady State Transfer Function

$$\frac{V_o}{V_d} = -CA^{-1}B$$

$$\frac{V_o}{V_d} = D\frac{R + r_C}{R + (r_C + r_L)}$$

Case Study (Mohan 10-1)

Steady State Transfer Function

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If parasitic resistances are small

Case Study (Mohan 10-1)

Steady State Transfer Function

$$\frac{V_o}{V_d} = -CA^{-1}B$$

$$\frac{V_o}{V_d} = D\frac{R + r_C}{R + (r_C + r_L)}$$

If parasitic resistances are small

$$\frac{V_o}{V_d} \approx D$$

Case Study (Mohan 10-1)

AC Transfer Function

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$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

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AC Transfer Function

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$$= V_d \frac{1 + s r_C C}{LC[s^2 + s(1/RC + (r_C + r_L)/L) + 1/L\cdots}$$

Case Study (Mohan 10-1)

AC Transfer Function

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}\,(s)}$$

$$= V_d \frac{1 + sr_C C}{LC[s^2 + s(1/RC + (r_C + r_L)/L) + 1/L}$$

Remember this equation?

Case Study (Mohan 10-1)

AC Transfer Function

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$$= V_d \frac{1 + s r_C C}{LC[s^2 + s(1/RC + (r_C + r_L)/L) + 1/L$$

Remember this equation?

$$s^2 + 2\xi\omega_0 s + \omega_0^2$$

Case Study (Mohan 10-1)

AC Transfer Function

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$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Case Study (Mohan 10-1)

AC Transfer Function

$$s^2 + 2\xi\omega_0 s + \omega_0^2$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\xi = \frac{1/RC + (r_C + r_L)/L}{2\omega_0}$$

Case Study (Mohan 10-1)

AC Transfer Function Becomes

$$T_p(s) = V_d \frac{\omega_0^2}{\omega_z} \frac{s + \omega_z}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

where $\omega_z = \dfrac{1}{r_C C}$

Example (Mohan 10-1)

Put the parameters into the equation

$V_d = 8V$

$V_o = 5V$

$r_L = 20m\Omega$

$L = 5\mu H$

$r_C = 10m\Omega$

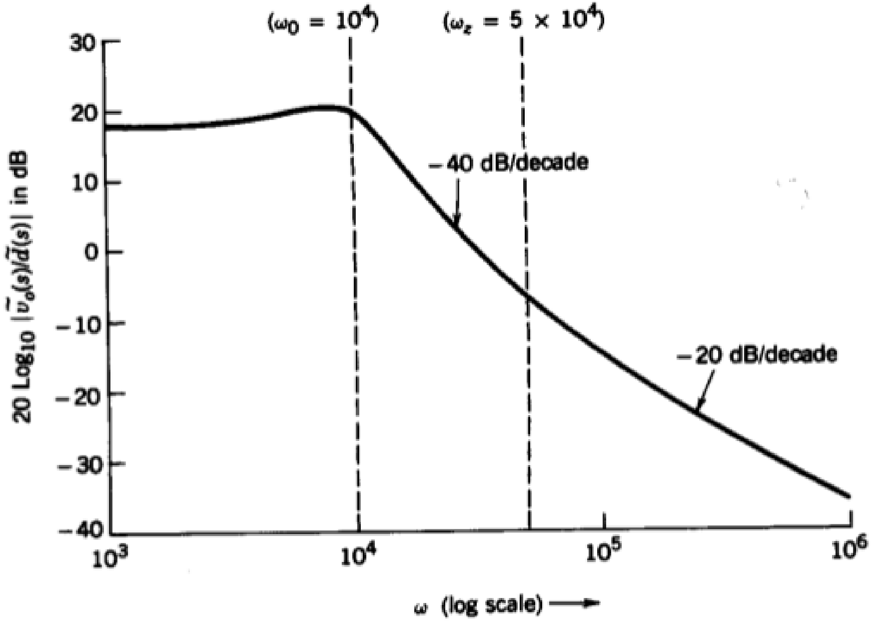
$C = 2mF$

$R = 200m\Omega$

$f_s = 200kHz$

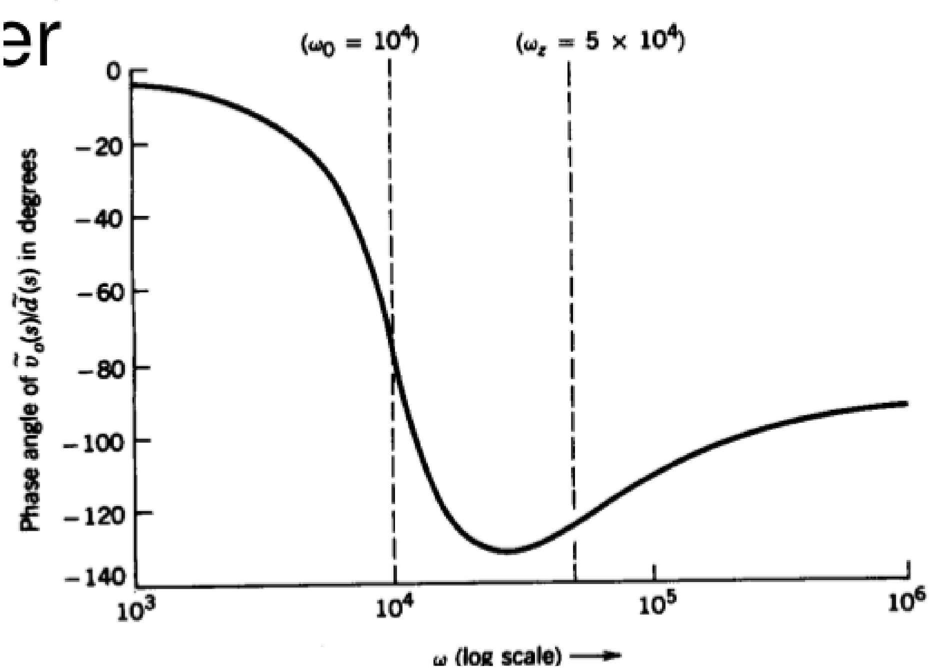
Example (Mohan 10-1)

Bode Plot (Gain)



Example (Mohan 10-1)

Bode Plot (Phase)



Flyback Converter

Flyback Converter

Equation 10-86

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

Flyback Converter

Equation 10-86

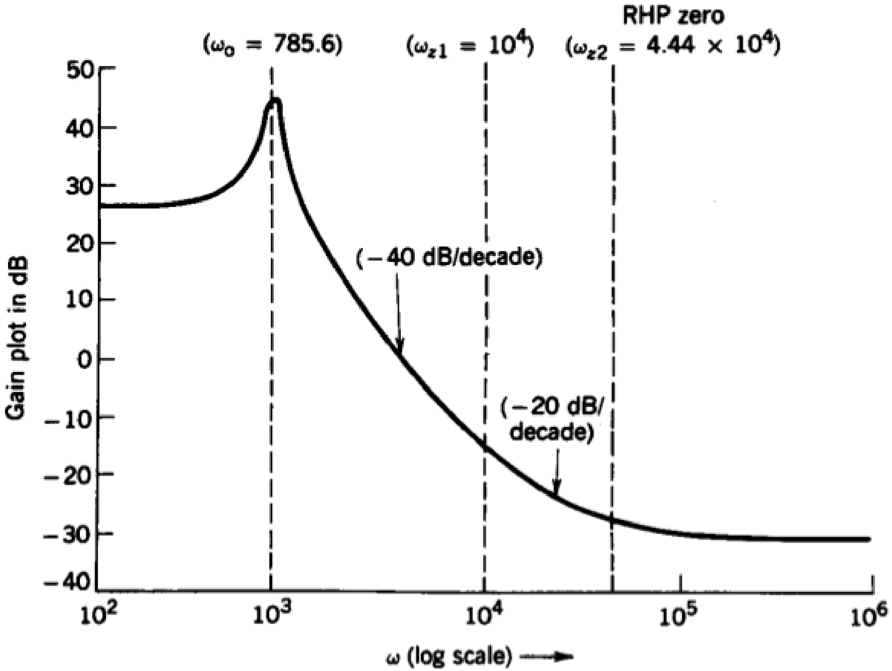
$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

$$T_p(s) = V_d f(D) \frac{(1 + s/\omega_{z1})(1 - s/\omega_{z2})}{as^2 + bs + c}$$

Flyback Converter

Flyback Converter

Bode Plot (Gain)



Flyback Converter

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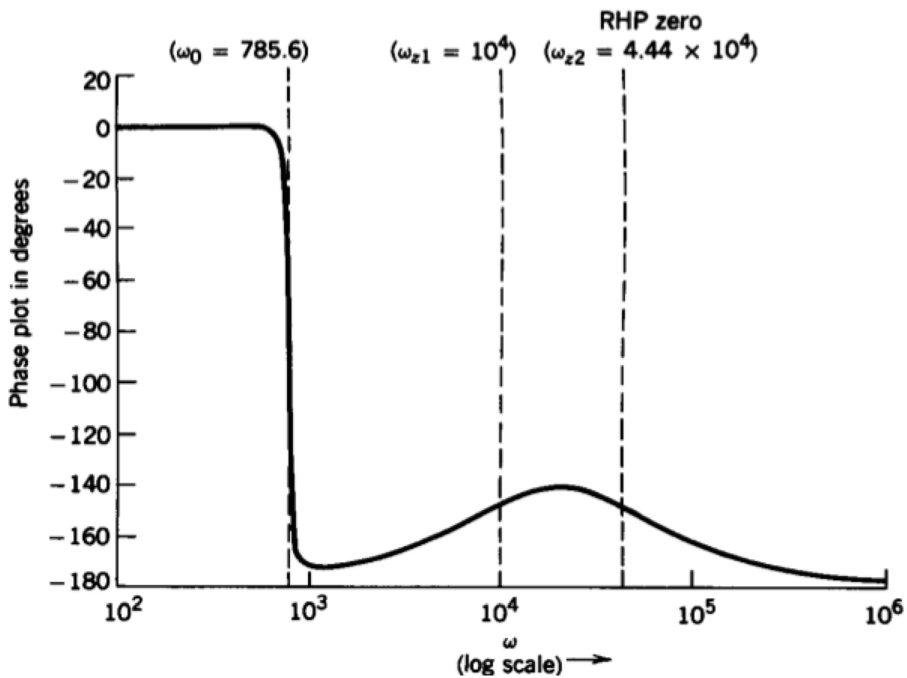
Bode Plot (Phase)



A few readings for controller design

A few readings for controller design

- Control Design of a Boost Converter Using Frequency Response Data
- PID Control Tuning for Buck Converter
- Design digital controllers for power electronics using simulation
- Bode Response of Simulink Model
- How to Run an AC Sweep with PSIM?
- Peak Current Control with PSIM
- Plexim-Frequency Analysis of Buck Converter

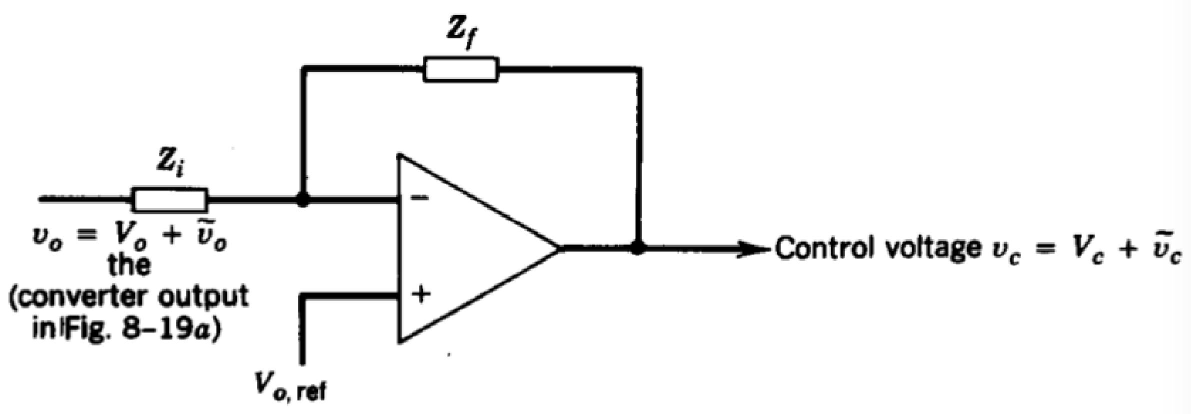
Controller Design

Controller Design

Generalized Compensated Error Amplifier

Controller Design

Generalized Compensated Error Amplifier



Types of Error Amplifier

Types of Error Amplifier

Common Ones:

- Type-1
- Type-2
- Type-3

Type-1 Error Amplifier

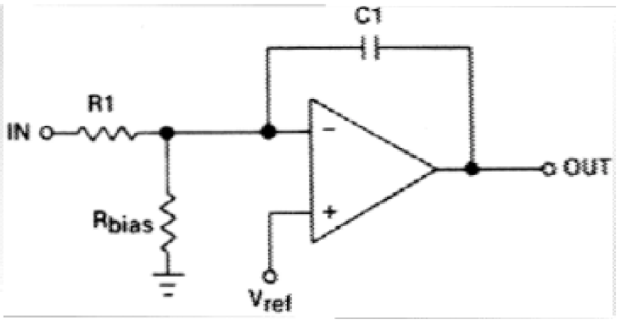


Figure 5. Schematic Diagram of a Type 1 Amplifier

Simple Integrator

Has one pole at the origin

Type-1 Error Amplifier

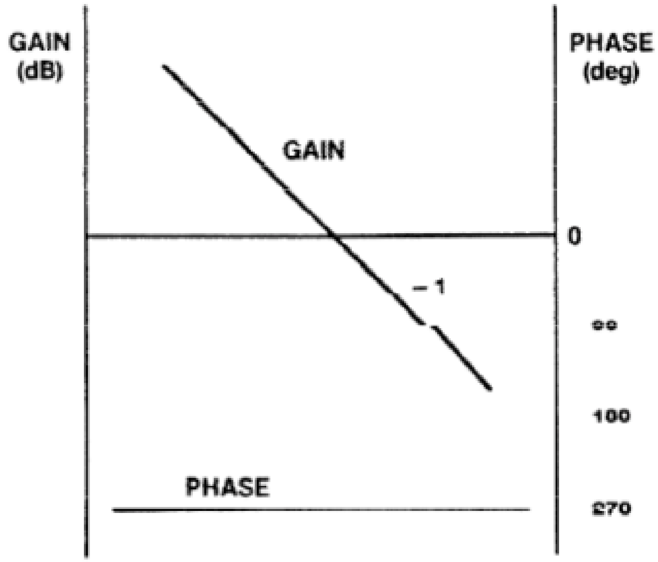


Figure 6. Transfer Function of a Type 1 Amplifier

Type-2 Error Amplifier (Most Common Type)

Type-2 Error Amplifier (Most Common Type)

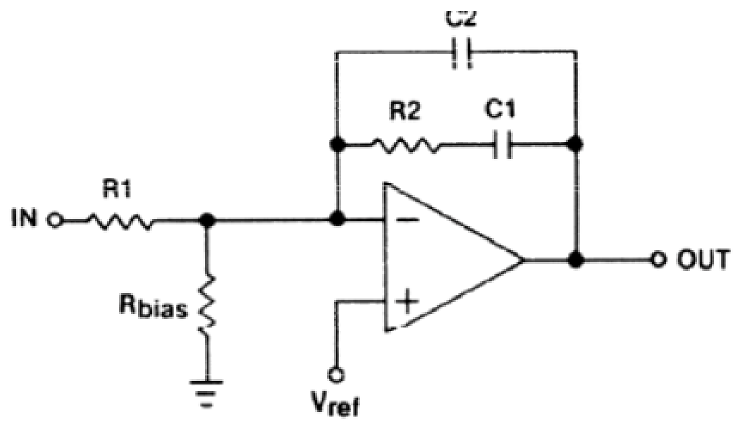


Figure 7. Schematic Diagram of a Type 2 Amplifier

Has two poles: at origin and one at zero-pole pair

90 degrees phase boost can be obtained due to single zero

Type-2 Error Amplifier (Most Common Type)

Type-2 Error Amplifier (Most Common Type)

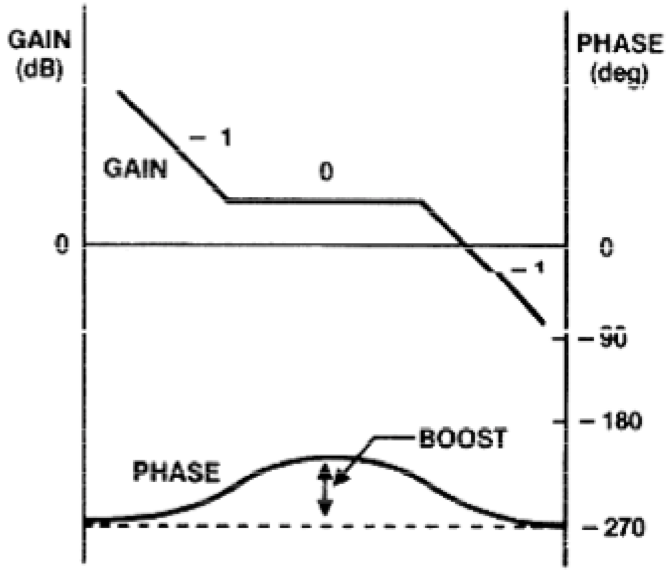


Figure 8. Transfer Function of a Type 2 Amplifier

Note the phase boost

Type-3 Error Amplifier

Type-3 Error Amplifier

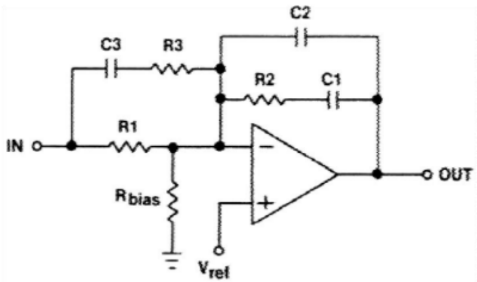
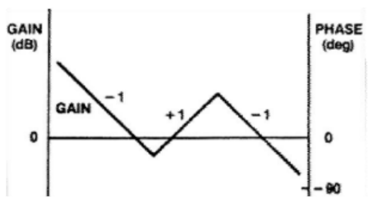


Figure 9. Schematic Diagram of a Type 3 Amplifier



has two zeros can can boost up to 180 degrees

A Few Examples

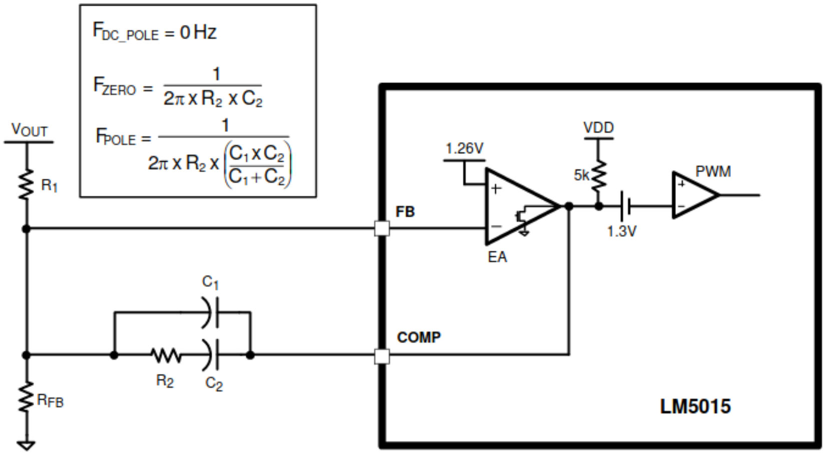


Figure 15. Type II Compensator

- TL494, pg. 7, 15
- LM5015, Fig. 12, 15

Putting all Together

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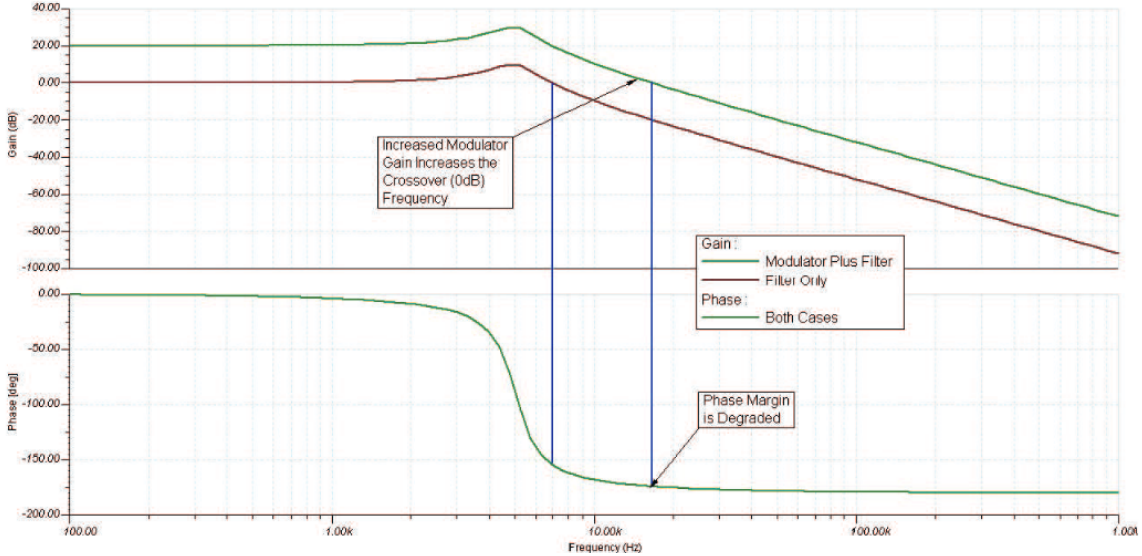
A controller just increases the gain (Proportional)

Putting all Together

A controller just increases the gain (Proportional)



Increasing gain usually reduces phase margin (and reduces stability)

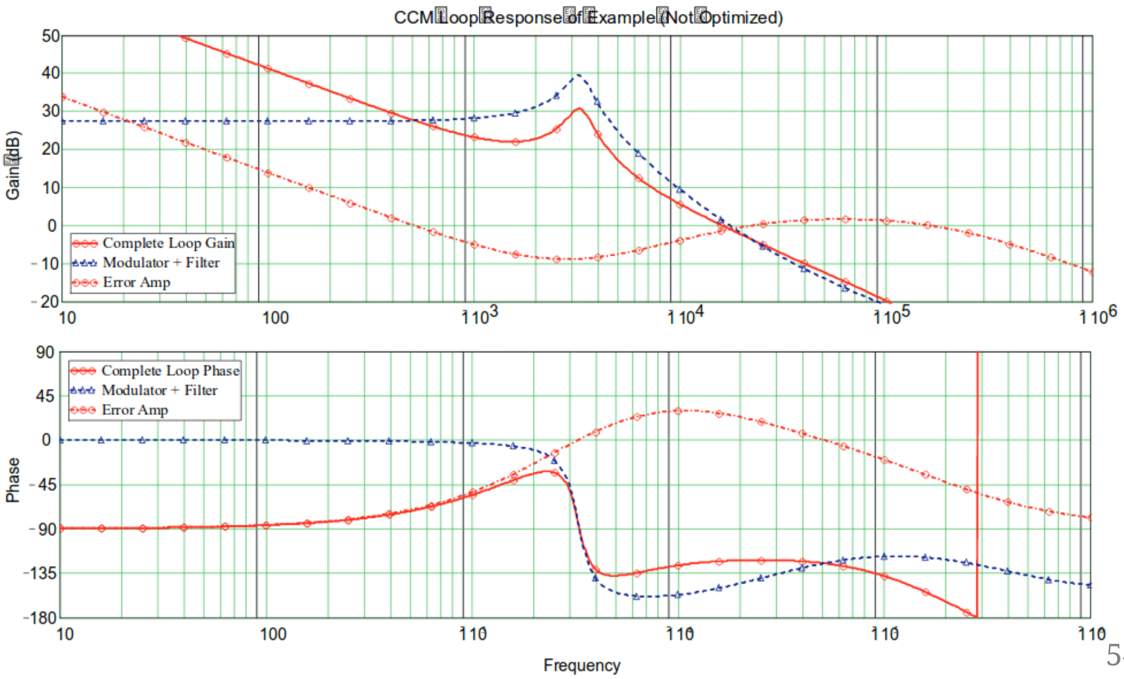
Putting all Together

Putting all Together

A proper controller (adjust gain and phase margin)

Putting all Together

A proper controller (adjust gain and phase margin)



More Information

More Information

- Fundamentals of Power Electronics, Erickson
- Phase Margin, Crossover Frequency, and Stability
- Loop Stability Analysis of Voltage Mode Buck Regulator
- DC-DC Converters Feedback and Control
- Modeling and Loop Compensation Design
- Compensator Design Procedure

You can download this presentation from:
keysan.me/ee464

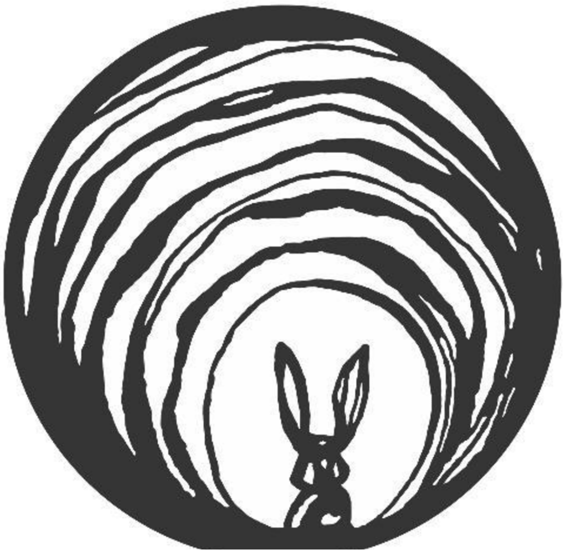
Saved for further reference

Ready?

Saved for further reference

Ready?

Down the rabbit hole



Linearization with State-Space Averaging

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- Represent everthing in matrix form

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- Inductor current, and capacitor voltage as state variables

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- Obtain two states (for switch on and siwtch off)

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- Inductor current, and capacitor voltage as state variables
- Obtain two states (for switch on and siwtch off)
- Find the weighted average

Linearization with State-Space Averaging

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Obtain two states (for switch on and siwtch off)

Linearization with State-Space Averaging

Obtain two states (for switch on and siwtch off)

$\dot{x} = A_1 x + B_1 v_d$ (for switch on, dTs)

Linearization with State-Space Averaging

Obtain two states (for switch on and siwtch off)

$\dot{x} = A_1 x + B_1 v_d$ (for switch on, dTs)

$\dot{x} = A_2 x + B_2 v_d$ (for switch off, (1-d)Ts)

where, A1 and A2 are state matrices

B1 and B2 are vectors

Example:

Mohan 10.1

Linearization with State-Space Averaging

Linearization with State-Space Averaging

Find weighted average

Linearization with State-Space Averaging

Find weighted average

$$A = dA_1 + (1-d)A_2$$

$$B = dB_1 + (1-d)B_2$$

Linearization with State-Space Averaging

Find weighted average

$$A = dA_1 + (1 - d)A_2$$

$$B = dB_1 + (1 - d)B_2$$

$\dot{x} = Ax + Bv_d$ (for switch off, (1-d)Ts)

Similar calculations for the output voltage

$v_o = C_1 x$ (for switch on, dTs)

$v_o = C_2 x$ (for switch off, (1-d)Ts)

where C1 and C2 are transposed vectors

Similar calculations for the output voltage

$$v_o = Cx$$

$$C = dC_1 + (1 - d)C_2$$

where C1 and C2 are transposed vectors

Use Small signal model

Equations 10.46-10.52

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$$x = X + \tilde{x}$$

Use Small signal model

Equations 10.46-10.52

$$x = X + \tilde{x}$$

$$\dot{\tilde{x}} = AX + BV_d + A\tilde{x}$$
$$+[(A_1 - A_2)X + (B_1 - B_2)V_d]\tilde{d}$$

Use Small signal model

Equations 10.46-10.52

$$x = X + \tilde{x}$$

$$\dot{\tilde{x}} = AX + BV_d + A\tilde{x}$$

$$+[(A_1 - A_2)X + (B_1 - B_2)V_d]\tilde{d}$$

In the steady state:

$\dot{X}$=0

Use derivations from eq.10.53-10.59

Steady State DC Voltage Transfer Function

Steady State DC Voltage Transfer Function

$$\frac{V_o}{V_d} = -CA^{-1}B$$

Small Signal Model to Get AC Transfer Function

Small Signal Model to Get AC Transfer Function

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

$$= C[sI - A]^{-1}[(A_1 - A_2)X + (B_1 - B_2)V_d]$$

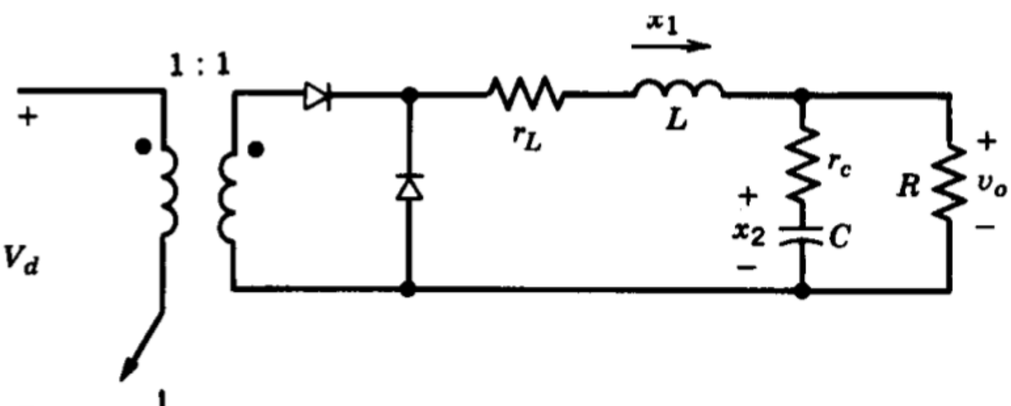
$$+(C_1 - C_2)X)]$$

Example (Mohan 10-1)

Find the transfer function of forward converter

Example (Mohan 10-1)

Find the transfer function of forward converter



(a)

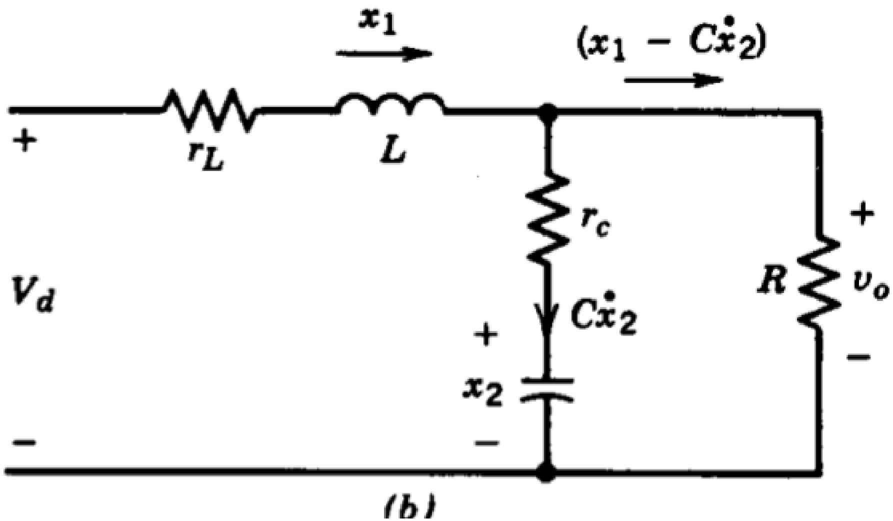
Note the state variables

Example (Mohan 10-1)

Switch ON

Example (Mohan 10-1)

Switch ON

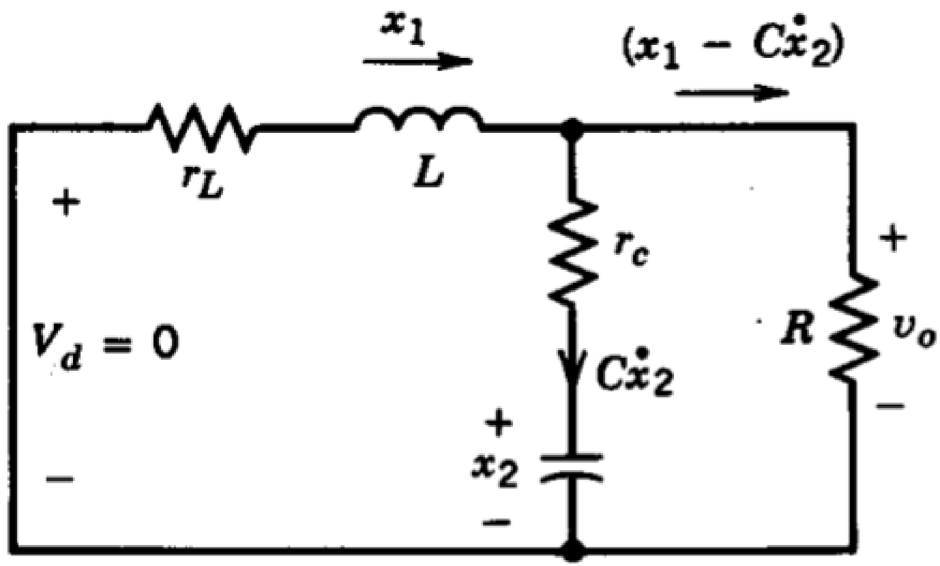


Example (Mohan 10-1)

Switch OFF

Example (Mohan 10-1)

Switch OFF



Example (Mohan 10-1)

Steady State Transfer Function

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Steady State Transfer Function

$$\frac{V_o}{V_d} = -CA^{-1}B$$

Example (Mohan 10-1)

Steady State Transfer Function

$$\frac{V_o}{V_d} = -CA^{-1}B$$

$$\frac{V_o}{V_d} = D\frac{R + r_C}{R + (r_C + r_L)}$$

Example (Mohan 10-1)

Steady State Transfer Function

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If parasitic resistances are small

Example (Mohan 10-1)

Steady State Transfer Function

$$\frac{V_o}{V_d} = -CA^{-1}B$$

$$\frac{V_o}{V_d} = D\frac{R + r_C}{R + (r_C + r_L)}$$

If parasitic resistances are small

$$\frac{V_o}{V_d} \approx D$$

Example (Mohan 10-1)

AC Transfer Function

Example (Mohan 10-1)

AC Transfer Function

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

Example (Mohan 10-1)

AC Transfer Function

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

$$T_p(s) = V_d \frac{1 + s r_C C}{LC[s^2 + s(1/RC + (r_C + r_L)/L) + 1/LC]}$$

Example (Mohan 10-1)

AC Transfer Function

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Remember this equation?

Example (Mohan 10-1)

AC Transfer Function

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

$$T_p(s) = V_d \frac{1 + s r_C C}{LC[s^2 + s(1/RC + (r_C + r_L)/L) + 1/LC]}$$

Remember this equation?

$$s^2 + 2\xi\omega_0 s + \omega_0^2$$

Example (Mohan 10-1)

AC Transfer Function

$$s^2 + 2\xi\omega_0 s + \omega_0^2$$

Example (Mohan 10-1)

AC Transfer Function

$$s^2 + 2\xi\omega_0 s + \omega_0^2$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Example (Mohan 10-1)

AC Transfer Function

$$s^2 + 2\xi\omega_0 s + \omega_0^2$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\xi = \frac{1/RC + (r_C + r_L)/L}{2\omega_0}$$

Example (Mohan 10-1)

AC Transfer Function Becomes

$$T_p(s) = V_d \frac{\omega_0^2}{\omega_z} \frac{s + \omega_z}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

where $\omega_z = \dfrac{1}{r_C C}$

Example (Mohan 10-1)

Put the parameters into the equation

$V_d = 8V$

$V_o = 5V$

$r_L = 20m\Omega$

$L = 5\mu H$

$r_C = 10m\Omega$

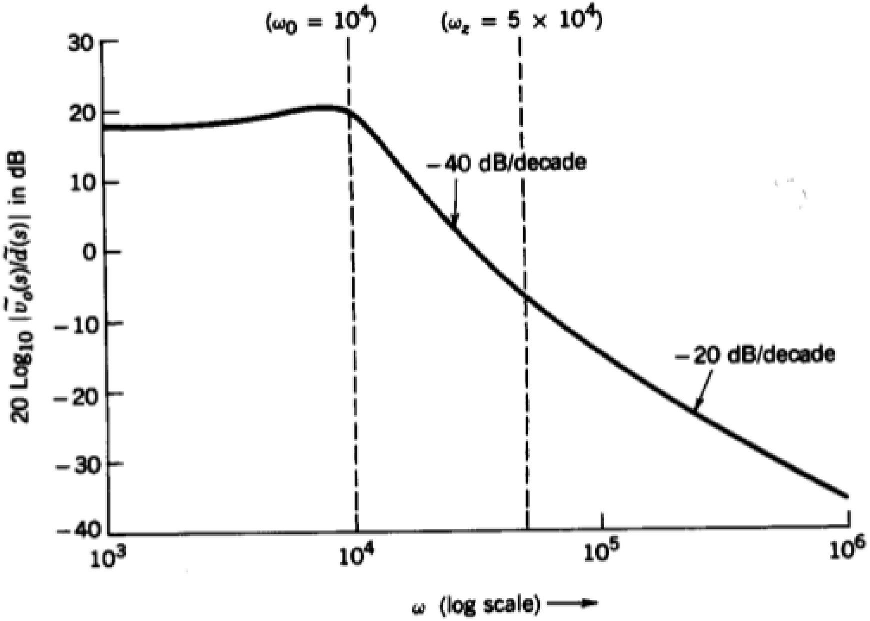
$C = 2mF$

$R = 200m\Omega$

$f_s = 200kHz$

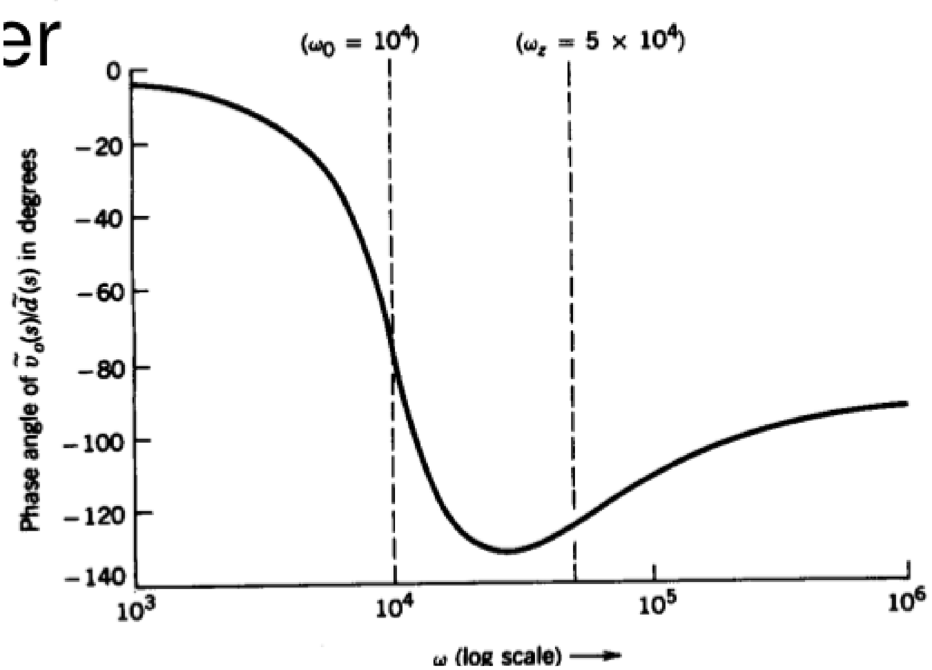
Example (Mohan 10-1)

Bode Plot (Gain)



Example (Mohan 10-1)

Bode Plot (Phase)



Flyback Converter

Flyback Converter

Equation 10-86

$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

Flyback Converter

Equation 10-86

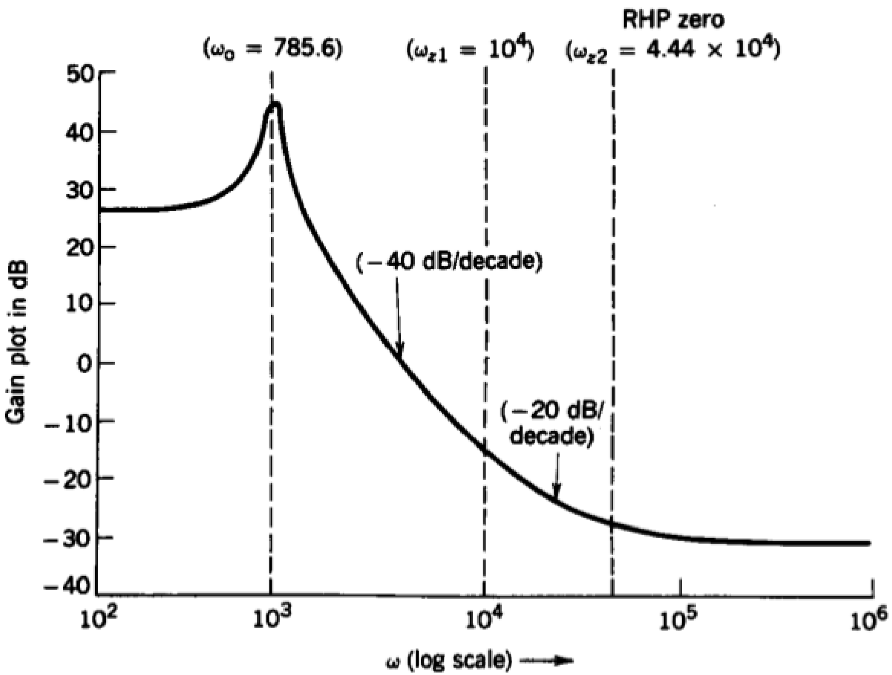
$$T_p(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)}$$

$$T_p(s) = V_d f(D) \frac{(1 + s/\omega_{z1})(1 - s/\omega_{z2})}{as^2 + bs + c}$$

Flyback Converter

Flyback Converter

Bode Plot (Gain)



Flyback Converter

Flyback Converter

Bode Plot (Phase)