ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

Dr. Ergin TÖNÜK

Department of Mechanical Engineering

Graduate Program of Biomedical Engineering

tonuk@metu.edu.tr

http://tonuk.me.metu.edu.tr

3.4 Rectilinear Motion

Particle idealization may be utilized if motion of center of mass of a real body is under consideration.

If possible, select x-axis along the motion direction and use rectangular (Cartesian) coordinates to resolve Newton's Second Law as:

$$\sum F_x = ma_x$$

$$\sum F_y = 0$$

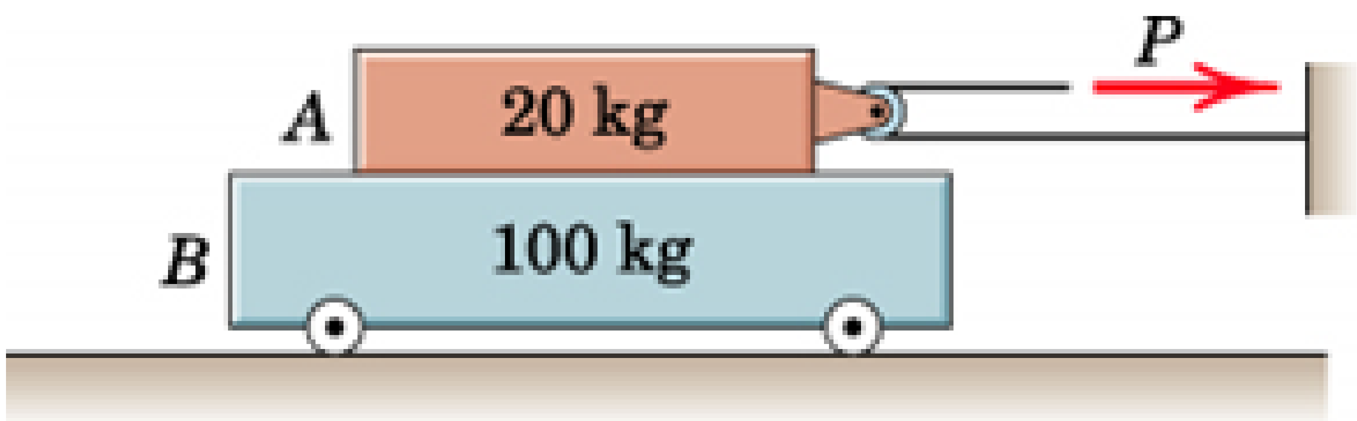
$$\sum F_z = 0$$

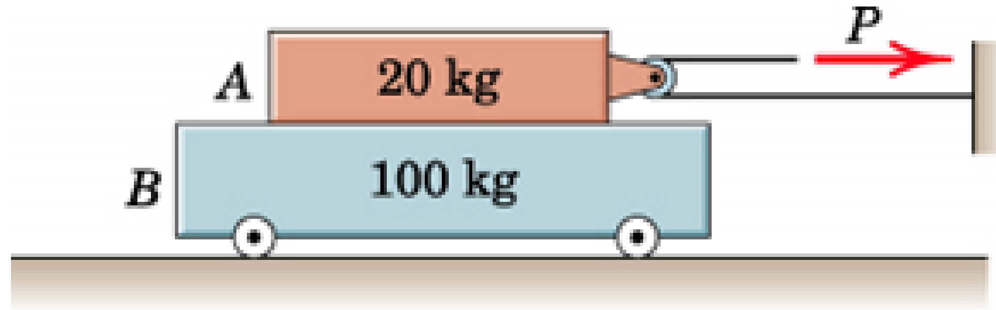
3/24 (4th), 3/23 (5th), 3/23 (6th), 3/24 (7th), None (8th)

If the coefficients of static and kinetic friction between the 20 kg block A and 100 kg cart B are both essentially the same value of 0.50, determine the acceleration of each part for:

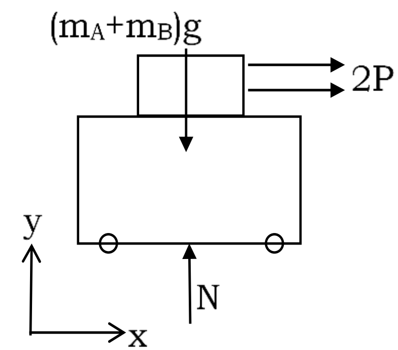
a. P = 60 N

b. P = 40 N.





a. Assume there is no sliding between the blocks and they move together:

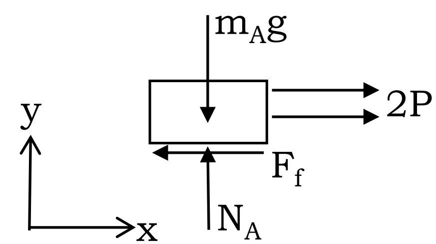


$$\sum F_y = 0\,,N - (m_a + m_b)g = 0$$

$$\sum F_x = (m_a + m_b)a_x$$

$$a_x = \frac{2P}{(m_a + m_b)} = 1\ m/s^2$$

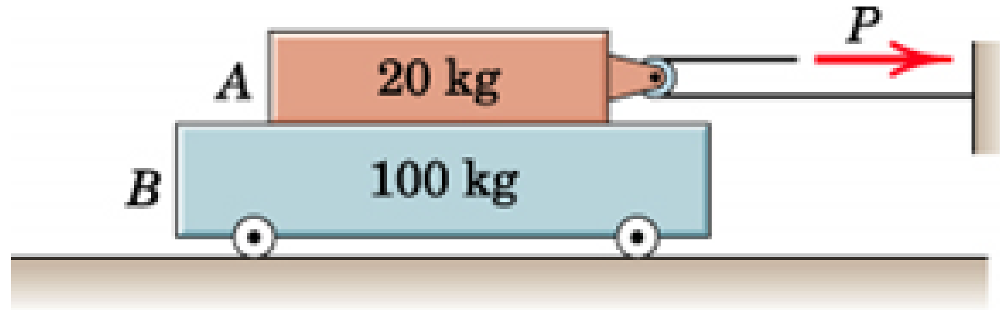
Check the assumption: Do we have enough friction force so that they can move together?



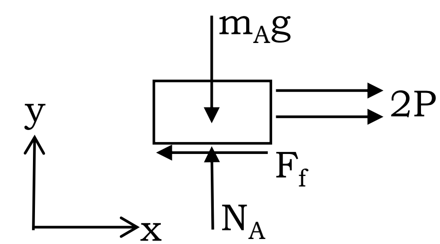
$$\sum F_y = 0,\ N_A - m_A g = 0, N_A = 196.2\ N$$

$$\sum F_x = m_A a,\ 2P - F_f = 20 * 1$$

$$F_f = 100\ N, F_{f_{max}} = \mu N_A = 0.5 * 196.2 = 98.1\ N < F_f = 100\ N!$$



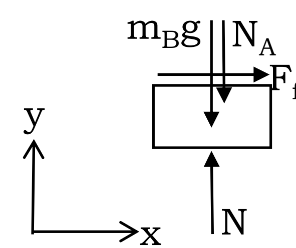
The assumption fails. Friction is not sufficient so that the two blocks can move together. Therefore there is sliding. For sliding the friction force is at its maximum value:



$F_{f_{max}} = \mu N_A = 98.1\ N,$
then for block A:

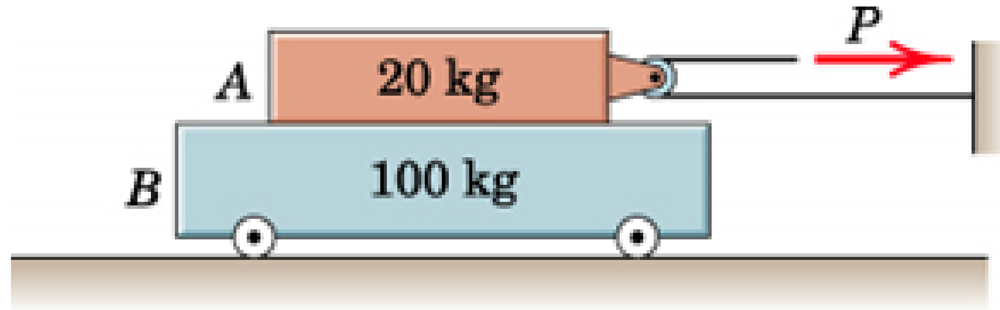
$$\sum F_x = m_A a_A$$

$2 * 60 - 98.1 = 20 * a_A$
$a_A = 1.095\ m/s^2$
For block B:

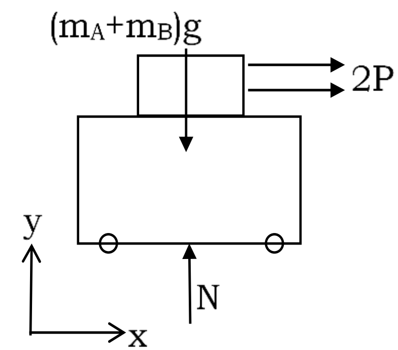


$$\sum F_x = m_B a_B$$

$F_F = 100 * a_B$
$a_B = 0.981\ m/s^2$



b. Assume they move together:



$$\sum F_x = (m_A + m_B)a\,, a = 0.667\ m/s^2$$

Check for body A

$$\sum F_x = m_A a,\ 2 * 40 - F_f = 20 * 0.667, F_f = 66.7N < \mu N_A = 98.1\ N$$

3.5 Curvilinear Motion

$$\sum \vec{F} = m\vec{a}$$

can be decomposed in one of the most appropriate coordinate system suitable for the problem.

- *Rectangular (Cartesian) Coordinates:*

$$\sum F_x = ma_x, \sum F_y = ma_y$$

where

$a_x = \ddot{x},\ a_y = \ddot{y}$

3.5 Curvilinear Motion

$$\sum \vec{F} = m\vec{a}$$

can be decomposed in one of the most appropriate coordinate system suitable for the problem.

- *Normal and Tangential Coordinates:*

$$\sum F_n = ma_n, \sum F_t = ma_t$$

where

$$a_n = \frac{v^2}{\rho}, a_t = \dot{v}$$

3.5 Curvilinear Motion

$$\sum \vec{F} = m\vec{a}$$

can be decomposed in one of the most appropriate coordinate system suitable for the problem.

- *Polar Coordinates:*

$$\sum F_r = ma_r, \sum F_\theta = ma_\theta$$

where

$$a_r = \ddot{r} - r\dot{\theta}^2, a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

3.5 Curvilinear Motion

The steps in solution of plane curvilinear motion problems are:

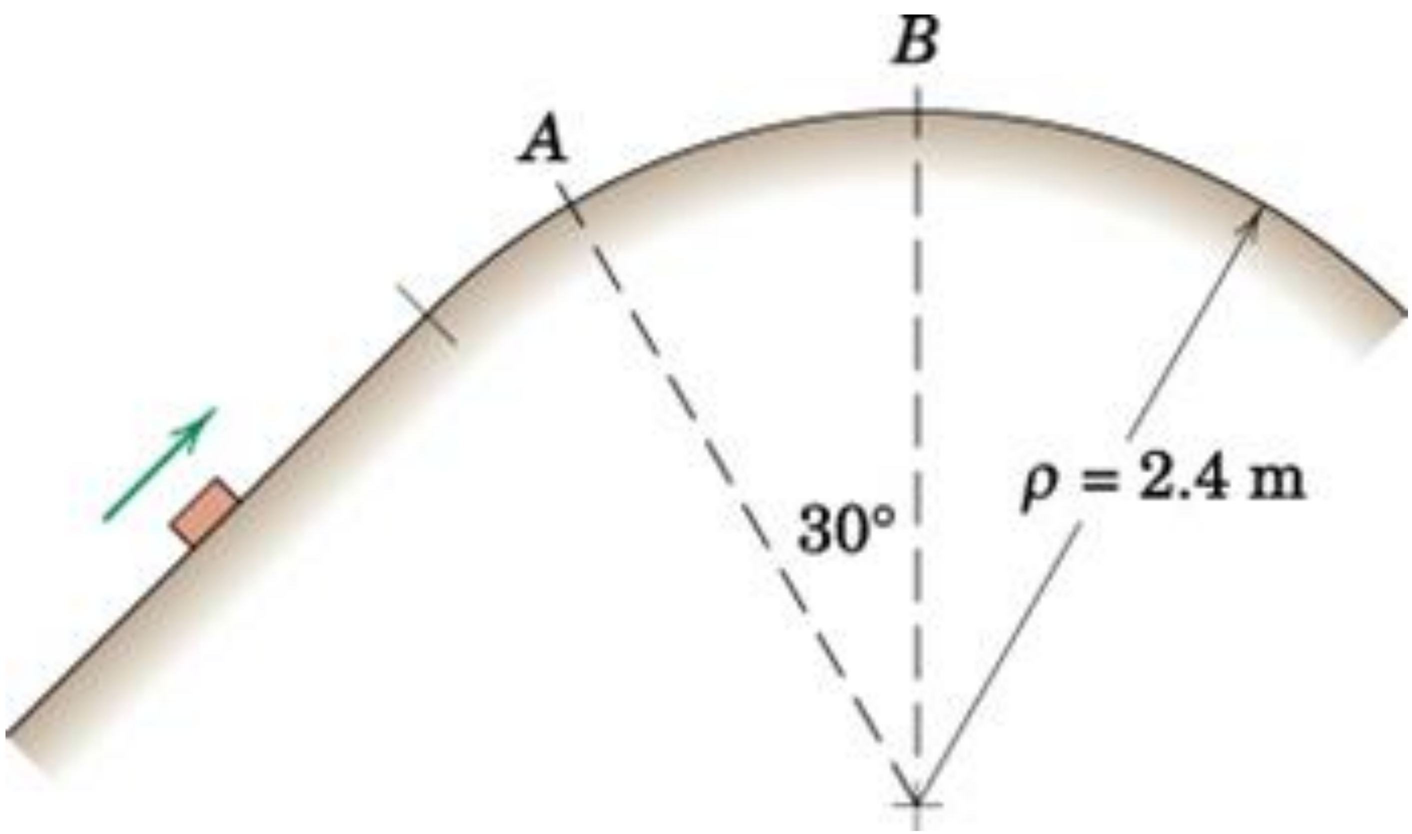
- Identify the most convenient coordinate system,
- Draw proper free body diagram(s),
- Apply Newton’s second law to each free body diagram.

Positive directions are the positive coordinate directions. Any force or acceleration in the positive coordinate direction is positive, any force or acceleration opposite to the positive coordinate direction is negative.

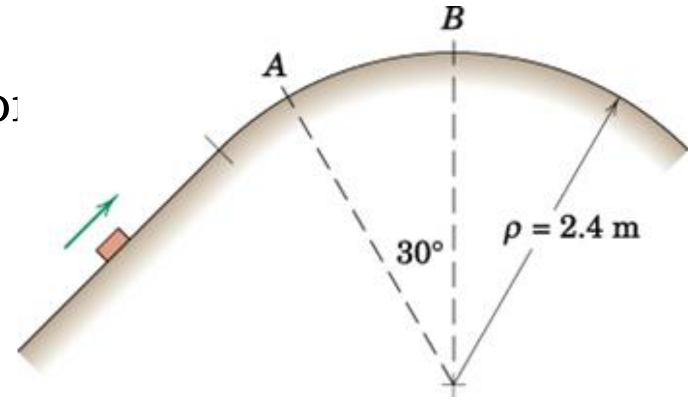
If an unknown force or acceleration is found as a positive number then it is in the assumed direction as in the free body diagram, if a negative number, opposite to the assumed direction.

3/52 (4th), 3/50 (5th), 3/51 (6th), None (7th), 3/48 (8th)

If the 2 kg block passes over the top B of the circular portion of the path with a speed of 3.5 m/s, calculate the magnitude, NB of the normal force exerted by the path on the block. Determine the maximum speed v which the block can have at A without losing contact with the path.

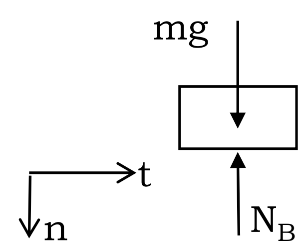


The most suitable coordinate system for this problem is n-t.



For N_B

$$\sum F_n = ma_n$$

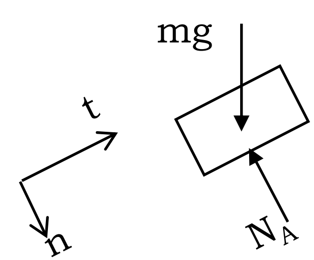


$$mg - N_B = m\frac{v^2}{\rho}$$

$$N_B = mg - m\frac{v^2}{\rho} = 2 * 9.81 - 2 * \frac{3.5^2}{2.4} = 9.41\ N$$

For $v_{A_{max}}$

$$\sum F_n = ma_n$$



$$mg\cos 30° - N_A = m\frac{{v_{max}}^2}{\rho}$$

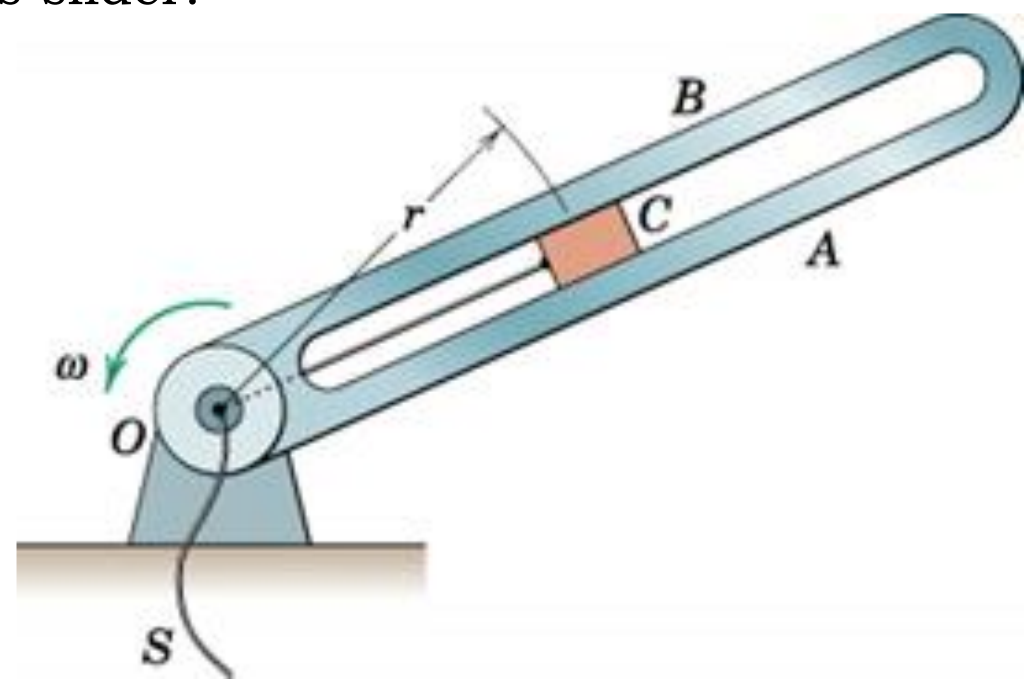
$$2 * 9.81cos30° - 0 = 2\frac{{v_{max}}^2}{2.4}$$

$$v_{max} = 4.52\ m/s$$

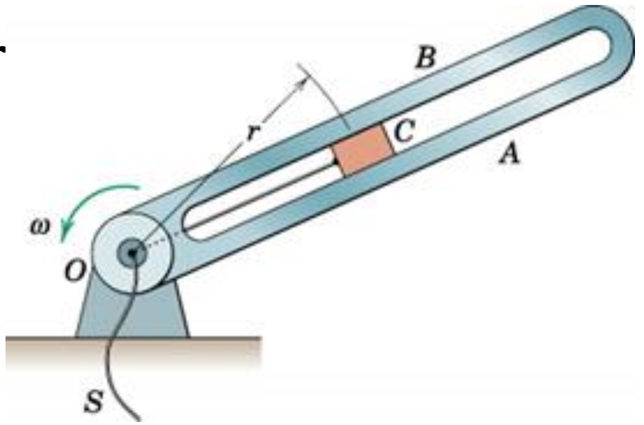
Also one may determine $\dot{v}$ by $\sum F_t = ma_t = m\dot{v}$ rate of decrease of speed at this position.

3/75 (4th), 3/81 (5th), 3/86 (6th), 3/80 (7th), 3/83 (8th)

The slotted arm revolves in the horizontal plane about the fixed vertical axis through point O. The 2 kg slider C is drawn toward O at a constant rate of 50 mm/s by pulling the cord at S. At the instant for which r = 225 mm the arm has a counterclockwise angular velocity of ω = 6 rad/s and is slowing down at a rate of 2 rad/s^2. For this instant, determine the tension, T in the cord and the magnitude N of the force exerted on the slider by the sides of smooth radial slot. Indicate which side, A or B, of the slot contacts slider.



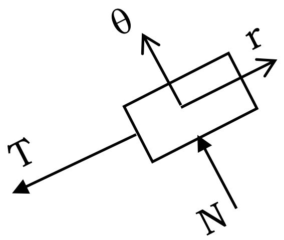
The suitable coordinate system is polar coordinates.



$r = 225\, m, \dot{r} = -0.05\, m/s = const, \ddot{r} = 0$

$\dot{\theta} = 6 rad/s, \ddot{\theta} = -2\, rad/s^2$

$$\sum F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$



$-T = 2(0 - 0.225 * 6^2), T = 16.20\ N$

$$\sum F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

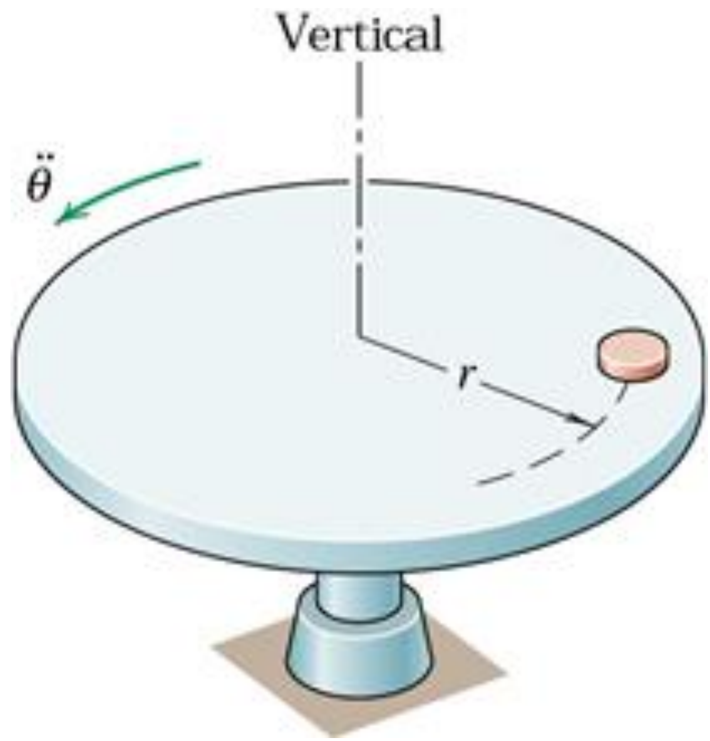
$N = 2[0.225 * (-2) + 2 * (-0.05) * 6]$

$N = -2.10\ N$

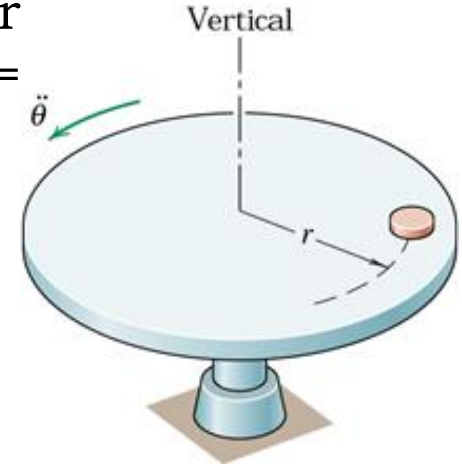
Contact is on side B.

3/86 (4th), 3/88 (5th), None (6th), 3/81 (7th), 3/85 (8th)

A small coin is placed on the horizontal surface of the rotating disk. If the disk starts from rest and is given a constant angular acceleration, α, determine an expression for the number of revolutions, N, through which the disk turns before the coin slips. The coefficient of static friction between the coin and the disk is μ_s.



One may use n-t or r-θ coordinates. For circular motion since $\dot{r} = \ddot{r} = 0$ they yield equivalent results (t = θ, r = -n). Let us use n-t:



$$\sum F_t = ma_t = mr\alpha$$

$$\sum F_n = ma_n = mr\omega^2 = m\frac{v^2}{r}$$

$$F_{f_{max}} = \mu_s N = \sqrt{{F_n}^2 + {F_t}^2}$$

$$\mu_s mg = mr\sqrt{\alpha^2 + \omega^4}$$

$$(\mu_s mg)^2 = m^2 r^2(\alpha^2 + \omega^4)$$

$$\omega^2 = \frac{\sqrt{{\mu_s}^2 g^2 - r^2\alpha^2}}{r}$$

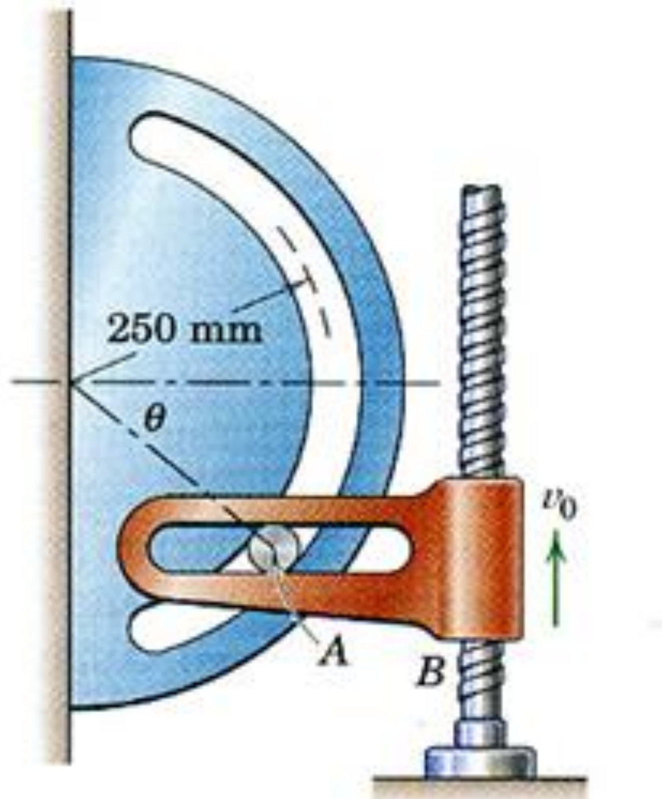
For constant α, $\omega d\omega = \alpha d\theta$ can be integrated to get $\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$ and $\theta = 2\pi N$ so

$$\frac{\sqrt{{\mu_s}^2 g^2 - r^2\alpha^2}}{r} = 4\pi\alpha N$$

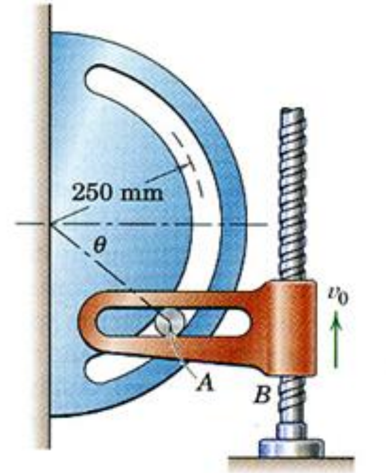
$$N = \frac{1}{4\pi}\sqrt{\left(\frac{\mu_s g}{r\alpha}\right)^2 - 1}$$

3/94 (4th), None (5th), None (6th), None (7th), None (8th)

At the instant when $\theta = 30°$ the horizontal guide is given a constant upward velocity $v_0 = 2$ m/s. For this instant calculate the force N exerted by the circular slot and the force P exerted by the horizontal slot on the 0.5 kg pin A. The width of the slots is slightly larger than the diameter of the pin, and friction is negligible.



Again n-t or r-θ will yield similar results. Let us use r-θ this time

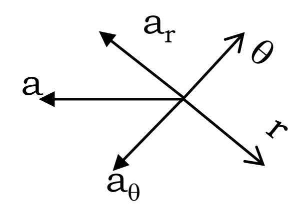


Remember velocity is *always* tangent to the path therefore velocity of the pin is tangent to the slot!

$$v = \frac{v_0}{sin30°} = 2.31\, m/s$$

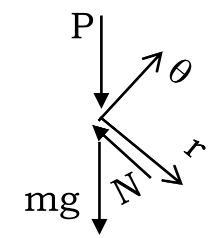
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.25 * \left(\frac{2.31}{0.25}\right)^2 = -21.3\, m/s^2$$

$v_0 = const$ so $\dot{v}_0 = 0$ (i.e. vertical acceleration of the pin then the pin only has a horizontal acceleration)



$$a_\theta = a_r tan30° = 12.32\, m/s^2$$

$$\sum F_\theta = ma_\theta\, , -0.5 * 9.81 * cos30° - Pcos30° = 0.5 * -12.32$$



$$P = 2.21\, N$$

$$\sum F_x = ma_x\, , -Ncos30° = 0.5\left(\frac{-21.3}{cos30°}\right)$$

$$N = 14.22\, N$$