ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

Dr. Ergin TÖNÜK
Department of Mechanical Engineering
Graduate Program of Biomedical Engineering
tonuk@metu.edu.tr
http://tonuk.me.metu.edu.tr
https://www.youtube.com/watch?v=nDDfti-RPMA

PART I: PARTICLES

Chapter 3: Kinetics of Particles

There are three different approaches to the kinetics problems:

A. Direct application of Newton's Second Law/Force-Mass Acceleration Method

B. Work – Energy Principles (integration of second law with respect to displacement)

C. Impulse and Momentum Methods (integration of second law with respect to time)

PART I: PARTICLES
Chapter 3: Kinetics of Particles
C. Impulse and Momentum Methods
3/10 Angular Impulse and Angular Momentum

$\vec{G} = m\vec{v}$ is linear momentum. Angular momentum is moment of linear momentum about an arbitrary or convenient point O and is defined as:

$\vec{H}_O = \vec{r} \times m\vec{v}, [kg \cdot m^2/s]\ or\ [N \cdot m \cdot s]$

Recall $\sum \vec{F} = \dot{\vec{G}}$ and is it true for $\sum \vec{M}_O = \dot{\vec{H}}_O$

$$\dot{\vec{H}}_O = \frac{d}{dt}(\vec{r} \times m\vec{v}) = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}} = \vec{0} + \vec{r} \times m\vec{a} = \vec{r} \times \sum \vec{F} = \sum \vec{M}_O$$

this relation can be integrated for a finite interval of time to obtain angular impulse and momentum relation:

$$\int_{t_1}^{t_2} \sum \vec{M}_O\, dt = \vec{H}_O(t_2) - \vec{H}_O(t_1) = \Delta\vec{H}_O$$

PART I: PARTICLES
Chapter 3: Kinetics of Particles
C. Impulse and Momentum Methods
3/10 Angular Impulse and Angular Momentum

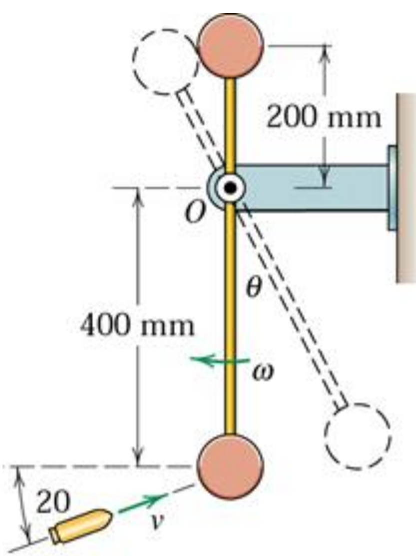
$$\int_{t_1}^{t_2} \sum \vec{M}_O \, dt = \vec{H}_O(t_2) - \vec{H}_O(t_1) = \Delta\vec{H}_O$$

In planar problems angular momentum vector is always normal to the plane of motion and its direction remains fixed, only magnitude changes. In 3-D problems the direction changes as well.

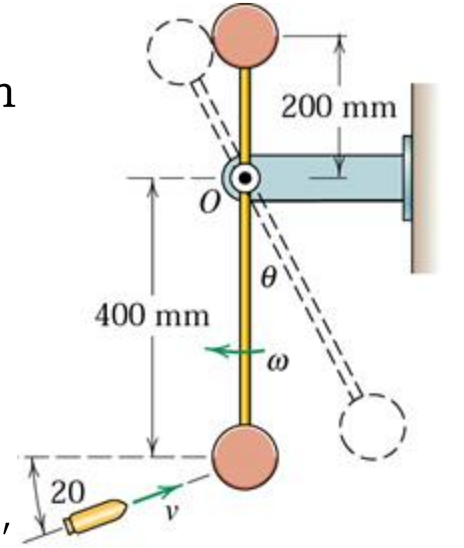
For a certain interval of time if $\sum \vec{M}_O = \vec{0}$ then angular momentum about point O is conserved.

3/238 (4th), 3/240 (5th), None (6th), 3/235 (7th), 3/235 (8th)

The pendulum consists of two 3.2 kg concentrated masses positioned as shown on a light but rigid bar. The pendulum is swinging through the vertical position with a clockwise angular velocity ω = 6 rad/s when a 50 g bullet traveling with a velocity of 300 m/s in the direction shown strikes the lower mass and becomes embedded in it. Calculate the angular velocity ω' with which the pendulum has immediately after impact and find the maximum angular deflection θ of the pendulum.



During impact of the bullet with the pendulum because of reaction forces at O linear momentum of the bullet-pendulum system is *not* conserved. However since moment of these unknown reaction forces about point O is zero, the angular momentum is conserved during impact.



$$\ell m_B v_B cos20° - \ell m v - \frac{\ell}{2} m \frac{v}{2} = \ell(m + m_B)v' + \frac{\ell}{2} m \frac{v'}{2}$$

$$\omega' = \frac{v'}{0.4}$$

$$0.4 * 0.05 * 300cos20° - 0.4 * 3.2 * 6 - 0.2 * 3.2 * 6 = 0.4 * 3.25 * 0.4\omega' + 0.2 * 3.2 * 0.2\omega'$$

$$\omega' = 2.77\ rad/s$$

After the impact energy of the bullet-pendulum system is conserved.

$$U_{A\to B} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{A\to B} = 0$$

$$\Delta T = 0 - \frac{1}{2}(m + m_B)(\ell\omega')^2 - \frac{1}{2} m \left(\frac{\ell}{2}\omega'\right)^2$$

$$\Delta T = 0 - \frac{1}{2}(3.2 + 0.05)(0.4 * 2.77)^2 - \frac{1}{2}3.2(0.2 * 2.77)^2 = -2.49\ J$$

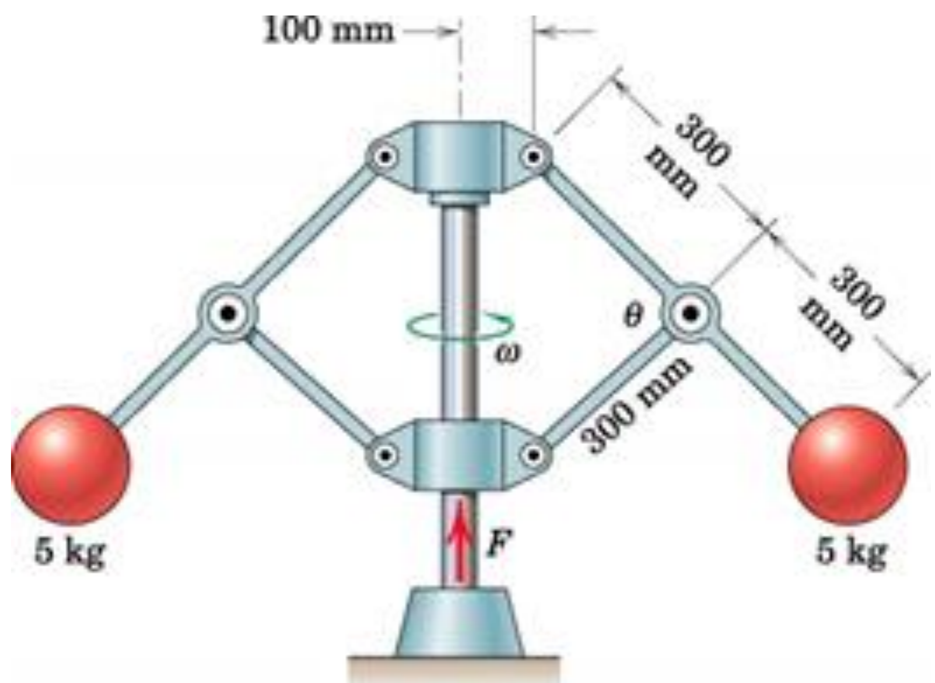
$$\Delta V_g = (m + m_B)g[0 - \ell(1 - cos\theta)] + mg\left[0 + \frac{\ell}{2}(1 - cos\theta)\right]$$

$$\Delta V_g = (3.2 + 0.05)9.81[0 - 0.4(1 - cos\theta)] + 3.2 * 9.81[0 + 0.2(1 - cos\theta)]$$

$$\theta = 52.1°$$

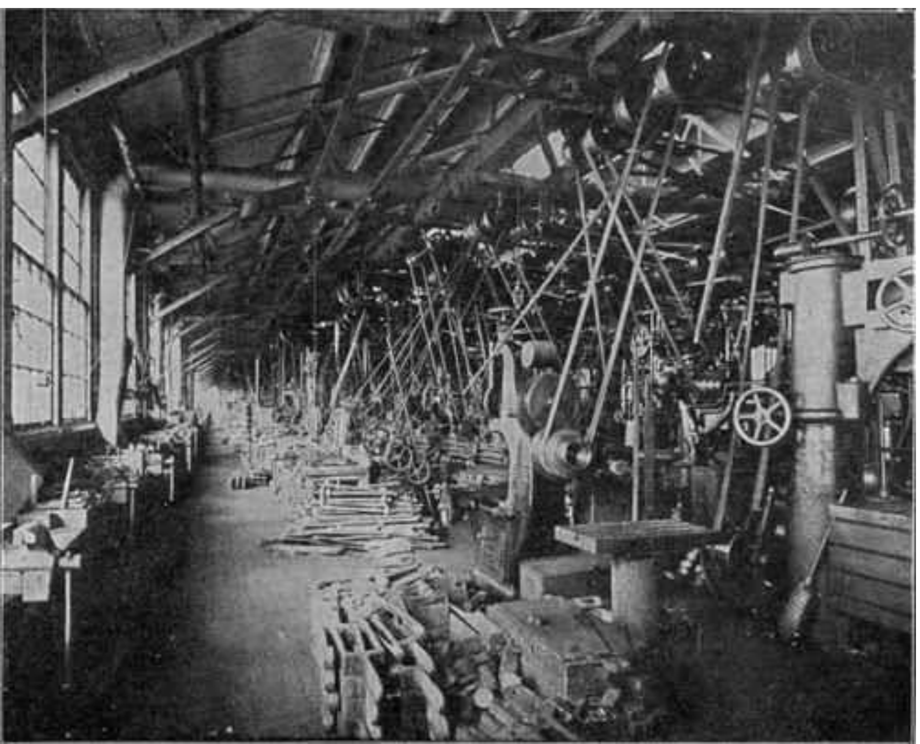
3/241 (4th), 3/243 (5th), 3/250 (6th), 3/238 (7th), 3/240 (8th)

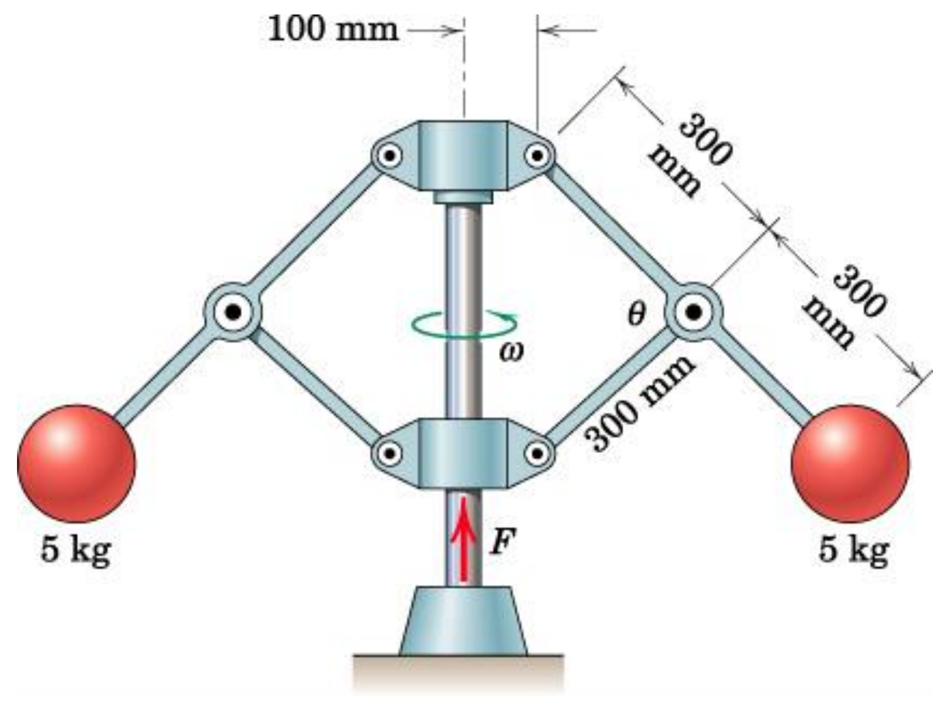
The assembly of two 5 kg spheres is rotating freely about the vertical axis at 40 rev/min with θ = 90°. If the force F which maintains the given position is increased to raise the base collar and reduce θ to 60°, determine the new angular velocity ω. Also determine the work done by F in changing the configuration of the system. Assume that the mass of the arms and collars is negligible.



This is called a centrifugal/flyball governor. It is used to control the speed of the steam engines during industrial revolution.

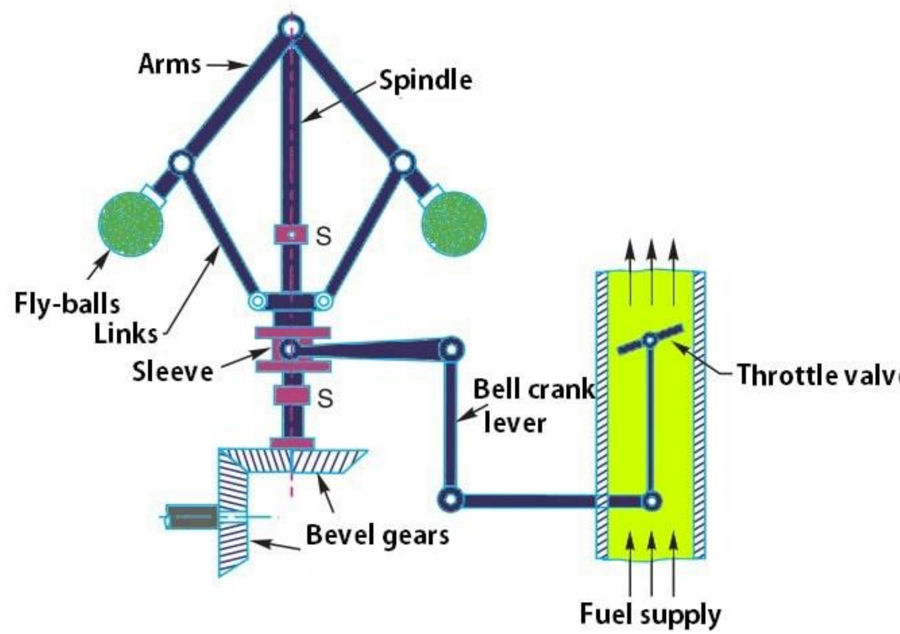
Unlike today, the factories at that time had a single steam engine to power many machines through overhead shafts. Therefore the load on the steam engine is very variable but the speed should be kept more or less constant. This was achieved by the governor.



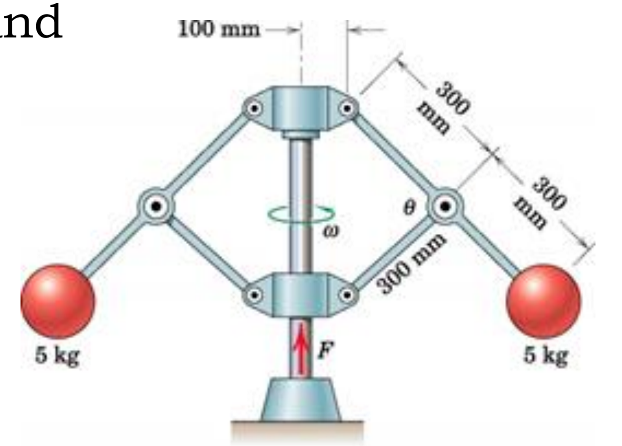


The vertical governor shaft is connected directly to the crank of the steam engine and rotates with it.

The collar on which the force acts is connected to steam throttle and when it rises, it throttles steam fed into the engine therefore reduce the speed when load on the steam engine is low. When load is heavy, the steam engine slows down lowering the balls and lowering the collar thus increasing the steam inflow to the engine, trying to keep its speed.



Moment of all forces, including weights of the balls and F about the rotation axis, O-O vanishes.



$$0 = 2mr_2{}^2\omega_2 - 2mr_1{}^2\omega_1$$

From figure

$$r_1 = 0.1 + 2 * 0.3cos45° = 0.524\, m$$

$$r_2 = 0.1 + 2 * 0.3cos30° = 0.620\, m$$

$$\omega_2 = \frac{r_1{}^2}{r_2{}^2}\omega_1 = \frac{0.524^{\,2}}{0.620^2}40 * \frac{\pi}{30} = 3.00\, rad/s \equiv 28.6rpm$$

$$U_{1\to2} = \Delta T + \Delta V_g + \Delta V_e$$

$$\Delta T = \frac{1}{2}2m(r_2\omega_2)^2 - \frac{1}{2}2m(r_1\omega_1)^2$$

$$\Delta T = 5[(0.630 * 3.00)^2 - (0.524 * 4.19)^2] = -6.85\, J$$

$$\Delta V_g = 2mg\Delta h$$

$$\Delta h = 2 * 0.3(sin45° - sin30°) = 0.1243\, m$$

$$\Delta V_g = 2 * 5 * 9.81 * 0.1243 = 12.19J$$

$$\Delta V_e = 0$$

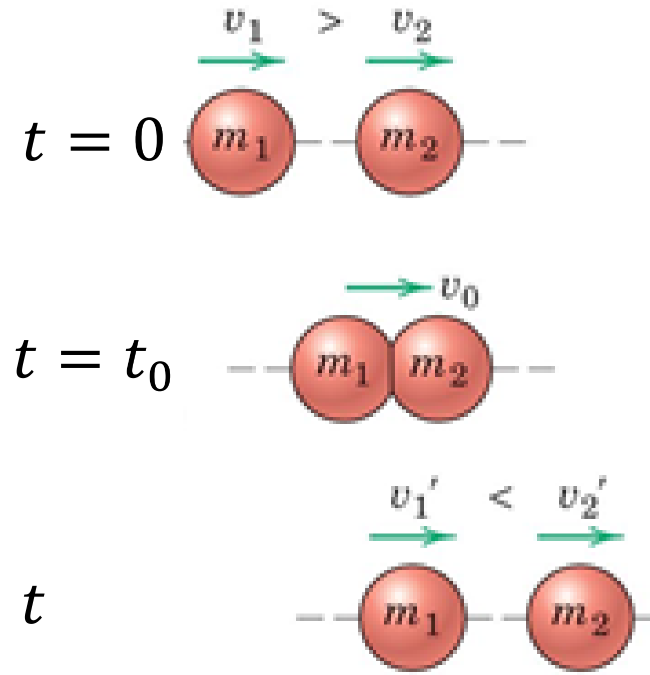
$$U_{1\to2} = -6.85 + 12.19 = 5.34\, J$$

D. Special Applications
3/12 Impact

Impact is collusion between two bodies and is characterized by *relatively large contact forces* in a *very short time interval*.

Impact is a complex phenomena involving *large deformations and recovery*, also generation of heat and sound. Therefore the predictions of mathematical models should not be relied on heavily.

D. Special Applications
3/12 Impact



Direct Central Impact

During direct central impact the impulse of forces other than the impact forces may be neglected therefore assuming conservation of linear momentum of the system during impact is an accurate approximation.

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

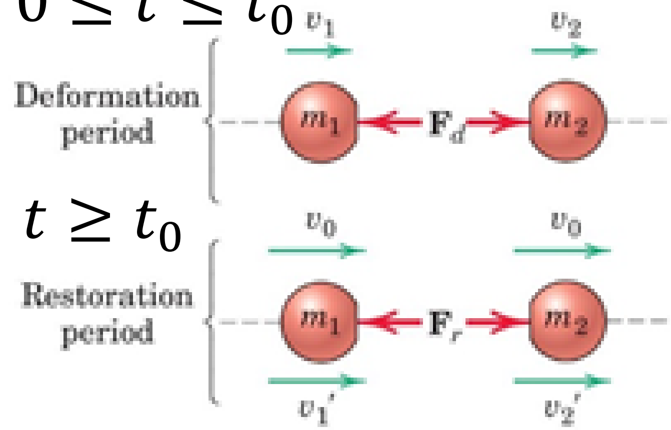
One equation however two final velocities which cannot be determined individually.

D. Special Applications
3/12 Impact

The capacity of contacting bodies to recover from the impact is defined by *coefficient of restitution*, *e*, which is the ratio of impulse of restoring forces to the impulse of deforming forces. Of course these impulses are equal to change of momentum of individual particles.

$0 \leq t \leq t_0$

$t \geq t_0$



D. Special Applications
3/12 Impact

For particle 1

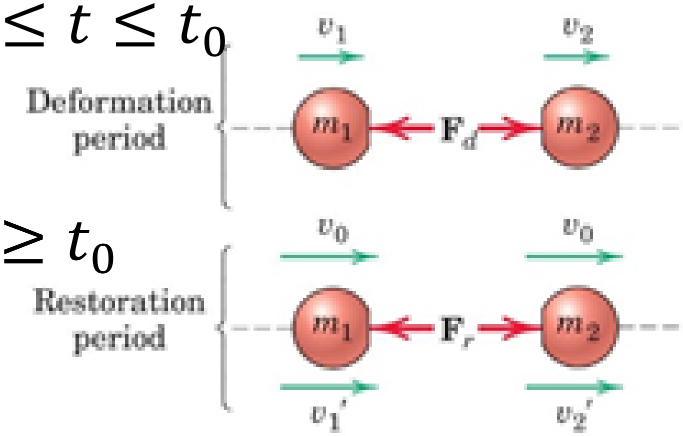
$$e = \frac{\int_{t_0}^{t} F_r\,dt}{\int_0^{t_0} F_d\,dt} = \frac{m_1[-v'_1 - (-v_0)]}{m_1[-v_0 - (-v_1)]} = \frac{v_0 - v'_1}{v_1 - v_0}$$

For particle 2

$$e = \frac{\int_{t_0}^{t} F_r\,dt}{\int_0^{t_0} F_d\,dt} = \frac{m_2(v'_2 - v_0)}{m_1(v_0 - v_2)} = \frac{v'_2 - v_0}{v_0 - v_2}$$

$0 \leq t \leq t_0$

$t \geq t_0$



where the deformation period is from t = 0 to t_0 and recovery period is from t_0 to t. Elimination of instantaneous common velocity, v_0, from both equations yield:

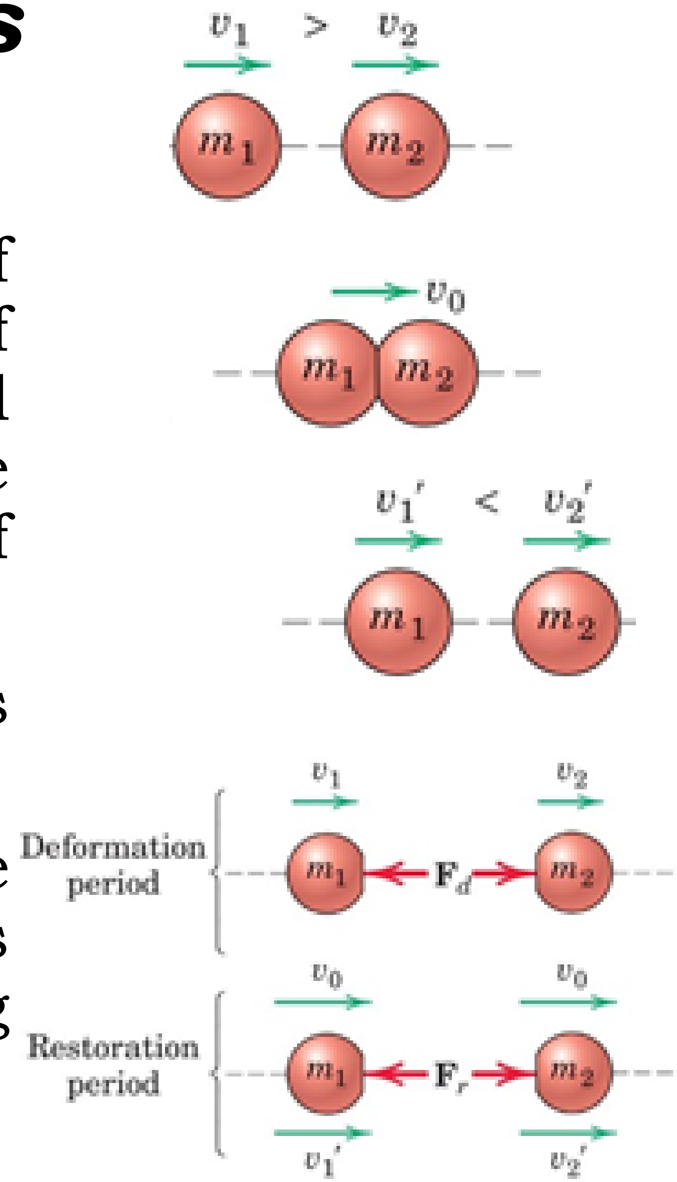
$$e = \frac{v'_2 - v'_1}{v_1 - v_2} = \frac{|Relative\ velocity\ of\ seperation|}{|Relative\ velocity\ of\ approach|}$$

D. Special Applications
3/12 Impact

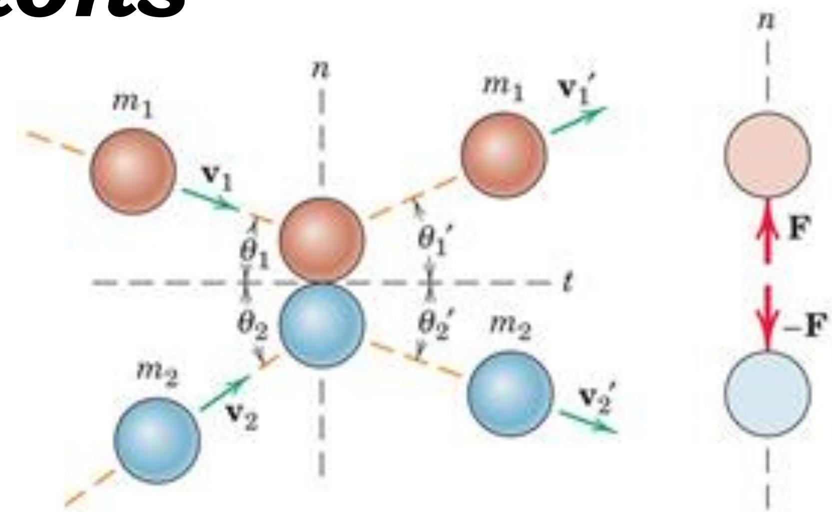
By utilizing conservation of momentum and coefficient of restitution, e, equations the final velocities of impacting bodies are determined. However coefficient of restitution, *e*, value is not reliable!

$e = 1$ is elastic impact where there is no mechanical energy loss.

$e = 0$ is inelastic impact where there is maximum mechanical energy loss and the two particles continue moving together without separating.



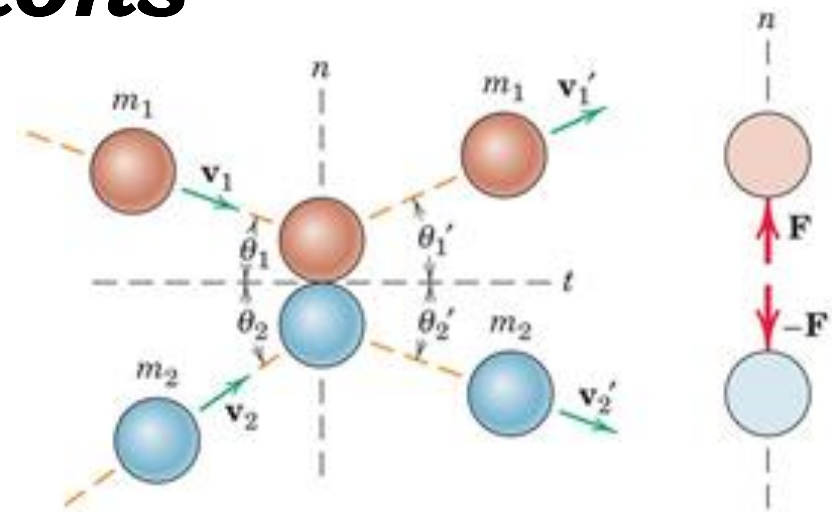
D. Special Applications
3/12 Impact



Oblique Central Impact

If initial and final velocities of the two particles are not on the same line then oblique central impact occurs. The normal direction is the normal of the deformed surfaces and the impulsive force is in this direction. The tangent direction is perpendicular to the normal direction.

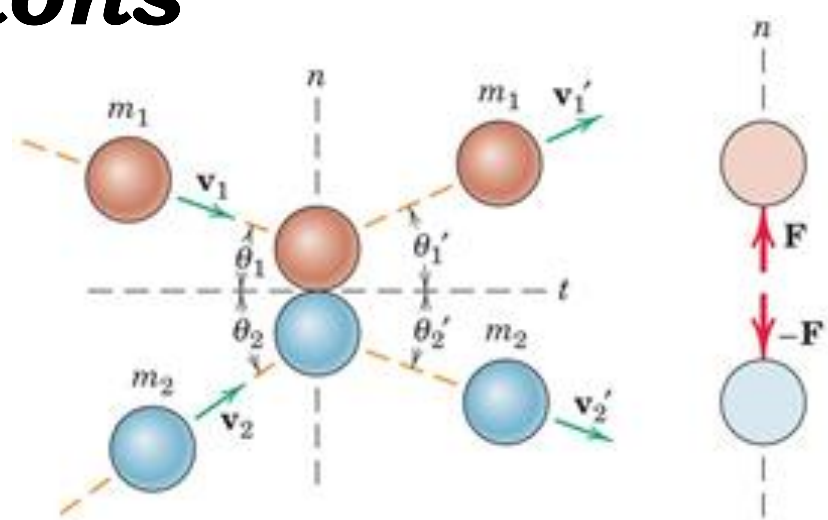
D. Special Applications
3/12 Impact



Oblique Central Impact

Oblique central impact is a direct central impact in n-direction and since there is no force in t-direction during impact the individual momentum of each of the particles is conserved in t-direction.

D. Special Applications
3/12 Impact



Oblique Central Impact

$$m_1 v_{1_n} + m_2 v_{2_n} = m_1 v'_{1_n} + m_2 v'_{2_n}$$

$$e = \frac{v'_{2_n} - v'_{1_n}}{v_{1_n} - v_{2_n}} = \frac{|Relative\ velocity\ of\ seperation\ in\ n - dir|}{|Relative\ velocity\ of\ approach\ in\ n - dir|}$$

$$m_1 v_{1_t} = m_1 v'_{1_t}$$

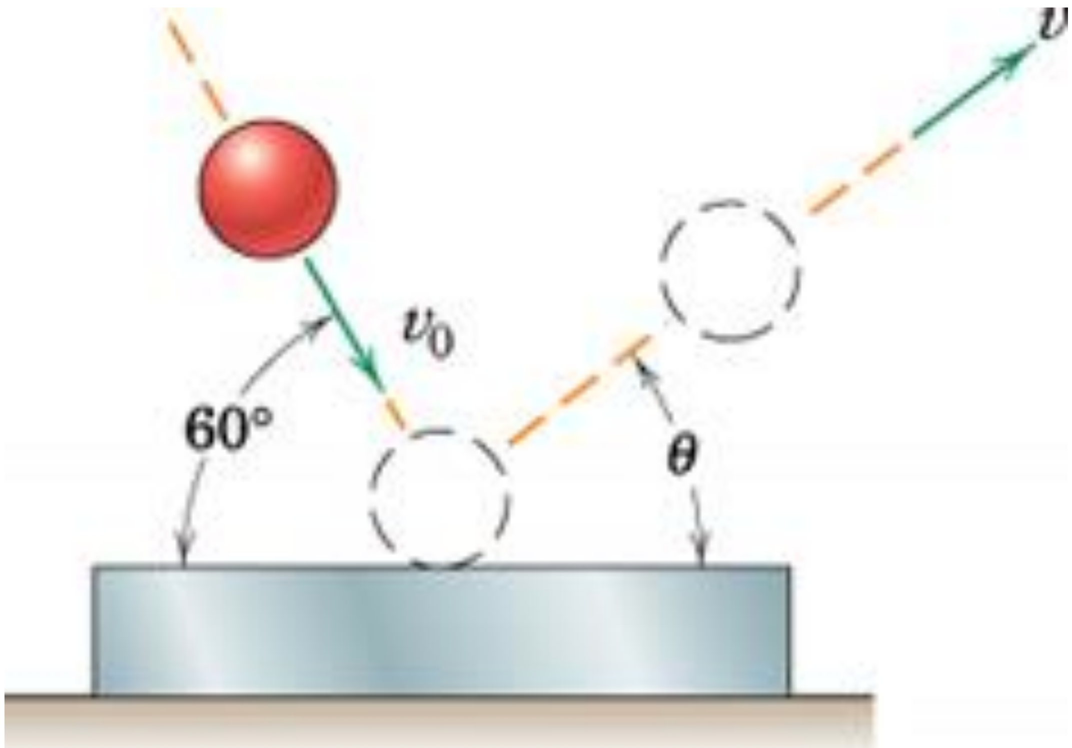
$$m_2 v_{2_t} = m_2 v'_{2_t}$$

That is actually

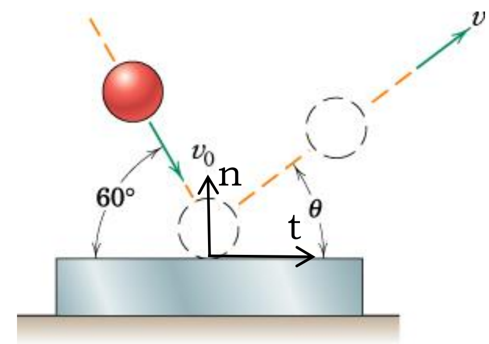
$v_{1_t} = v'_{1_t}$ and $v_{2_t} = v'_{2_t}$

3/250 (4th), 3/254 (5th), 3/260 (6th), None (7th), None (8th)

The steel bar strikes heavy steel plate with a velocity v_0 = 24 m/s at an angle of 60° with the horizontal. If the coefficient of restitution is 0.8, compute the velocity and direction with which the ball rebounds from the plate.



Since collusion is with ground conservation of momentum cannot be used because impact of this little mass with earth *does not* cause any change in its momentum.



This is an oblique central impact since the ball does not rebound from the location it came.

$$v'_t = v_t = v_0 \cos 60° = 12 \text{ m/s}$$

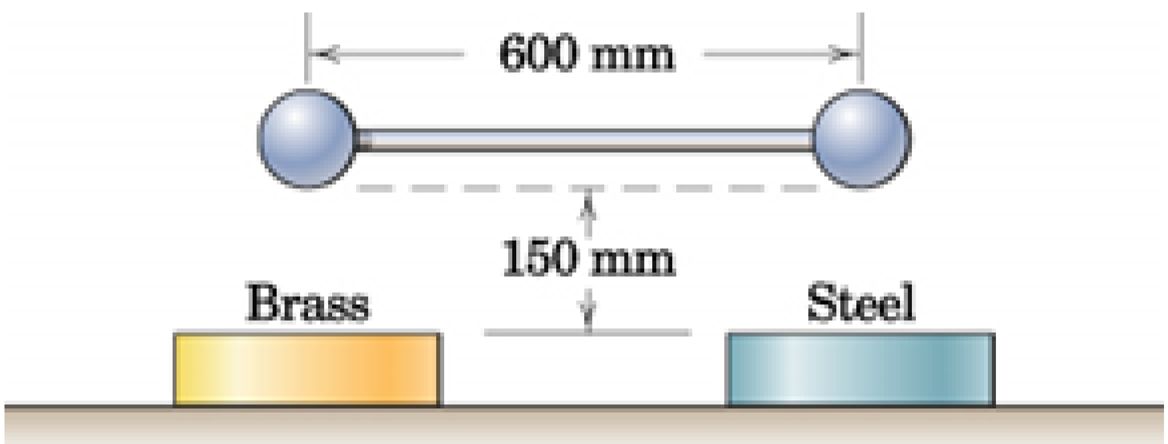
$$e = \frac{v'_n}{24 sin60°} = 0.8, v'_n = 16.63\, m/s$$

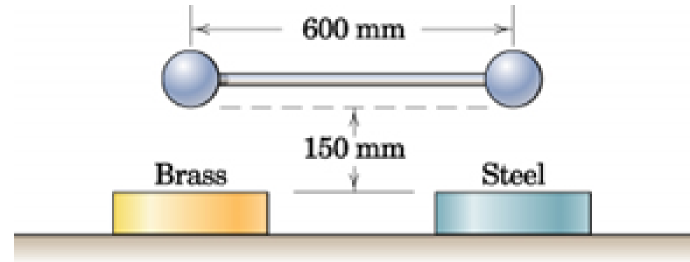
$$v', \theta = Pol(v'_t, v'_n) = Pol(12{,}16.63)$$

$$v', \theta = 20.5\, m/s, \qquad 54.2°$$

3/258 (4th), 3/261 (5th), 3/268 (6th), None (7th), (8th)

Two steel balls of the same diameter are connected by a rigid bar of negligible mass as shown and are dropped in the horizontal position from a height of 150 mm above the heavy steel and brass base plates. If the coefficient of restitution between the ball and the steel base is 0.6 and that between the other ball and the brass base is 0.4, determine the angular velocity ω of the bar immediately after impact. Assume that the two impacts are simultaneous.





This is direct central impact since the balls fall and rebound along the same line. Again since this is impact with earth conservation of momentum cannot be utilized.

During free fall, neglecting air resistance, the energy is conserved so velocity before impact can be calculated:

$$U_{1\to 2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to 2} = \Delta V_e = 0$$

$$\Delta T = \frac{1}{2} m v^2$$

$$\Delta V_g = mg(0 - h)$$

$$v = \sqrt{2gh} = \sqrt{2 * 9.81 * 0.15} = 1.716\, m/s$$

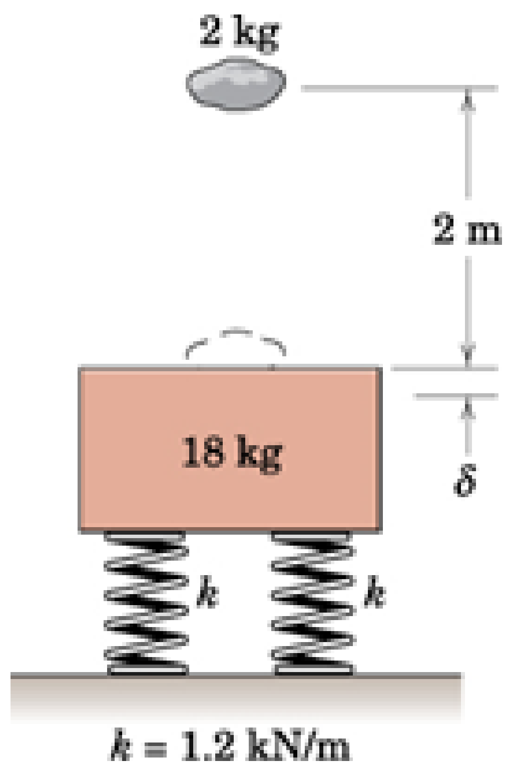
$$e_b = \frac{v'_b}{v} = 0.4, v'_b = 0.686\, m/s$$

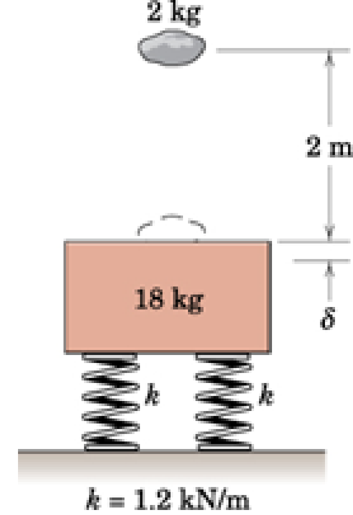
$$e_s = \frac{v'_s}{v} = 0.6, v'_s = 1.029\, m/s$$

$$\omega = \frac{v'_s - v'_b}{r} = \frac{1.029 - 0.686}{0.6} = 0.572\, rad/s$$

3/336 (4th), 3/345 (5th), 3/353 (6th), 3/335 (7th), 3/336 (8th)

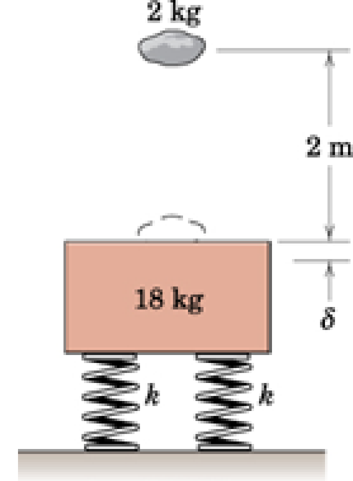
The 2 kg piece of putty is dropped 2 m onto the 18 kg block initially at rest on the two springs, each with a stiffness k = 1.2 kN/m. Calculate the additional deflection δ of the springs due to impact of the putty, which adheres to the block upon contact.





1. During free fall, neglecting air friction, energy of the putty is conserved.
2. During impact of putty with the mass linear momentum is conserved in vertical direction (the impulse of additional spring deflection due to induced velocity is negligible and weight is balanced by spring deflection initially).
3. After impact energy of the system is conserved again till the putty-mass system comes to a rest at the maximum deflection δ.

1. During free fall, neglecting air friction, energy of the putty is conserved.
2. During impact of putty with the mass linear momentum is conserved in vertical direction (the impulse of additional spring deflection due to induced velocity is negligible and weight is balanced by spring deflection initially).
3. After impact energy of the system is conserved again till the putty-mass system comes to a rest at the maximum deflection δ.



$$U_{1\to2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to2} = 0$$

$$\Delta T = \frac{1}{2} m_P v^2 - 0$$

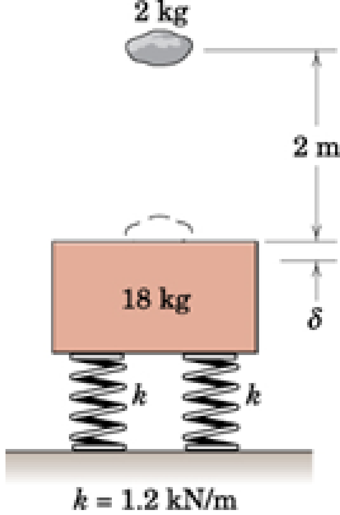
$$\Delta V_g = -m_P g h$$

$$\Delta V_e = 0$$

so

$$v = \sqrt{2gh} = \sqrt{2 * 9.81 * 2} = 6.26\, m/s$$

1. During free fall, neglecting air friction, energy of the putty is conserved.
2. During impact of putty with the mass linear momentum is conserved in vertical direction (the impulse of additional spring deflection due to induced velocity is negligible and weight is balanced by spring deflection initially).
3. After impact energy of the system is conserved again till the putty-mass system comes to a rest at the maximum deflection δ.

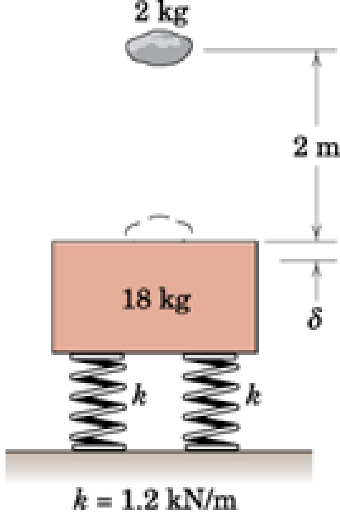


$$m_P v = (m + m_P)v'$$

$$2 * 6.26 = (2 + 18)v'$$

$$v' = 0.626\ m/s$$

1. During free fall, neglecting air friction, energy of the putty is conserved.
2. During impact of putty with the mass linear momentum is conserved in vertical direction (the impulse of additional spring deflection due to induced velocity is negligible and weight is balanced by spring deflection initially).
3. After impact energy of the system is conserved again till the putty-mass system comes to a rest at the maximum deflection δ.



$$U_{1\to2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to2} = 0$$

$$\Delta T = 0 - \frac{1}{2}(m + m_P)v'^2 = -\frac{1}{2}20 * 0.626^2 = -3.92\,J$$

$$\Delta V_g = -(m + m_P)g\delta = -20 * 9.81 * \delta = -196\delta\,J$$

$$\Delta V_e = \frac{1}{2}k({x_2}^2 - {x_1}^2)$$

$x_2 = x_1 + \delta$ where x_1 is the static deflection of the spring due to weight of 18 kg.

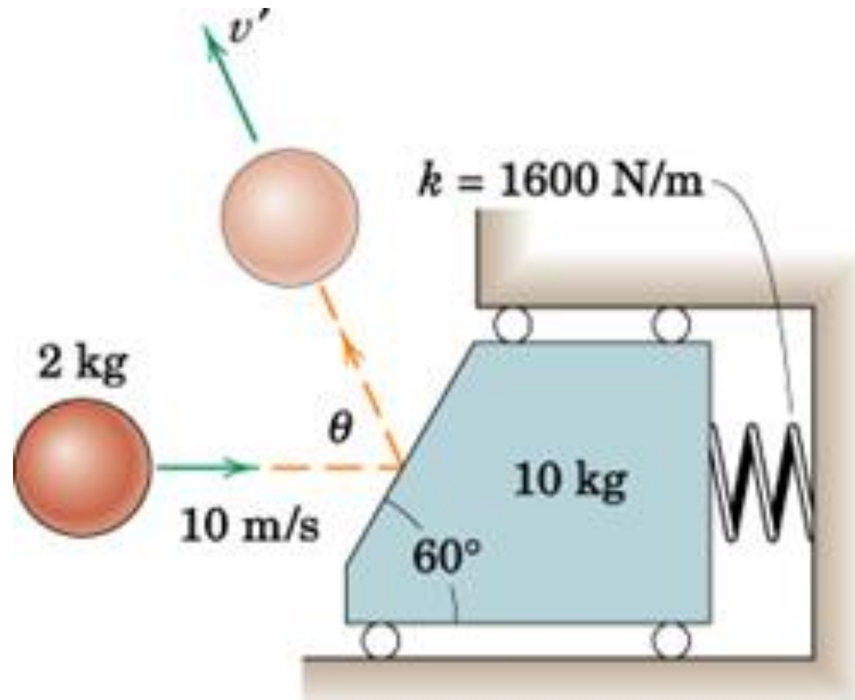
$$mg = kx_1, x_1 = \frac{mg}{k} = \frac{18 * 9.81}{2 * 1200} = 0.0736\,m$$

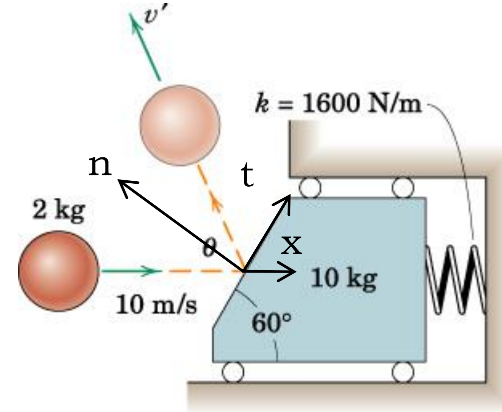
$$0 = -3.92 - 196\delta + 1200[(\delta + 0.0736)^2 - 0.0736^{\,2}]$$

$$\delta = \begin{cases} 65.9\,mm \\ \cancel{-49.6\,mm} \end{cases}$$

None (4th), 3/270 (5th), 3/278 (6th), None (7th), 3/267 (8th)

The 2 kg sphere is projected horizontally with a velocity of 10 m/s against the 10 kg carriage which is backed up by the spring with stiffness of 1600 N/m. The carriage is initially at rest with the spring uncompressed. If the coefficient of restitution is 0.6, calculate the rebound velocity, v', the rebound angle, θ, and the maximum travel of the carriage, δ, after impact.





Linear momentum of the ball is conserved along t:

$$m_b v_b cos60° = m_b v'_b sin(\theta - 30)$$

$$2 * 10cos60° = 2v'_b sin(\theta - 30)$$

Linear momentum of the ball and the carriage is conserved in horizontal direction (impulse of spring force is negligible):

$$m_b v_b = -m_b v_b cos\theta + m_c v'_c$$

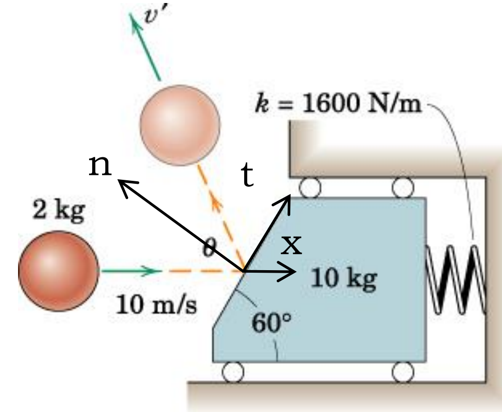
$$2 * 10 = -2v_b cos\theta + 10v'_c$$

Coefficient of restitution in normal direction

$$e = \frac{|-v'_c cos30° - v'_b cos(\theta - 30)|}{|-v_b cos30°|}$$

$$0.6 = \frac{|-v'_c cos30° - v'_b cos(\theta - 30)|}{|-10cos30°|}$$

$$v'_b = 6.04\, m/s, v'_c = 2.09\, m/s, \theta = 85.9°$$



Energy of the carriage is conserved after the impact until it stops at maximum spring deflection, δ.

$$U_{1\to 2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to 2} = \Delta V_g = 0$$

$$\Delta T = 0 - \frac{1}{2} m_c {v'_c}^2 = -\frac{1}{2} 10 * 2.09^2 = -21.8\, J$$

$$\Delta V_e = \frac{1}{2} k\delta^2 = \frac{1}{2} 1600\delta^2$$

$$\delta = 165\, mm$$