ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

Dr. Ergin TÖNÜK

Department of Mechanical Engineering

Graduate Program of Biomedical Engineering

tonuk@metu.edu.tr

http://tonuk.me.metu.edu.tr

1. Introduction to Dynamics

Mechanical Engineering

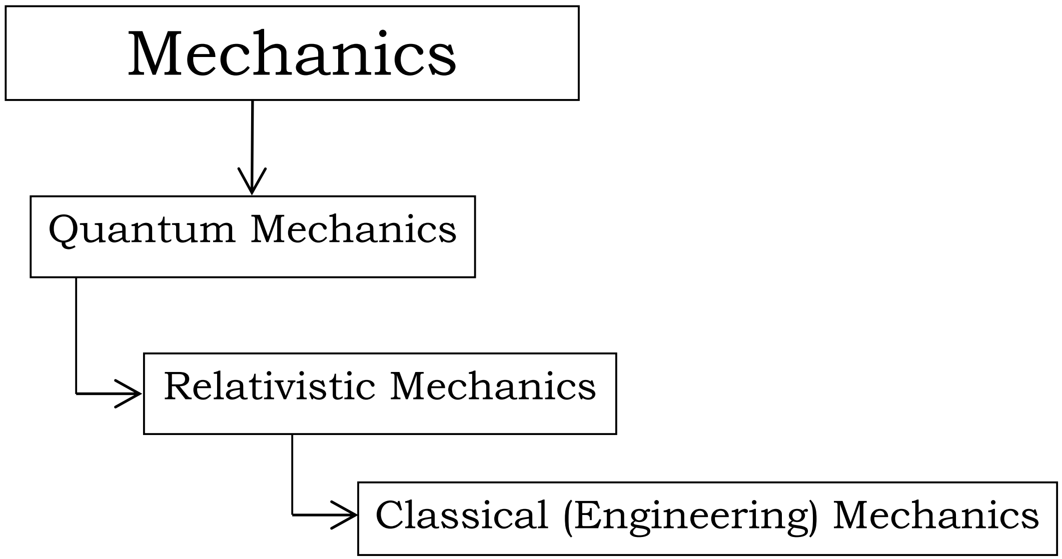
Mechanical Sciences

Thermal Sciences

MECHANICS (Greek *Μηχανική*)

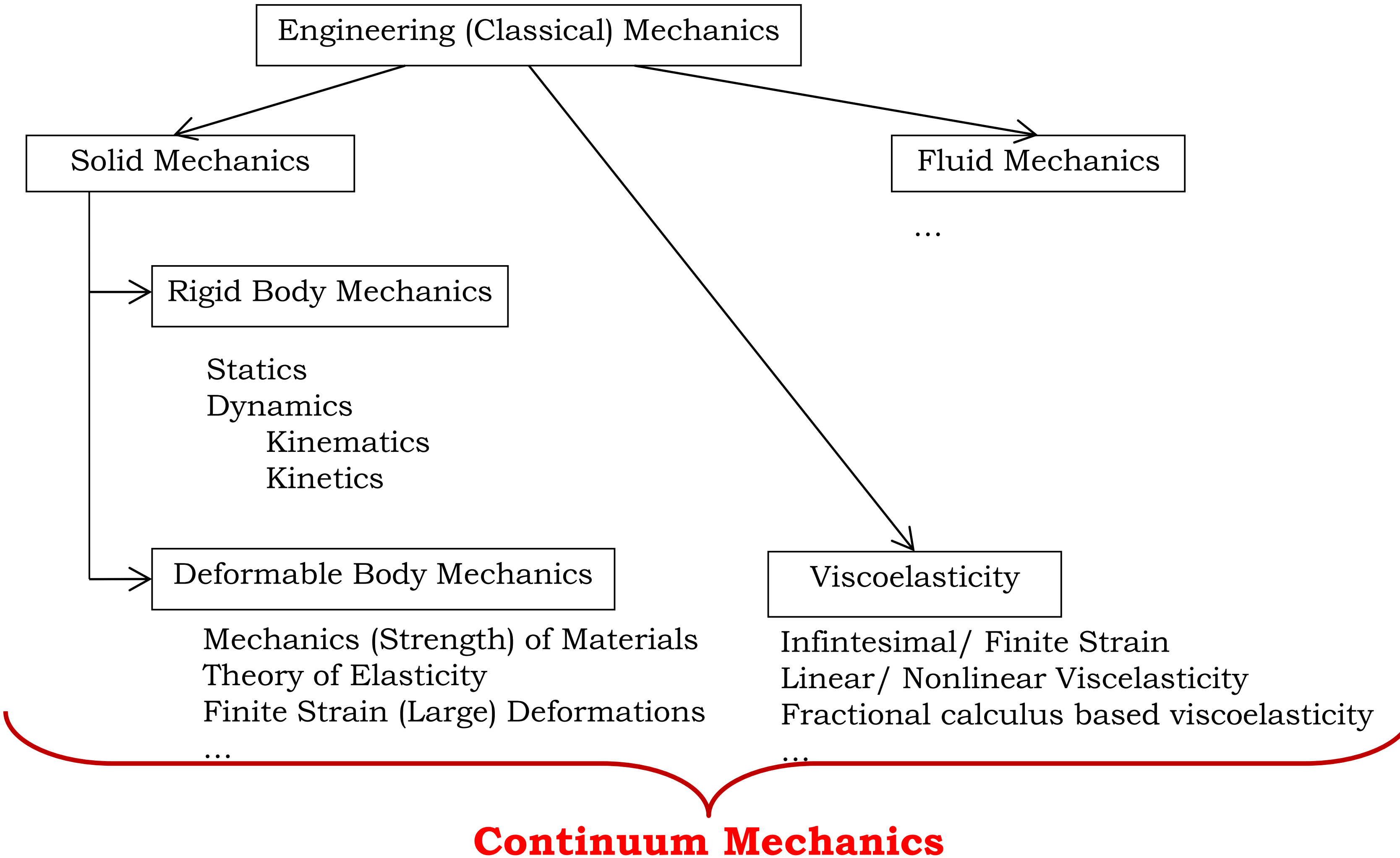
The branch of physical science concerned with the **behavior** (i.e. motion and/or deformation) of bodies under the action of **forces**.

It is an exact science based on experimental evidence.



Vector Mechanics
- Newtonian Mechanics
- D'Alembert's Principle
- …

Analytic Mechanics
- Lagrangian Mechanics
- Hamiltonian Mechanics
- …



Engineering (Classical) Mechanics
Solid Mechanics
Fluid Mechanics
...
Rigid Body Mechanics
Statics
Dynamics
Kinematics
Kinetics
Deformable Body Mechanics
Mechanics (Strength) of Materials
Theory of Elasticity
Finite Strain (Large) Deformations
...
Viscoelasticity
Infintesimal/ Finite Strain
Linear/ Nonlinear Viscelasticity
Fractional calculus based viscoelasticity
...
Continuum Mechanics

DYNAMICS

The branch of mechanics dealing with the motion of *rigid* bodies.

Kinematics is the study of the geometry of the motion. It relates displacement, velocity, acceleration and time without reference to cause of motion.

Kinetics relates forces on a body to its motion using mass. Kinetics may be used to predict the motion under known forces or forces required for certain motion.

1.1. Brief History of Dynamics

Galileo Galilei (1564-1642)

Observed free fall, motion on inclined plane, motion of pendulum.
Did not have a clock to keep time!

Isaac Newton (1642-1727)

Accurate formulation of laws of motion.

Leonhard Euler (1707-1783)

Jean le Rond d'Alembert (1717-1783)

Joseph-Louis Lagrange (1736-1813)

Pierre-Simon Laplace (1749-1827)

Louis Poinsot (1777-1859)

Gaspard-Gustave de Coriolis (1792-1843)

Albert Einstein (1879-1955)

...

1.2. Basic Concepts

Space: Geometric region occupied by the bodies. *Position of a body is determined by means of linear and angular measurements relative to a reference frame (coordinate system).*

The basic frame of reference for classical mechanics is Primary Inertial System

Time: A measure of succession of events. *Time is common for all reference frames in classical mechanics.*

1.2. Basic Concepts

Mass: Measure of resistance to change of motion of a body.

Force: Vector action (push or pull) of a body on another.

Particle: A body of negligible dimensions compared to motion.

Rigid Body: A body whose change of shape under the forces are negligible.

1.2. Basic Concepts

Scalar: A physical quantity with a magnitude.

Vector: A physical quantity with a magnitude and a direction.

Vectors should satisfy the law of parallelogram addition!

Scalars and vectors have dimensions and units!

1.2. Basic Concepts

Dimension: It is the property of a physical quantity.

In mechanics the dimensions used are mass [M], length [L], time [T] and force [F].

Unit: It is the reference scale to measure a certain dimension.

In SI units mass is measured in kg, length in m, time in s and force in N.

1.2. Basic Concepts

Law of Dimensional Homogeneity: Every *term* in a physical equation should have the same dimensions and units.

Term: Entity separated by a + , - or = sign.

$$\vec{F} = m\vec{a}$$

$$[N] \equiv [kg.m.s^{-2}] = [kg]\left[{}^{m}/{}_{s^2}\right]$$

$$v(t) = v_0 + at$$

1.3. Newton's *Postulates* of Motion

1. $\sum \vec{F} = \vec{0}$ then $\vec{a} = \vec{0}$

2. $\sum \vec{F} = \frac{d}{dt}(m\vec{v})$ becomes $\sum \vec{F} = m\vec{a}$ for constant mass particles

3. Action-reaction principle

Law of universal gravitation: $F = G\frac{m_1 m_2}{r^2}$

$m_1 = m_e$, $r = r_e$ then $G\frac{m_1}{r^2} = g$ and $W = mg$

1.3. Newton's *Postulates* of Motion

- Based on pure observation and cannot be derived or proven mathematically.
- First two laws are valid when expressed with respect to an *inertial reference frame*.
- Law of universal gravitation predicts the mutual attractive forces of two masses (weight of a body) but does not question the reason and the action is *immediate*.

Newton's Notation

$$\frac{d}{dt} = \dot{}$$

$$\frac{d^2}{dt^2} = \ddot{}$$

Example: $\vec{v} = \dot{\vec{r}}$, $\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$

Course Content

Particle

Kinematics (Chapter 2)

Kinetics (Chapter 3)

Rigid Body

Kinematics (Chapter 5)

Kinetics (*Chapter 4*, *Appendix B* and Chapter 6)

PART I: PARTICLES
Chapter 2: Kinematics of Particles

Particle: A body whose dimensions can be neglected compared to its path then it can be treated like a point at its center of mass.

Kinematics should be studied first in order to relate the motion to forces.

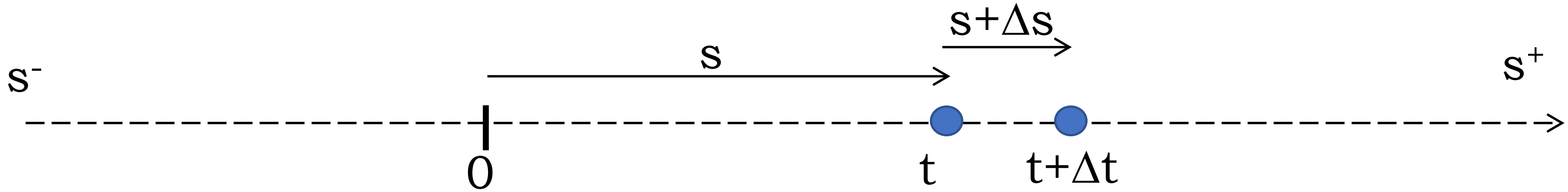
PART I: PARTICLES
Chapter 2: Kinematics of Particles

- **Constrained Motion:** If a particle is confined to a specified path (due to physical guides for example) then it is constrained motion.
- **Unconstrained Motion:** If the particle is free from any guides the motion is unconstrained.

Photo by Instructor 15.03.2012
"*Ankara Garı Doğu Makasları*"

2.2 Rectilinear Motion

Motion along a straight line.



$$v_{av} = \frac{\Delta s}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} = \dot{s}$$

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \dot{v}$$

$$v = \frac{ds}{dt} = \dot{s}$$

$$a = \frac{dv}{dt} = \dot{v}$$

2.2 Rectilinear Motion

Substituting definition of velocity into acceleration relation yields

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2} = \ddot{s}$$

Using definition of velocity and acceleration and solving for *dt* from both yields:

$$dt = \frac{ds}{v} = \frac{dv}{a}$$

$$\boxed{v dv = a ds}$$

This equation is *implicit* in time.

2.2 Rectilinear Motion

$$\boxed{\begin{aligned} &v = \frac{ds}{dt} = \dot{s} \\ &a = \frac{dv}{dt} = \dot{v} = \frac{d^2 s}{dt^2} = \ddot{s} \\ &v\,dv = a\,ds \end{aligned}}$$

- These **differential** equations are valid for any problem. Under certain conditions they may be integrated to obtain analytic expressions but as we will see later this is not true for all conditions.
- Please be careful about the directions therefore the signs for s, v and a!

2.2 Rectilinear Motion

a. Constant Acceleration (a=const)

$$dv = adt$$

$$\int_{v_0}^{v} dv = \int_{0}^{t} adt = a\int_{0}^{t} dt$$

$$\boxed{v(t) = v_0 + at}$$

$$ds = vdt$$

$$\int_{s_0}^{s} ds = \int_{0}^{t} vdt = \int_{v_0}^{v} (v_0 + at)dt$$

$$\boxed{s(t) = s_0 + vt + \frac{1}{2}at^2}$$

One may easily start integration from t_0 rather than $t = 0$.

2.2 Rectilinear Motion

b. Acceleration is a Function of Time a(t)

$$dv = adt$$

$$\int_{v_0}^{v} dv = \int_{0}^{t} a(t)dt$$

$$\boxed{v(t) = v_0 + \int_{0}^{t} a(t)dt}$$

$$ds = vdt \quad \int_{s_0}^{s} ds = \int_{0}^{t} v(t)dt = \int_{0}^{t} \left(v_0 + \int_{0}^{t} a(t)dt \right) dt$$

$$\boxed{s(t) = s_0 + \int_{0}^{t} \left(v_0 + \int_{0}^{t} a(t)dt \right) dt}$$

2.2 Rectilinear Motion

c. Acceleration is a Function of Velocity a(v)

$$dt = \frac{dv}{a(v)}$$

$$\int_0^t dt = \int_0^t \frac{dv}{a(v)}$$

$$\boxed{t = \int_{v_0}^{v} \frac{dv}{a(v)}}$$

This yields *v(t)*, using

$$ds = vdt, \int_{s_0}^{s} ds = \int_0^t v(t)dt$$

$$\boxed{s(t) = s_0 + \int_0^t v(t)dt}$$

2.2 Rectilinear Motion

c. Acceleration is a Function of Velocity a(v)

Alternative method:

$$v dv = a(v) ds$$

$$\frac{v dv}{a(v)} = ds$$

$$\boxed{\int_{v_0}^{v} \frac{v}{a(v)} dv = \int_{s_0}^{s} ds}$$

$$\boxed{s(v) = s_0 + \int_{v_0}^{v} \frac{v}{a(v)} dv}$$

2.2 Rectilinear Motion

d. Acceleration is a Function of Displacement a(s)

$$v\,dv = a(s)\,ds$$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a(s)\,ds$$

$$v^2 = {v_0}^2 + 2\int_{s_0}^{s} a(s)\,ds$$

$$\boxed{v(s) = \sqrt{{v_0}^2 + 2\int_{s_0}^{s} a(s)\,ds}}$$

$$dt = \frac{ds}{v(s)} = \frac{ds}{\sqrt{{v_0}^2 + 2\int_{s_0}^{s} a(s)\,ds}}$$

$$\boxed{\int_{0}^{t} dt = t = \int_{s_0}^{s} \frac{ds}{\sqrt{{v_0}^2 + 2\int_{s_0}^{s} a(s)\,ds}}}$$

2.2 Rectilinear Motion

2018-19 Midterm Exam 1 Questions

A particle, confined to move along a straight line has an acceleration a(s) = s^2 - 1 where acceleration, a, is in m/s^2 and displacement, s, is in m. Initially it was at s_0 = 1 m with a speed v_0 = -2 m/s. Determine the speed of the particle, v(s), as a function of displacement of the particle s.

Check whether your answer satisfies $v(s=1) = v_0$?

$$vdv = a(s)ds$$

$$vdv = (s^2 - 1)ds$$

$$\int_{v_0=-2}^{v} vdv = \int_{s_0=1}^{s} (s^2 - 1)ds$$

$$\left.\frac{v^2}{2}\right|_{v_0}^{v} = \left.\left(\frac{s^3}{3} - s\right)\right|_{s_0}^{s}$$

$$v^2 = {v_0}^2 + 2\left(\frac{s^3}{3} - s - \frac{{s_0}^3}{3} + s_0\right)$$

2.2 Rectilinear Motion

A particle, confined to move along a straight line has an acceleration $a(s) = s^2 - 1$ where acceleration, a, is in m/s^2 and displacement, s, is in m. Initially it was at $s_0 = 1$ m with a speed $v_0 = -2$ m/s. Determine the speed of the particle, v(s), as a function of displacement of the particle s.

Check whether your answer satisfies $v(s = 1) = v_0$?

$$v = \pm\sqrt{{v_0}^2 + 2\left(\frac{s^3}{3} - s - \frac{{s_0}^3}{3} + s_0\right)}$$

$$v(s = 1) = \pm\sqrt{(-2)^2 + 2\left(\frac{1^3}{3} - 1 - \frac{1^3}{3} + 1\right)} = \pm 2$$

so

$$v(s) = -\sqrt{{v_0}^2 + 2\left(\frac{s^3}{3} - s - \frac{{s_0}^3}{3} + s_0\right)}$$

2.2 Rectilinear Motion

A particle, confined to move along a straight line has an acceleration $a(v) = 1/v^2$ where acceleration, a, is in m/s^2 and speed, v, is in m/s. At time, t = 0 it was moving with an initial speed, $v_0 = 1$ m/s. Determine the speed of the particle, v(t), as a function of time t.

Check whether your answer satisfies $v(t=0) = v_0$?

$$a(v) = \frac{dv}{dt}$$

$$dt = \frac{dv}{a(v)} = \frac{dv}{1/v^2}$$

$$dt = v^2 dv$$

$$\int_{t_0=0}^{t} dt = \int_{v_0=1}^{v} v^2 dv$$

$$t\Big|_{t_0=0}^{t} = \frac{v^3}{3}\Big|_{v_0=1}^{v} = \frac{v^3}{3} - \frac{1}{3}$$

$$v(t) = \sqrt[3]{3t+1}, \qquad v(t=0) = v_0 = \sqrt[3]{1} = 1$$

2.2 Rectilinear Motion

A particle, confined to move along a straight line has an acceleration $a(v) = 1/v^2$ where acceleration, a, is in m/s^2 and speed, v, is in m/s. At time, t = 0 it was at $s_o = 0$ m and moving with an initial speed, $v_o = 1$ m/s. Determine the speed of the particle, v(s), as a function of position s.

Check whether your answer satisfies $v(s=0) = v_0$?

$$vdv = a(v)ds$$

$$\frac{v}{1/v^2}dv = v^3 dv = ds$$

$$\int_{v_0=1}^{v} v^3 dv = \int_{s_0=1}^{s} ds$$

$$\left.\frac{v^4}{4}\right|_{v_0}^{v} = \left.s\right|_{s_0}^{s}$$

$$v^4 = {v_0}^4 + 4s$$

2.2 Rectilinear Motion

A particle, confined to move along a straight line has an acceleration a(v) = $1/v^2$ where acceleration, a, is in m/s^2 and speed, v, is in m/s. At time, t = 0 it was at s_0 = 0 m and moving with an initial speed, v_0 = 1 m/s. Determine the speed of the particle, v(s), as a function of position s.

Check whether your answer satisfies $v(s=0) = v_0$?

$$v^4 = {v_0}^4 + 4s$$

$$v = \pm\sqrt[4]{{v_0}^4 + 4s}$$

$$v(s=0) = \pm\sqrt[4]{1^4} = \pm 1$$

so

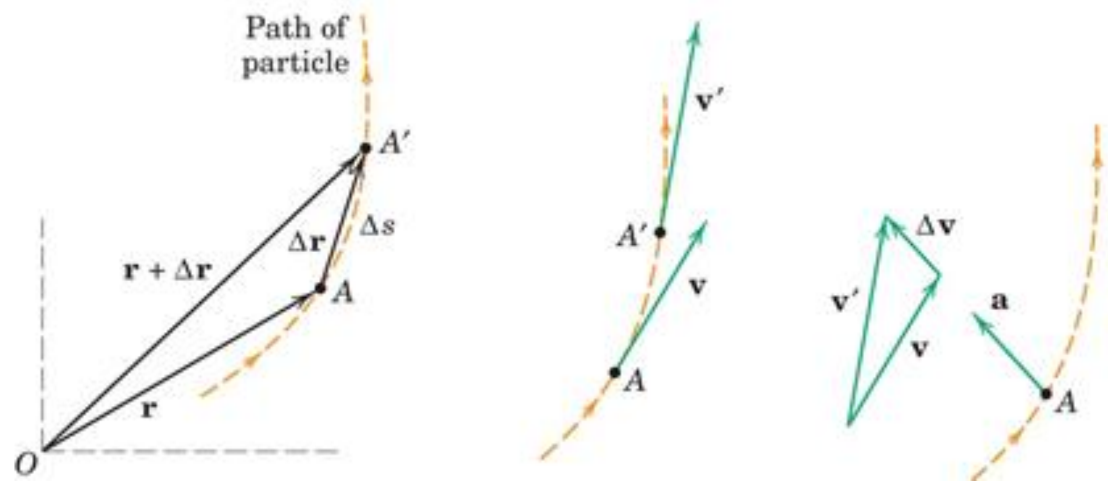
$$v(s) = \sqrt[4]{{v_0}^4 + 4s}$$

2.3 Plane Curvilinear Motion

Motion along a curved path in a single plane.

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$



Velocity is <u>always</u> tangent to the path of the particle!

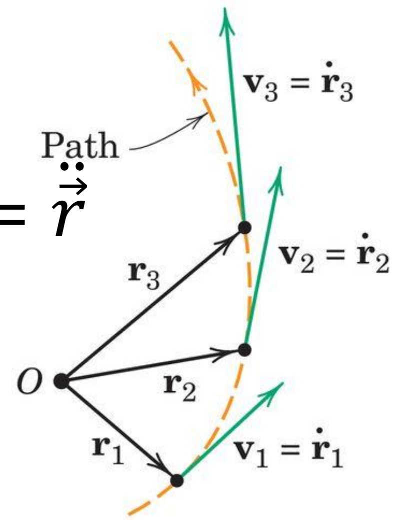
$$\left|\frac{d\vec{r}}{dt}\right| \neq \frac{d|\vec{r}|}{dt}$$

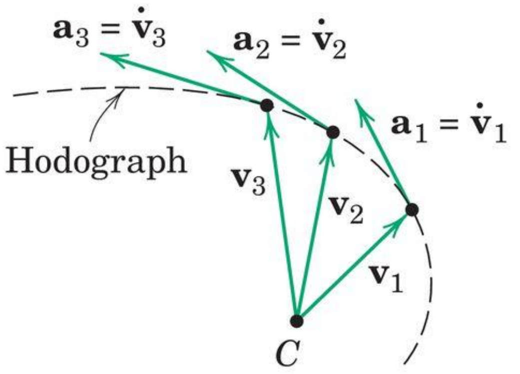
2.3 Plane Curvilinear Motion

Motion along a curved path in a single plane.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \ddot{\vec{r}}$$





As we will see later in detail acceleration vector has two sources:

- due to magnitude change of velocity vector,
- due to direction change of velocity vector.

2.3 Plane Curvilinear Motion

Three different coordinate systems to analyze plane curvilinear motion are:

- Rectangular (Cartesian) coordinates
- Normal-tangential (path) coordinates
- Polar coordinates