ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

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PART I: PARTICLES
Chapter 3: Kinetics of Particles
3/14 Relative Motion

All the analyses we did up to now were relative to a *fixed* coordinate system. Newton's second law, work-energy and impulse-momentum were used to analyze absolute motion. If the frame of reference has non-zero acceleration relative to the fixed coordinate system which cannot be neglected then the treatment is different.

We assumed earth to be fixed. Acceleration of center of earth with respect to sun is 0.00593 m/s^2 and acceleration of a point on the equator relative to the center of earth is 0.0339 m/s^2. Compared to g, 9.81 m/s^2 these values are negligible so for most purposes earth fixed reference frame is accurate enough.

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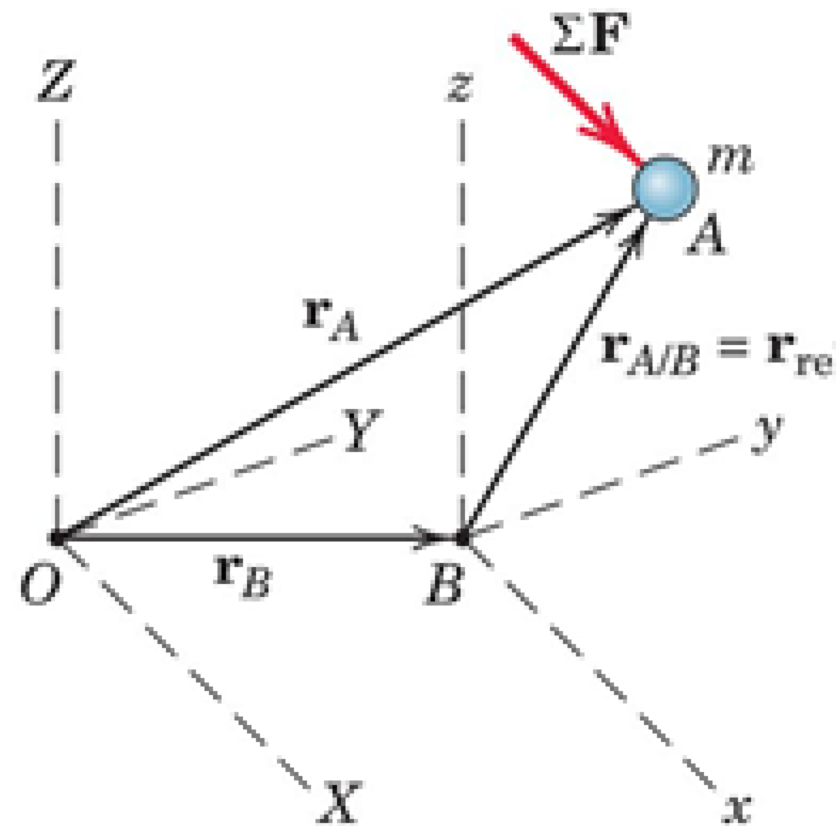
Relative Motion (Non-rotating Axes)

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$



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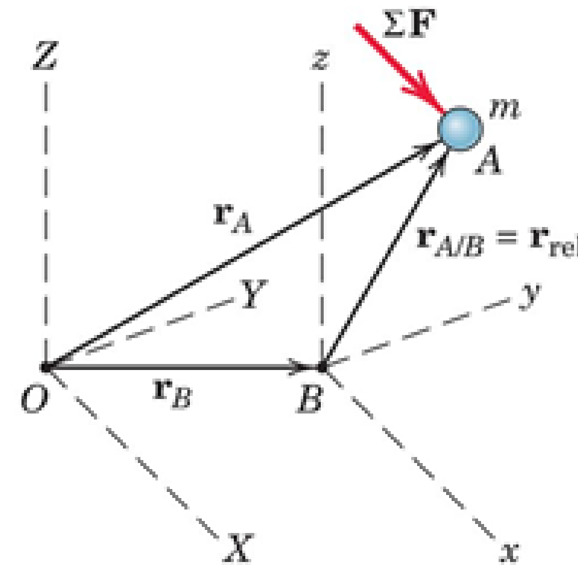

Newton's second law for a constant mass particle:

$$\sum \vec{F} = m\vec{a}_A = m(\vec{a}_B + \vec{a}_{A/B})$$

Unless $\vec{a}_B = \vec{0}$ Newton's second law is not valid in x-y-z coordinate system. Coordinate systems where $\vec{a}_B = \vec{0}$ are known as *inertial coordinate systems* which have zero acceleration relative to the primary inertial reference frame which is *fixed in the universe*. Newton's second law is valid only in inertial reference frames.

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D'Alembert's Principle

Newton 1687, Principa

D'Alembert 1743 Traité de Dynamique

D'Alembert introduced so called *inertia force* which is a resistance of mass to acceleration and converted dynamics problem into a kinetostatic problem or dynamic equilibrium.

$$\sum \vec{F} + \vec{F}^i = \vec{0}, \vec{F}^i = -m\vec{a}$$

We will **not** utilize D'Alembert's principle in this course but it proves to be very useful in certain applications in dynamics of machinery.

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Constant Velocity Translating Systems

These systems are also called inertial reference frames or Newtonian reference frames because $\vec{a}_B = \vec{0}$ and Newton's second law holds in this reference frame.

Work-Energy Relation

Since this is an inertial reference frame $\sum \vec{F} = m\vec{a}_{rel}$ holds.

$$dU_{rel} = \sum \vec{F} \cdot d\vec{r} = m\vec{a}_{rel} \cdot d\vec{r} = m\vec{v}_{rel} \cdot d\vec{v}_{rel} = mv_{rel}dv_{rel} = d\left(\frac{mv_{rel}^{\;2}}{2}\right)$$

$$dU_{rel} = dT_{rel}, U_{1\to 2_{rel}} = \Delta T_{rel}$$

Although in an inertial reference frame $\vec{a} = \vec{a}_{rel}$, $\vec{v} \neq \vec{v}_{rel}$ and $\vec{r} \neq \vec{r}_{rel}$ therefore $dU \neq dU_{rel}$ and $dT \neq dT_{rel}$.

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Constant Velocity Translating Systems

Linear Impulse-Momentum Relation

$$\sum \vec{F}\,dt = m\vec{a}dt = m\vec{a}_{rel}dt$$

$$\vec{a}_{rel}dt = d\vec{v}_{rel}$$

$$\sum \vec{F}\,dt = d(m\vec{v}_{rel})$$

Define $\vec{G}_{rel} = m\vec{v}_{rel}$

$$\sum \vec{F} = \dot{\vec{G}}_{rel}, \int_{t_1}^{t_2} \sum \vec{F}dt = \Delta\vec{G}_{rel}$$

Since $\vec{v} \neq \vec{v}_{rel}$, $\vec{G} \neq \vec{G}_{rel}$.

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Constant Velocity Translating Systems

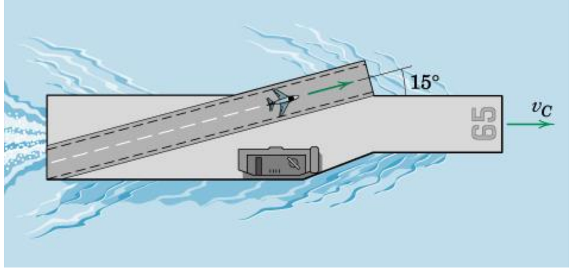
Angular Impulse-Momentum Relation

$$\vec{H}_{B_{rel}} = \vec{r}_{rel} \times \vec{G}_{rel}$$

$$\dot{\vec{H}}_{B_{rel}} = \dot{\vec{r}}_{rel} \times \vec{G}_{rel} + \vec{r}_{rel} \times \dot{\vec{G}}_{rel} = \vec{v}_{rel} \times m\vec{v}_{rel} + \vec{r}_{rel} \times m\vec{a}_{rel} = \sum \vec{M}_B$$

Since $\vec{v} \neq \vec{v}_{rel}$ and $\vec{r} \neq \vec{r}_{rel}$, $\vec{H}_B \neq \vec{H}_{B_{rel}}$.

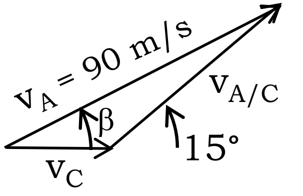
3/306 (4th), 3/310 (5th), 3/318 (6th), 3/304 (7th), 3/306 (8th)
The launch catapult of the aircraft carrier gives the 7 Mg jet airplane a constant acceleration and launches the airplane in a distance of 100 m measured along the angled takeoff ramp. The carrier is moving at a steady speed v_C = 16 m/s. If an absolute aircraft speed of 90 m/s is desired for takeoff, determine the net force, F, supplied by the catapult and the aircraft engines.



$$\vec{v}_A = 90(cos\beta\hat{\imath} + sin\beta\hat{\jmath})\ m/s$$

$$\vec{v}_C = 16\hat{\imath}\ m/s$$

$$\vec{v}_{A/C} = v_{A/C}(cos15°\hat{\imath} + sin15°\hat{\jmath})\ m/s$$



$$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}$$

$$90(cos\beta\hat{\imath} + sin\beta\hat{\jmath}) = 16\hat{\imath} + v_{A/C}(cos15°\hat{\imath} + sin15°\hat{\jmath})$$

$$v_{A/C} = 74.4\ m/s, \beta = 12.36°$$

$$U_{rel_{1\to2}} = \Delta T_{rel}$$

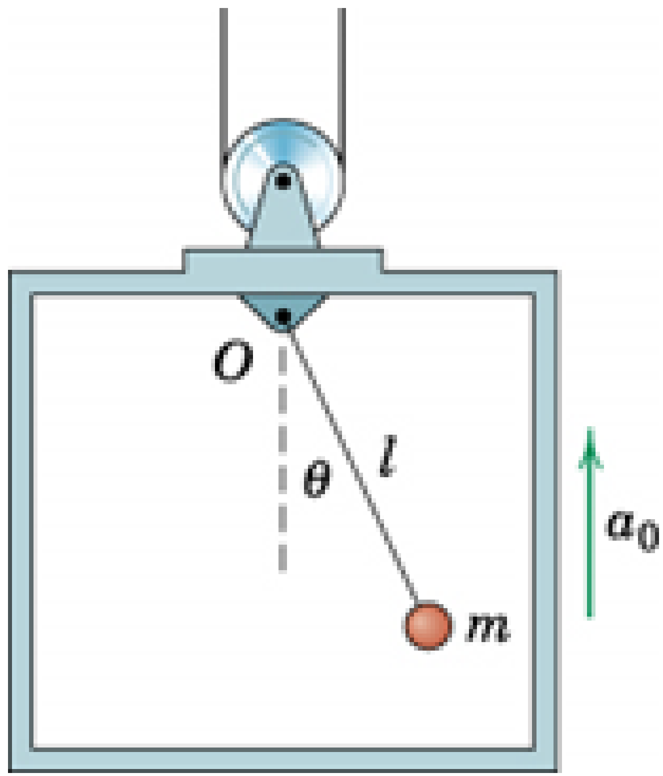
$$F_{av}s_{rel} = \frac{1}{2}m_A({v_{rel}}^2 - 0)$$

$$F_{av}100 = \frac{1}{2}7000 * 74.4^2$$

$$F_{av} = 193.7\ kN$$

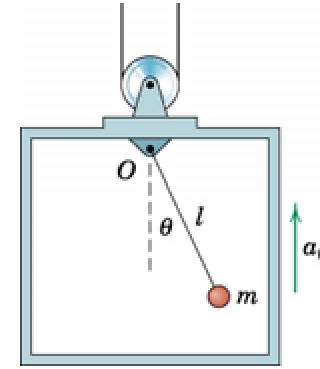
3/313 (4th), 3/317 (5th), 3/325 (6th), 3/311 (7th), 3/313 (8th)

A simple pendulum is placed on an elevator, which accelerates upwards as shown. If the pendulum is displaced by an amount θ_0 and released from the rest relative to the elevator, find the tension in the supporting rod when $\theta = 0$.



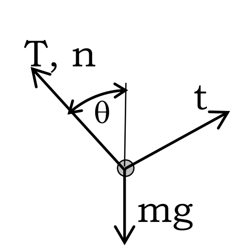
$$\sum F_t = ma_t, -mg\sin\theta = m\left(\ell\ddot{\theta} + a_0\sin\theta\right)$$

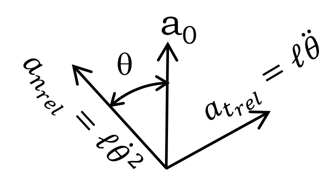
$$\sum F_n = ma_n, T - mg\cos\theta = m\left(\ell\dot{\theta}^2 + a_0\cos\theta\right)$$



$$T = m\left(g\cos\theta + \ell\dot{\theta}^2 + a_0\cos\theta\right) = m\left[(g + a_0)\cos\theta + \ell\dot{\theta}^2\right]$$

From F_t equation





$$\ddot{\theta} = -\frac{g + a_0}{\ell}\sin\theta$$

$$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$$

$$\int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \int_{\theta_0}^{\theta} -\frac{g + a_0}{\ell}\sin\theta\,d\theta$$

$$\dot{\theta} = \sqrt{2\frac{g + a_0}{\ell}(\cos\theta - \cos\theta_0)}$$

$$T = m[(g + a_0)(3\cos\theta - 2\cos\theta_0)]$$

PART II: RIGID BODIES
Chapter 5: Plane Kinematics of Rigid Bodies
5/1 Introduction

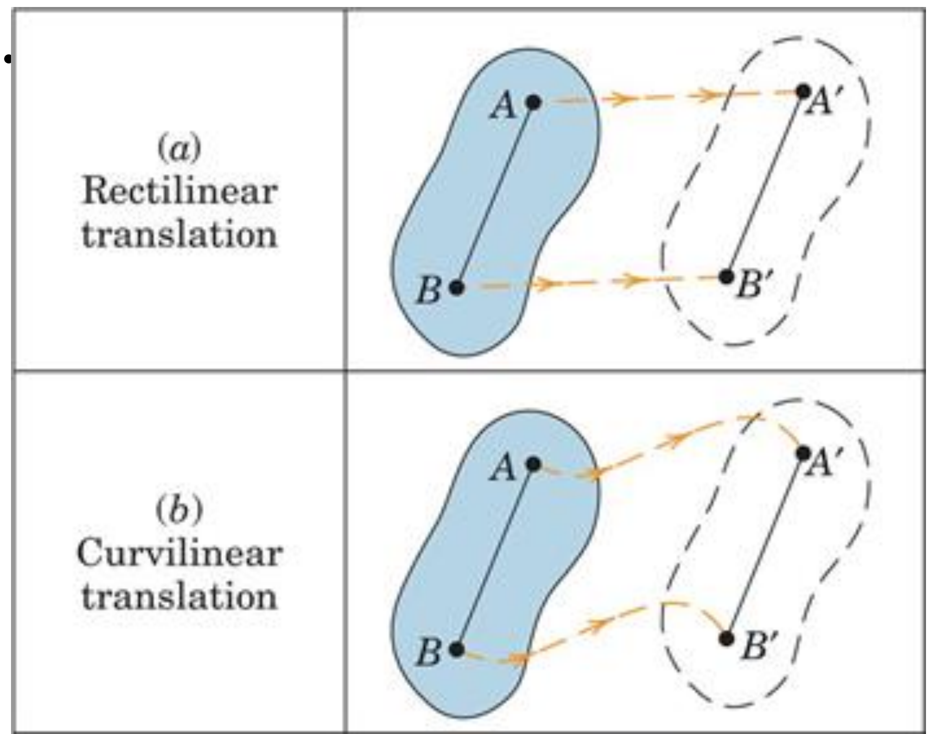
Rigid Body: It is a continuous collection of infinitely many particles of infinitesimal mass. The relative positions of particles remain unchanged under applied forces (no *deformation* or *strain*).

Kinematics of rigid bodies should be studied first in order to relate the motion to forces or forces needed for a desired motion.

5/1 Introduction

Types of Plane Motion of Rigid Bodies

1. Translation: Every line on the rigid body remains parallel to its initial position, there is no *rotation* therefore motion of the body can be specified by the motion of any single point on the body. Particle treatment is sufficient since there is no rotation.

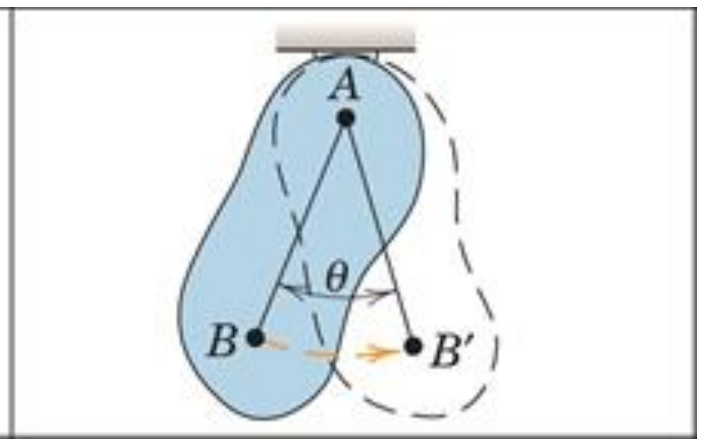


5/1 Introduction

Types of Plane Motion of Rigid Bodies

2. Rotation: Every particle moves in concentric circles centered at the axis of rotation.

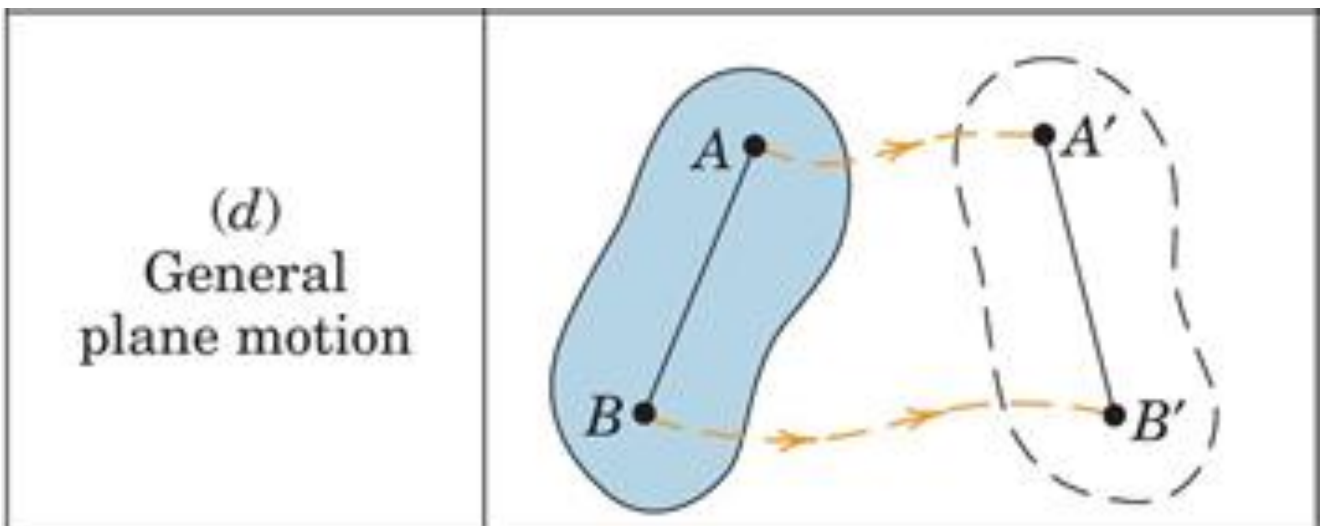
(c)
Fixed-axis rotation



5/1 Introduction

Types of Plane Motion of Rigid Bodies

3. General Plane Motion: This is a combination of translation and rotation. The motion can be analyzed either by *absolute displacements and their time derivatives* or by *relative motion* concept.



5/2 Rotation

Any two lines on a rigid body that have angular positions θ_1 and θ_2 with respect to any arbitrary reference may be used to designate the rotation.

$$\theta_2 = \theta_1 + \beta$$

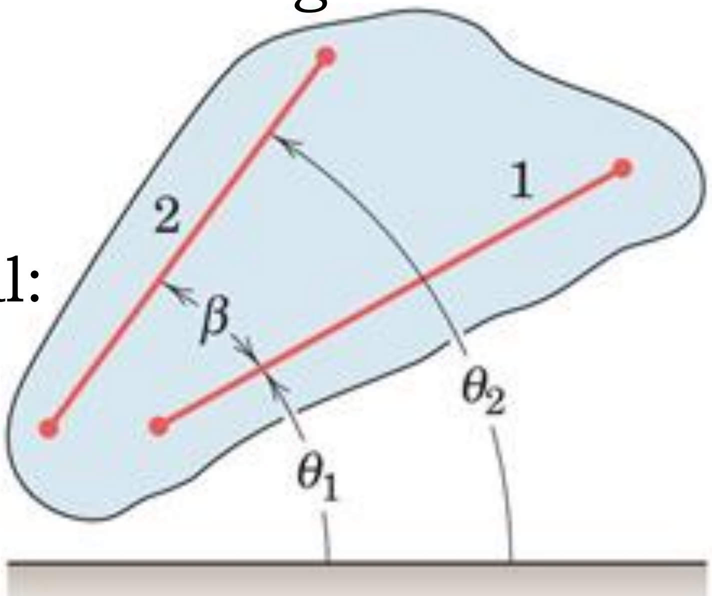
Because body is rigid, β is a constant angle. Therefore angular displacements are equal:

$$\Delta\theta_1 = \Delta\theta_2$$

Also time rates of changes are equal:

$$\dot{\theta}_1 = \dot{\theta}_2 \equiv \omega$$

$$\ddot{\theta}_1 = \ddot{\theta}_2 \equiv \alpha$$



Therefore for a rigid body there is **a single angular velocity,** ω, and **a single angular acceleration,** α.

5/2 Rotation

Angular Motion Relations

$$\omega \equiv \frac{d\theta}{dt} = \dot{\theta}$$

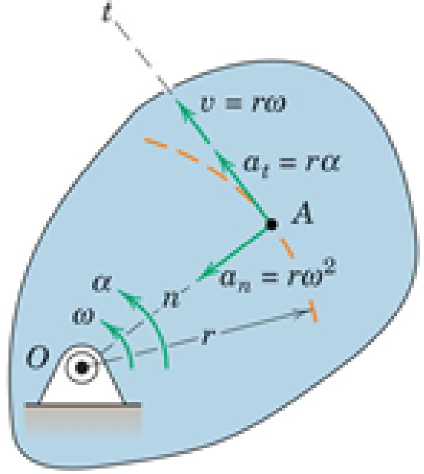
$$\alpha \equiv \frac{d\omega}{dt} = \dot{\omega} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

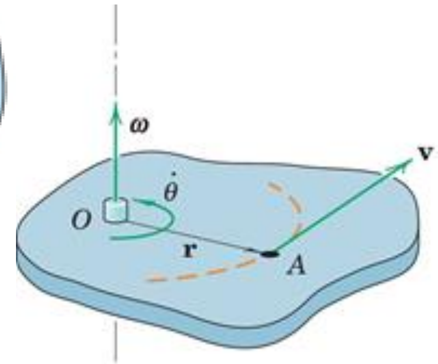
Solving dt from the two definitions yields:

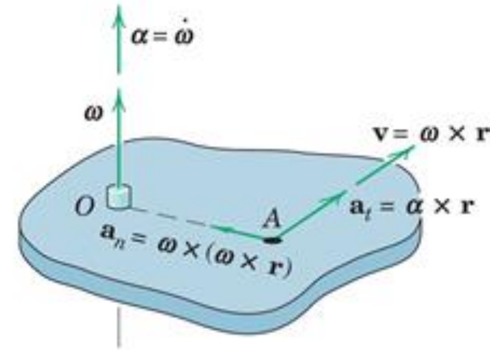
$\omega d\omega = \alpha d\theta$ or $\dot{\theta} d\dot{\theta} = \ddot{\theta} d\theta$

Please recognize that if θ is measured counter clockwise (i.e. increasing in counter clockwise direction), positive directions for angular velocity and acceleration are the counter clockwise as well.

5/2 Rotation







Rotation about a Fixed Axis

Consider an arbitrary particle, A, on the rigid body. $\vec{r}$ is the position vector of A relative to the rotation axis O. From kinematics of particles it is known that

$$v_A = r\omega$$

$$a_{A_t} = r\alpha$$

$$a_{A_n} = r\omega^2 = \frac{{v_A}^2}{r} = v_A\omega$$

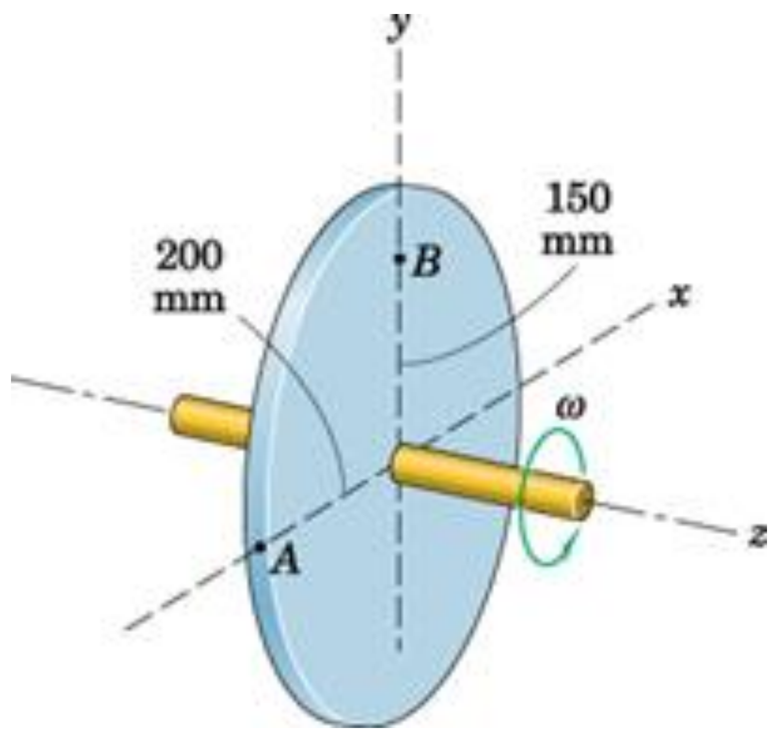
These relations can be expressed as vectors if cross product is utilized:

$$\vec{v}_A = \dot{\vec{r}}_A = \vec{\omega} \times \vec{r}_A$$

$$\vec{a}_A = \dot{\vec{v}}_A = \dot{\vec{\omega}} \times \vec{r}_A + \vec{\omega} \times \dot{\vec{r}}_A = \vec{\alpha} \times \vec{r}_A + \vec{\omega} \times (\vec{\omega} \times \vec{r}_A) = \vec{\alpha} \times \vec{r}_A - \omega^2 \vec{r}_A$$

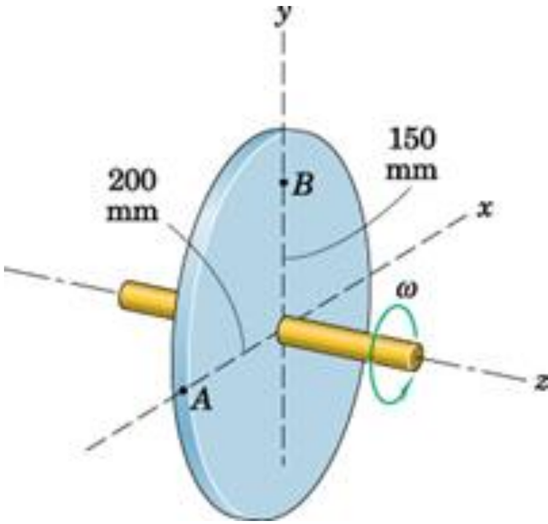
5/21 (4th), None (5th), 5/23 (6th), None (7th), None (8th)

The circular disk rotates about its z axis with an angular velocity in the direction shown. At a certain instant the magnitude of velocity of point A is 3 m/s and is decreasing at a rate of 7.2 m/s^2. Write the vector expressions for the angular acceleration $\vec{\alpha}$ of the disk and the total acceleration of point B at this instant.



$$\vec{v}_A = \vec{\omega} \times \vec{r}_A, -3\hat{j} = \omega\hat{k} \times -0.2\hat{\imath}$$

$$\omega = \frac{-0.2}{-3} = 15\, rad/s, \vec{\omega} = 15\hat{k}\, rad/s$$



$$\vec{a}_{A_t} = \alpha\hat{k} \times \vec{r}_A, 7.2\hat{j} = \alpha\hat{k} \times\ -0.2\hat{\imath}$$

$$\alpha = \frac{-0.2}{7.2} = -36\, rad/s^2, \vec{\alpha} = -36\hat{k}\, rad/s^2$$

$$\vec{a}_B = \vec{\alpha} \times \vec{r}_B - \omega^2\vec{r}_B = -36\hat{k} \times 0.15\hat{j} - 15^2 * 0.15\hat{j} = (5.4\hat{\imath} - 33.75\hat{j})m/s^2$$

Analysis of General Plane Motion

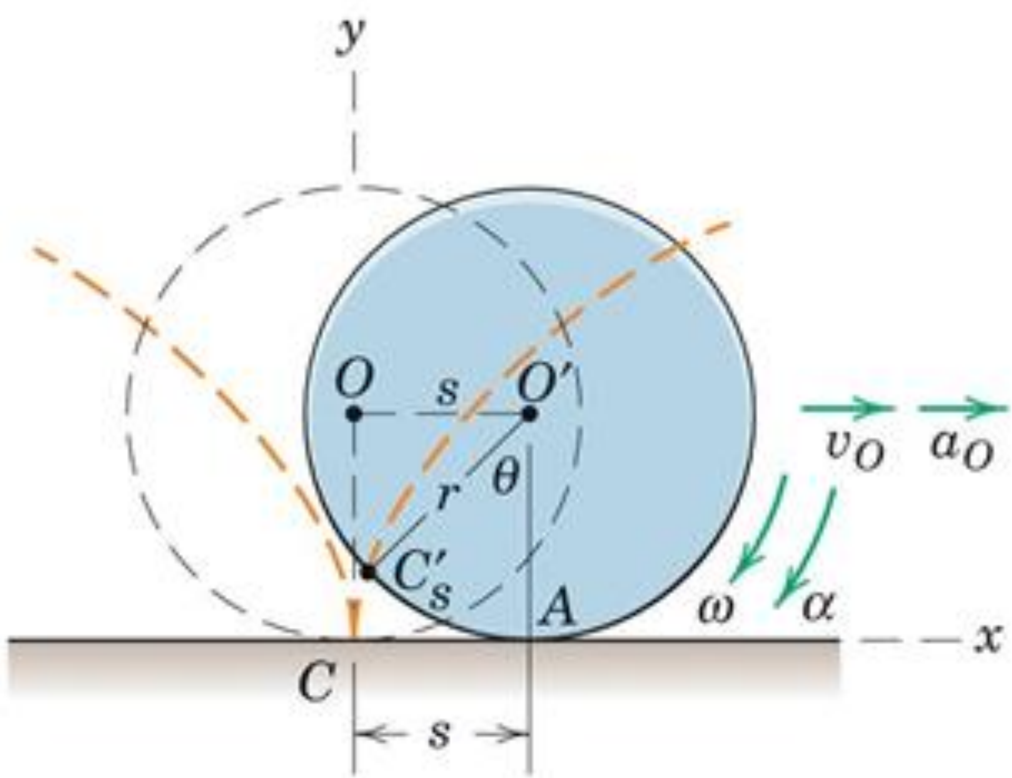
Absolute / Relative Motion Analyses

5/3 Absolute Motion

A geometric relation (which is actually a constraint equation for the motion of the particles on the rigid body we are interested in) is written for the rigid body. Time derivatives of this constraint equation yields velocities and accelerations.

Sample Problem 5/4

A wheel of radius r rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center, O. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.

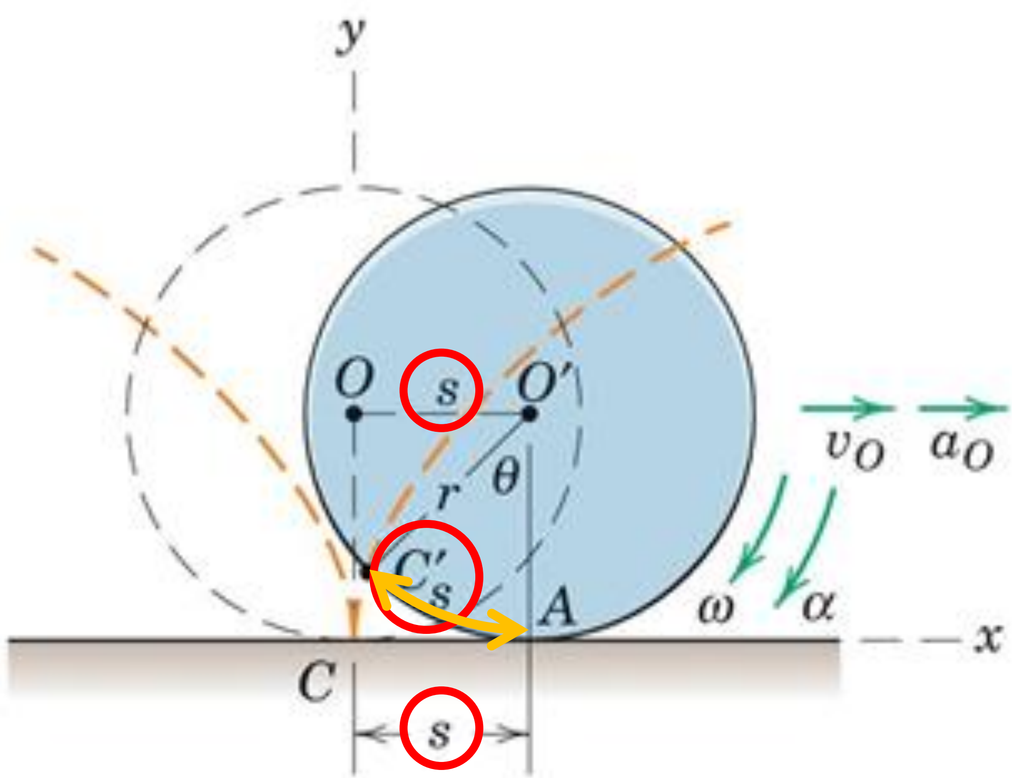


Since this is rolling without slipping point of contact with ground has zero velocity momentarily and arc length on the rim is equal to the distance travelled.

$s = s_O = r\theta$

$\dot{s} = v_O = r\dot{\theta} = r\omega$

$\ddot{s} = a_O = r\ddot{\theta} = r\alpha$

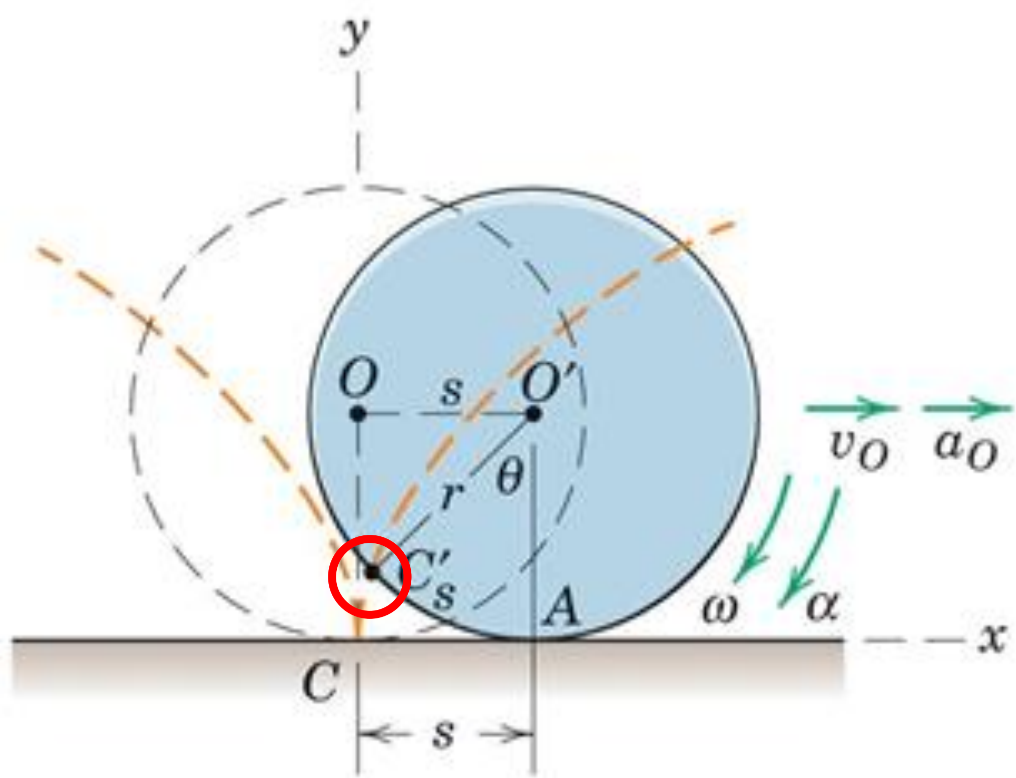


To determine the acceleration of a point on the rim of the wheel for any position, θ, just select the origin of the coordinate axis at C when θ = 0. Position of point C for any θ would be:

$x_C = s - r\sin\theta = r\theta - r\sin\theta = r(\theta - \sin\theta)$

$y_C = r - r\cos\theta = r(1 - \cos\theta)$

which are constraint equations!



$x_C = r(\theta - sin\theta)$

$y_C = r(1 - cos\theta)$

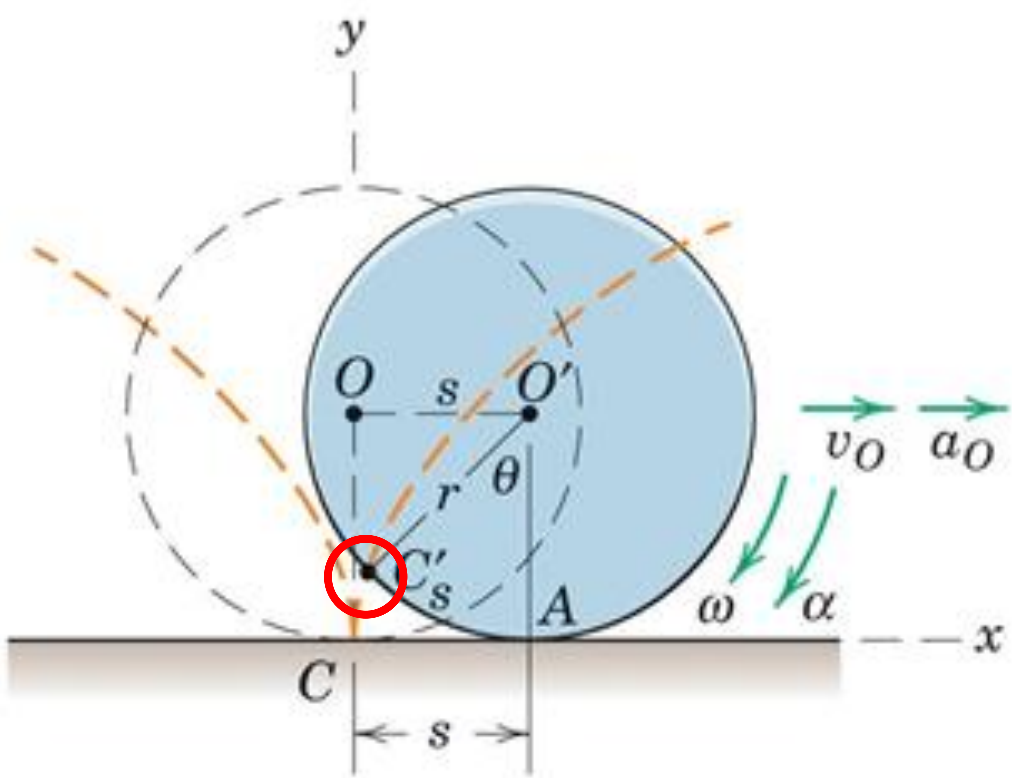
Derivatives yield velocity and acceleration of C:

$v_{C_x} = \dot{x}_C = r\left(\dot{\theta} - \dot{\theta}cos\theta\right) = r\dot{\theta}(1 - cos\theta)$

$v_{C_y} = \dot{y}_C = r\dot{\theta}sin\theta$

$a_{C_x} = \ddot{x}_C = r\ddot{\theta}(1 - cos\theta) + r\dot{\theta}^2 sin\theta$

$a_{C_y} = \ddot{y}_C = r\ddot{\theta}sin\theta + r\dot{\theta}^2 cos\theta$



$$a_{C_x} = \ddot{x}_C = r\ddot{\theta}(1 - cos\theta) + r\dot{\theta}^2 sin\theta$$

$$a_{C_y} = \ddot{y}_C = r\ddot{\theta} sin\theta + r\dot{\theta}^2 cos\theta$$

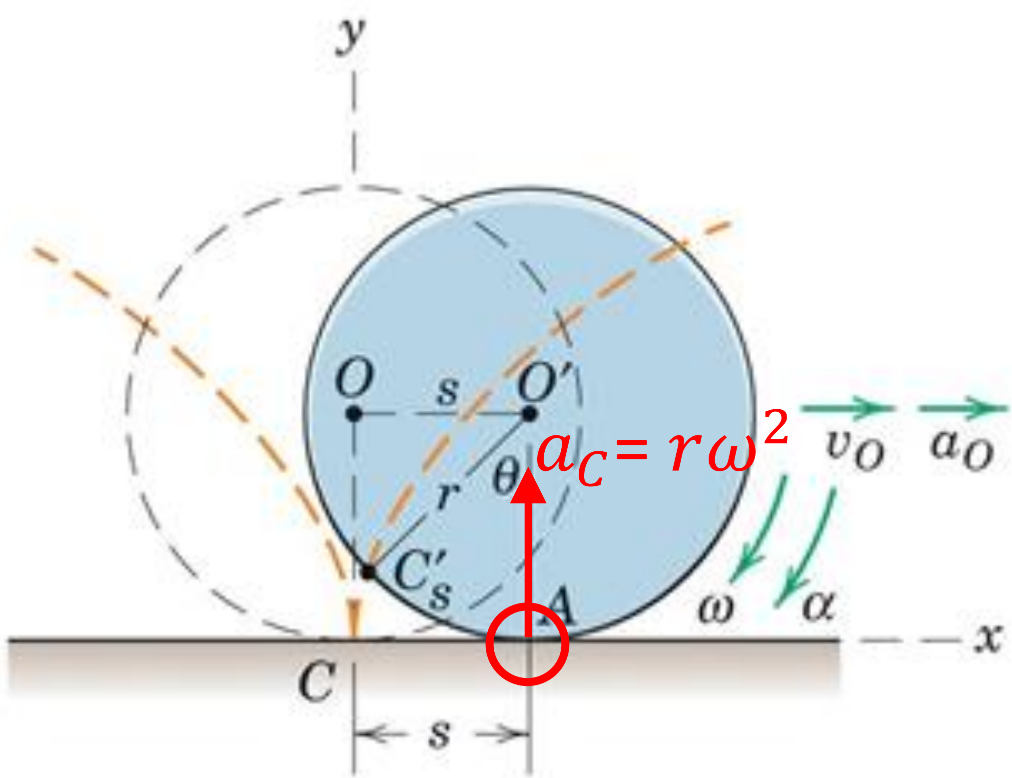
For C in contact with ground,

$$\theta = 2k\pi, k = \cdots, -2, -1, 0, 1, 2, \dots, sin\theta = 0, cos\theta = 1$$

$$a_{C_x} = \ddot{x}_C = r\ddot{\theta}(1 - 1) + r\dot{\theta}^2 * 0 = 0$$

$$a_{C_y} = \ddot{y}_C = r\ddot{\theta} * 0 + r\dot{\theta}^2 * 1 = r\omega^2$$

Please also go over the solution in the textbook!



Plot trajectory of point C in Excel:

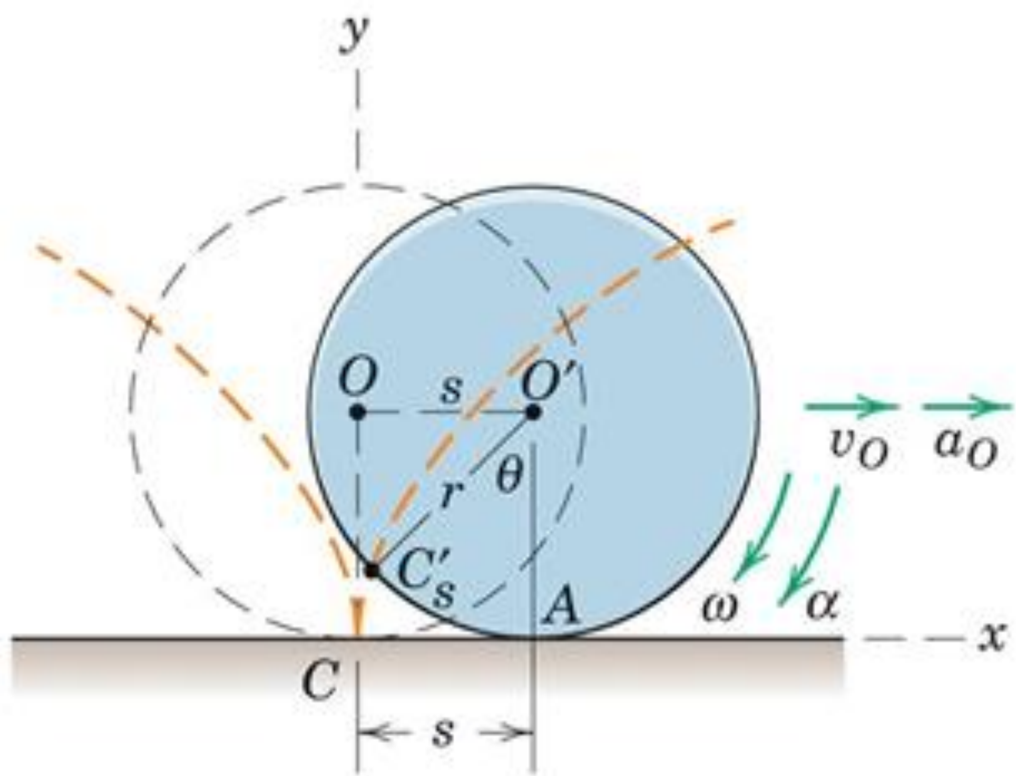
$$x_C = r(\theta - sin\theta)$$

$$y_C = r(1 - cos\theta)$$

Let $r = 2$

Create a column for θ in degrees:

	A	
1	Theta	
2		
3	deg	ra
4	0	
5	2	0
6	4	0
7	6	
8	8	0
9	10	0
10	12	
11	14	0
12	16	0

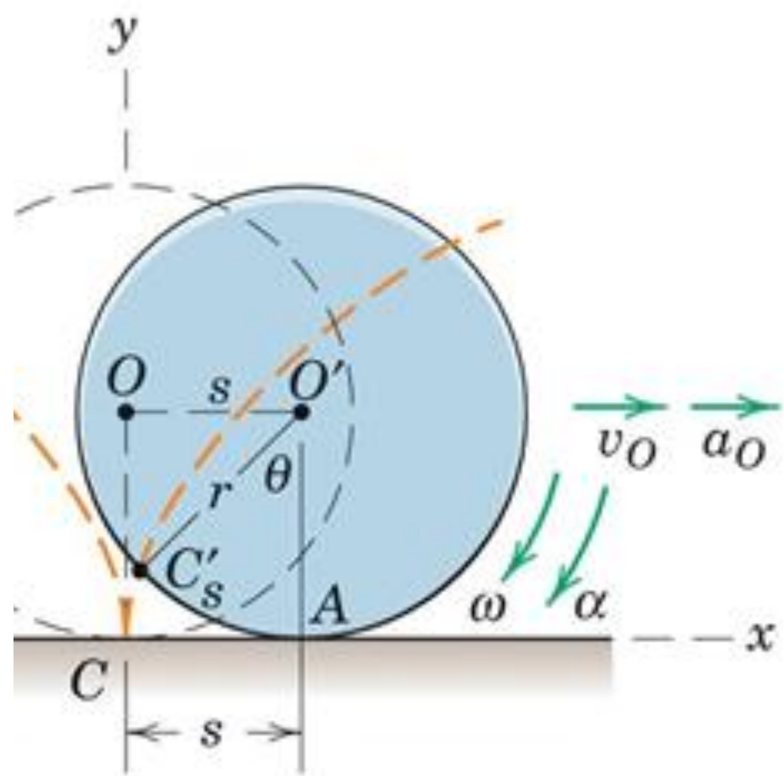


$$x_C = r(\theta - sin\theta)$$
$$y_C = r(1 - cos\theta)$$

Excel works in radians so convert the angle into radians:

B5 =A5*PI()/180

	A	B
1	Theta	
2		
3	deg	rad
4	0	0
5	2	0.034907
6	4	0.069813
7	6	0.10472
8	8	0.139626
9	10	0.174533
10	12	0.20944
11	14	0.244346



$$x_C = r(\theta - sin\theta)$$
$$y_C = r(1 - cos\theta)$$

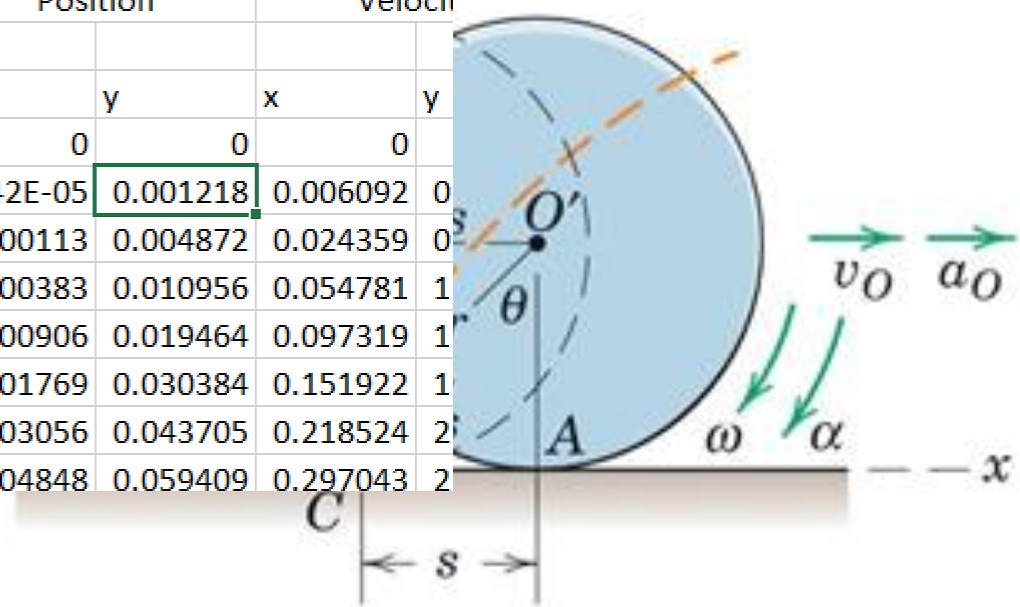
Type x and y coordinates of C as:

C5 =2*(B5-SIN(B5))

	A	B	C
1	Theta		
2			
3	deg	rad	x
4	0	0	
5	2	0.034907	1
6	4	0.069813	0
7	6	0.10472	0
8	8	0.139626	0
9	10	0.174533	0
10	12	0.20944	0
11	14	0.244346	0

D5 =2*(1-COS(B5))

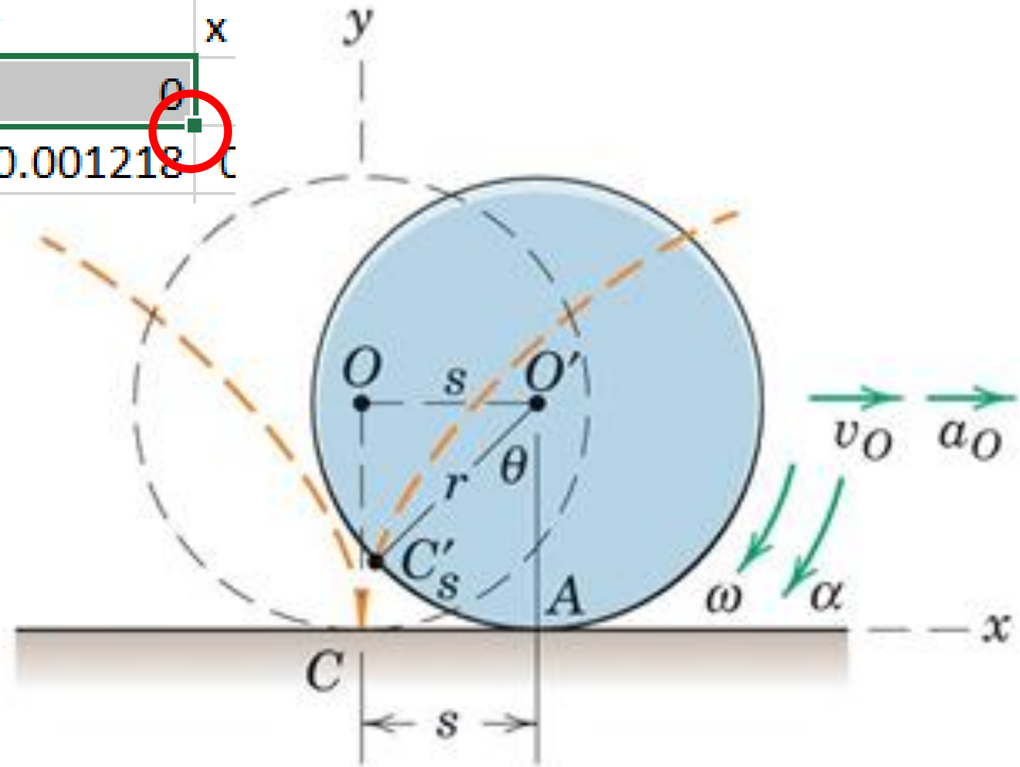
	A	B	C	D	E	F
1	Theta		Position		Velocit	
2						
3	deg	rad	x	y	x	y
4	0	0	0	0	0	
5	2	0.034907	1.42E-05	0.001218	0.006092	0
6	4	0.069813	0.000113	0.004872	0.024359	0
7	6	0.10472	0.000383	0.010956	0.054781	1
8	8	0.139626	0.000906	0.019464	0.097319	1
9	10	0.174533	0.001769	0.030384	0.151922	1
10	12	0.20944	0.003056	0.043705	0.218524	2
11	14	0.244346	0.004848	0.059409	0.297043	2



$$x_C = r(\theta - sin\theta)$$
$$y_C = r(1 - cos\theta)$$

Copy everything down by dragging the little box down till θ = 360 degrees

	A	B	C	D	
1	Theta		Position		
2					
3	deg	rad	x	y	x
4	0	0	0	0	
5	2	0.034907	1.42E-05	0.001218	



$$x_C = r(\theta - sin\theta)$$
$$y_C = r(1 - cos\theta)$$

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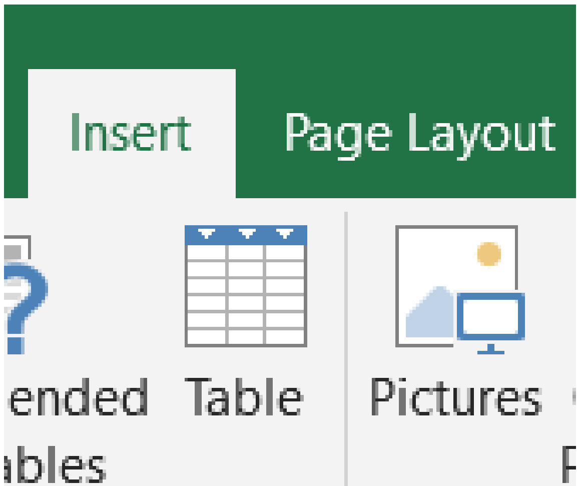
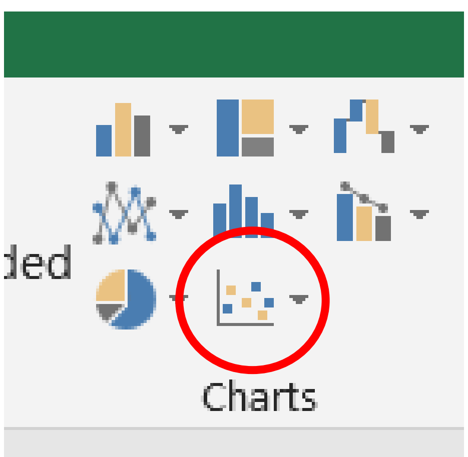
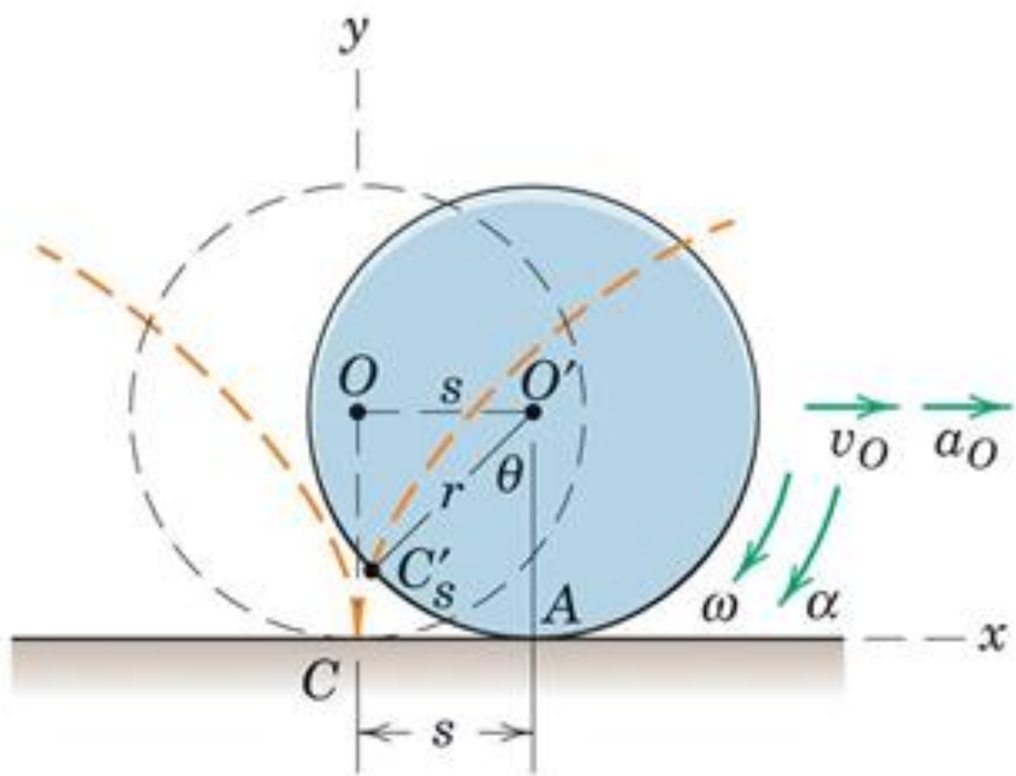


...Charts select scatter





$$x_C = r(\theta - sin\theta)$$
$$y_C = r(1 - cos\theta)$$

Stretch the plot so that x and y distances are equal: