ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

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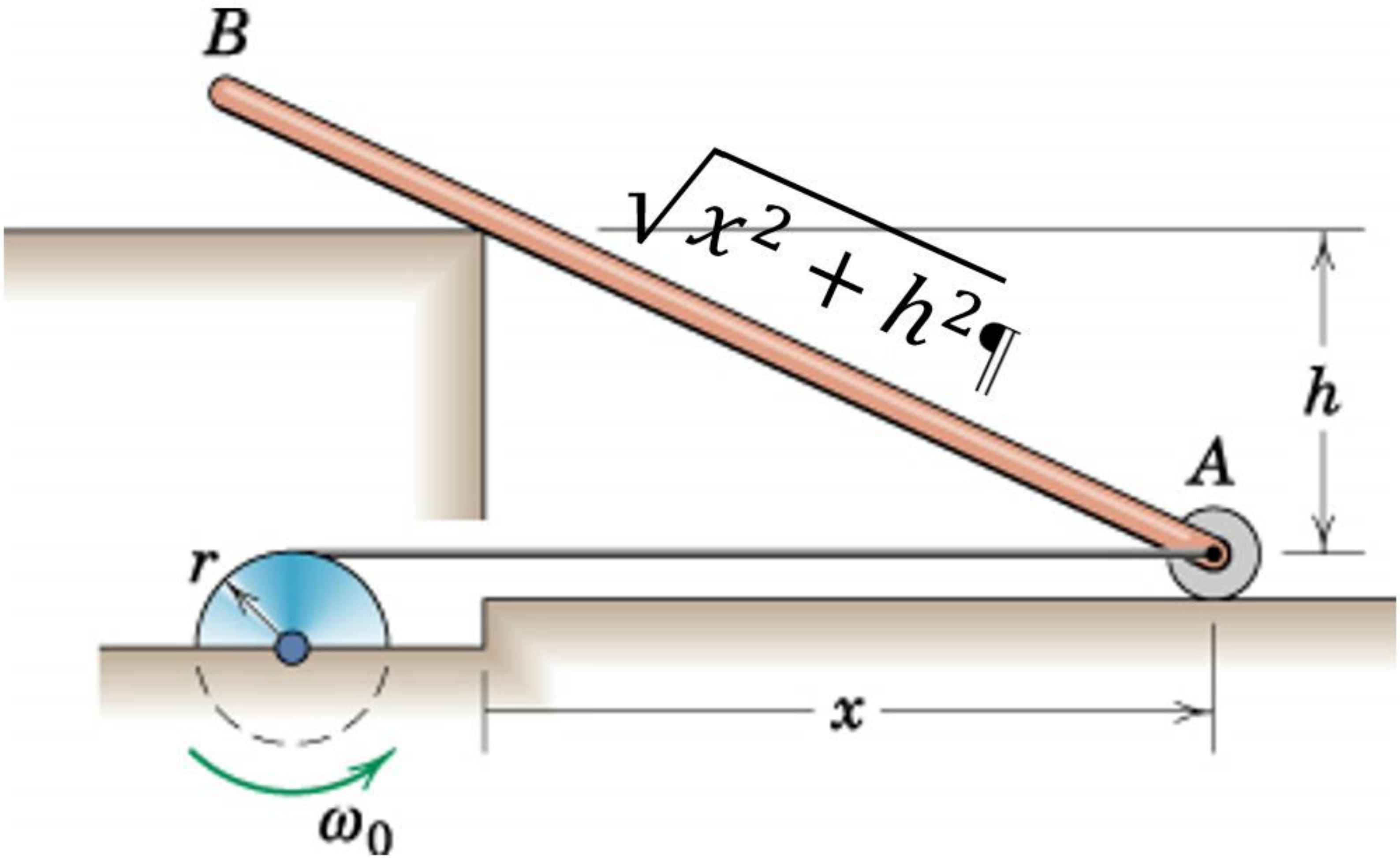
5/33 (4th & 5th), 5/37 (6th), 5/42 (7th), None (8th)

Calculate the angular velocity of the slender bar AB as a function of distance x and the constant angular velocity, ω_0 of the drum.



$$v_A = r\omega_0 = -\dot{x}$$

$$\theta = \sphericalangle OAB,\ \omega = \dot{\theta}$$

$$h = x tan\theta = x\frac{sin\theta}{cos\theta}$$

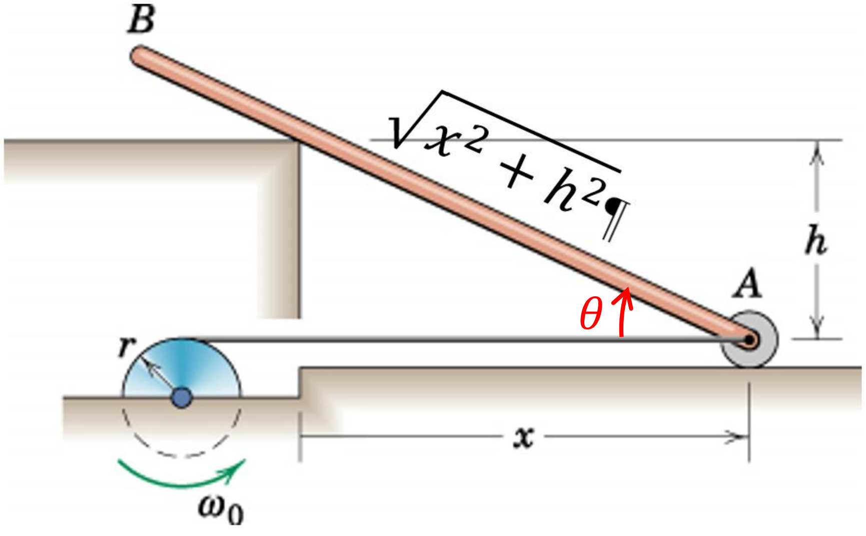
$$0 = \dot{x} tan\theta + x\frac{cos^2\theta + sin^2\theta}{cos^2\theta}\dot{\theta} = \dot{x} tan\theta + x\dot{\theta} sec^2\theta$$

$$\omega = \dot{\theta} = -\frac{\dot{x} tan\theta}{x sec^2\theta} = -\frac{\dot{x}}{x} sin\theta cos\theta$$

$$sin\theta = \frac{h}{\sqrt{x^2 + h^2}}$$

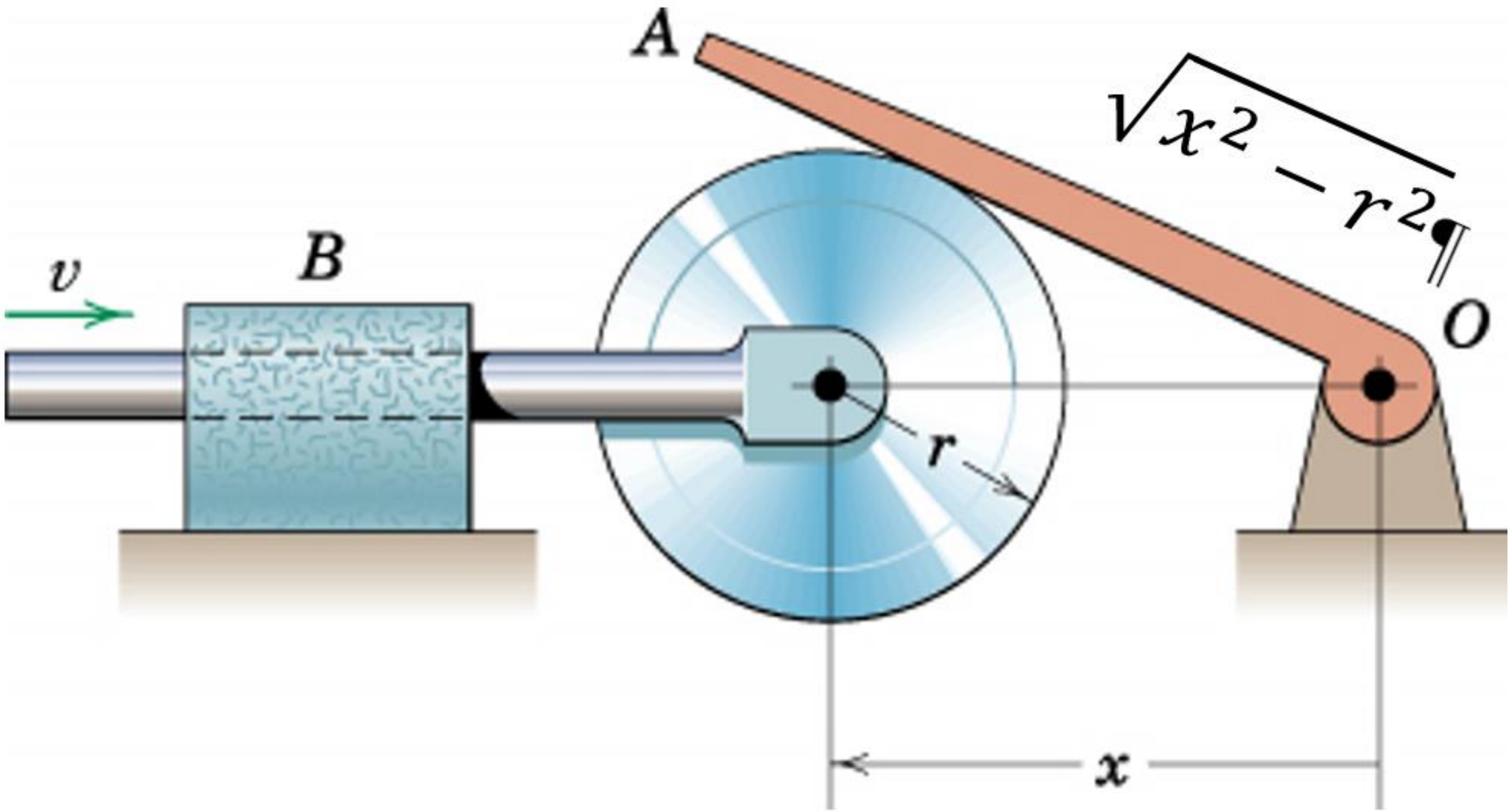
$$cos\theta = \frac{x}{\sqrt{x^2 + h^2}}$$

$$\omega = \dot{\theta} = \frac{-\dot{x}h}{x^2 + h^2} = \frac{r\omega_0 h}{x^2 + h^2}$$



5/34 (4th) 5/35 (5th), 5/39 (6th), 5/44 (7th), None (8th)

Rotation of the lever OA is controlled by the motion of the contacting circular disk whose center is given a horizontal velocity v. Determine the expression for the angular velocity ω of the lever OA in terms of x.



$$r = x\sin\theta$$

$$\theta = \sphericalangle BOA,\ \omega = \dot{\theta}$$

$$0 = \dot{x}\sin\theta + x\dot{\theta}\cos\theta$$

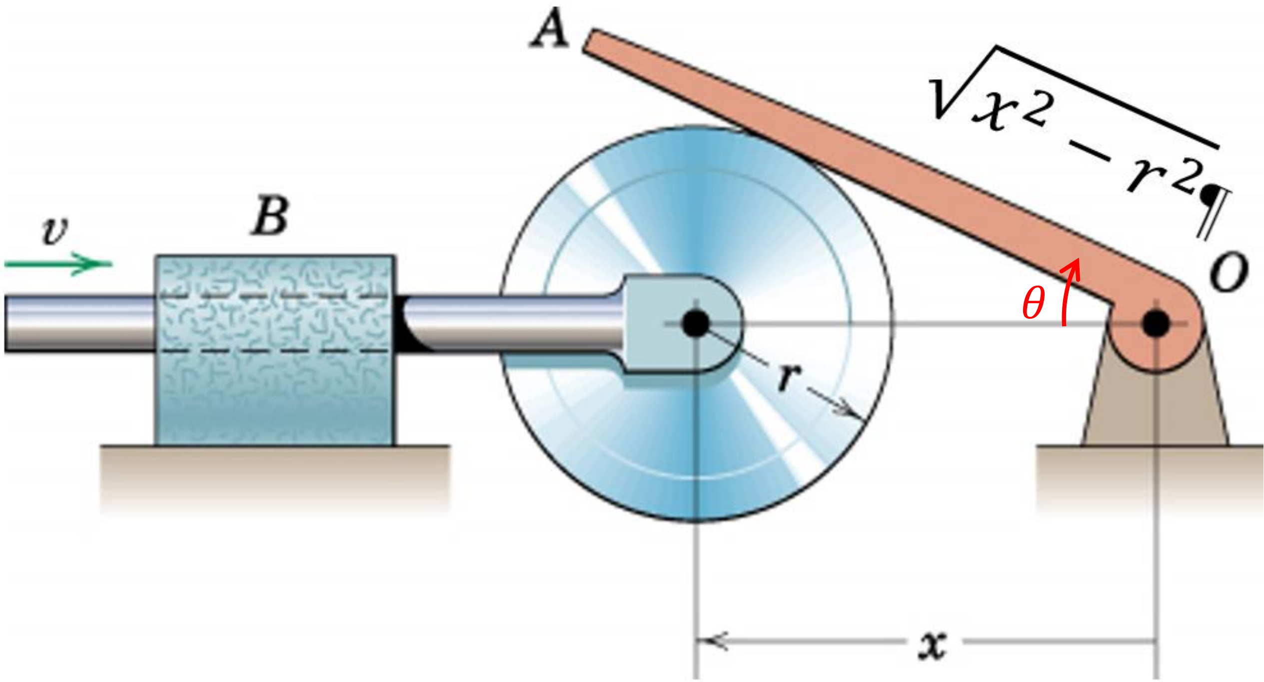
$$\omega = \dot{\theta} = -\frac{\dot{x}}{x}\tan\theta$$

$$\dot{x} = -v$$

$$\omega = \dot{\theta} = \frac{v}{x}\tan\theta$$

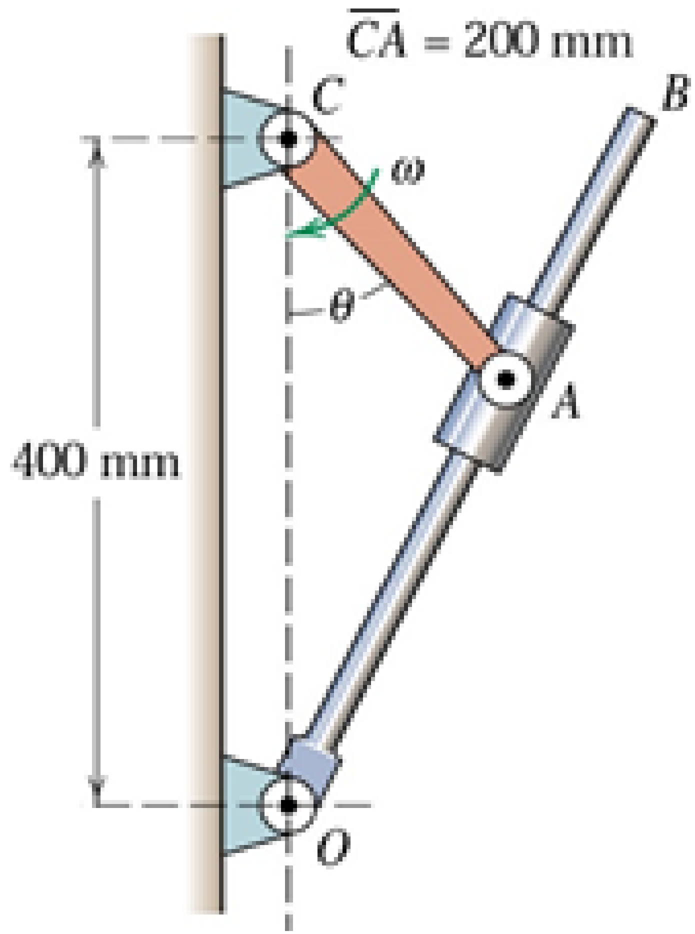
$$\tan\theta = \frac{r}{\sqrt{x^2 - r^2}}$$

$$\omega = \frac{v}{x\sqrt{\left(\frac{x}{r}\right)^2 - 1}}$$



5/42 (4th) 5/44 (5th), None (6th), None (7th), 5/44 (8th)

The rod OB slides through the collar pivoted to the rotating link at A. If CA has an angular velocity $\omega = 3$ rad/s for an interval of motion, calculate the angular velocity of OB when $\theta = 45°$.



$\beta = \sphericalangle COA$

$|AC| sin\theta = |OA| sin\beta$

$|AC| cos\theta + |OA| cos\beta = 0.4$

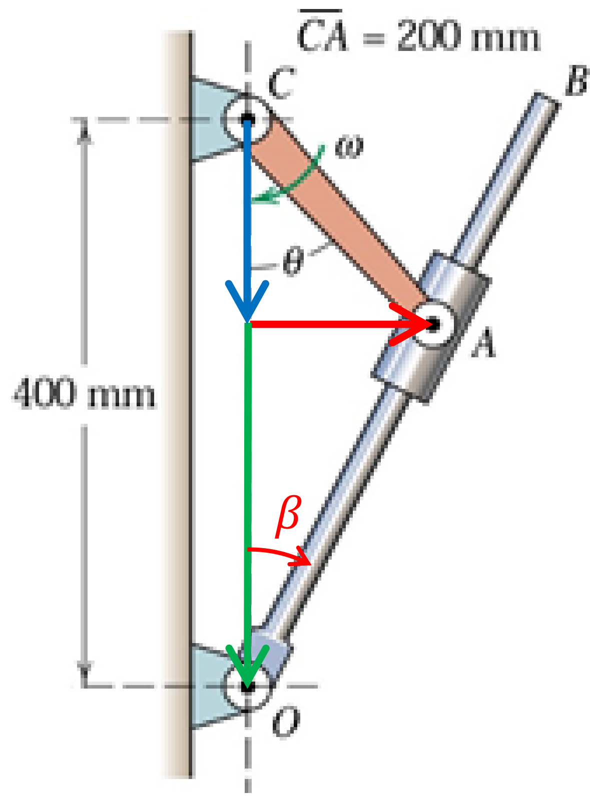
$|OA|$ and β unknown, eliminating $|OA|$ yields

$$tan\beta = \frac{|AC| sin\theta}{0.4 - |AC| cos\theta}$$

$$\dot{\beta} sec^2\beta = \frac{\dot{\theta}|AC| cos\theta(0.4 - |AC| cos\theta) - \dot{\theta}|AC| sin\theta |AC| sin\theta}{(0.4 - |AC| cos\theta)^2}$$

$$\dot{\beta} = \frac{2cos\theta - 1}{(2 - cos\theta)^2} cos^2\beta\dot{\theta}$$

For $\theta = 45°, \beta = 28.7°, \dot{\beta} = -0.572\, rad/s$

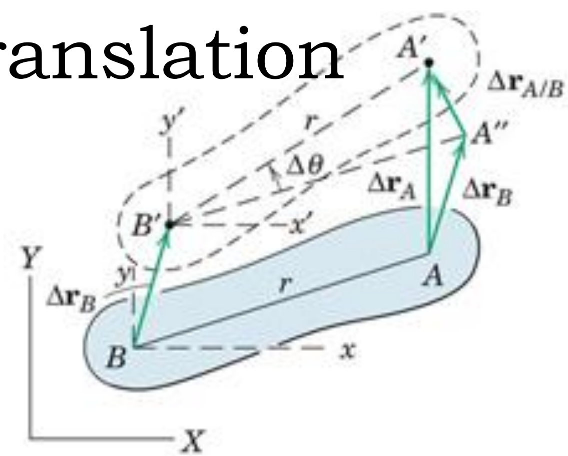


Analysis of General Plane Motion
Relative Motion Analysis
5/4 Relative Velocity

Recall relative velocity equation for two particles we have seen before:

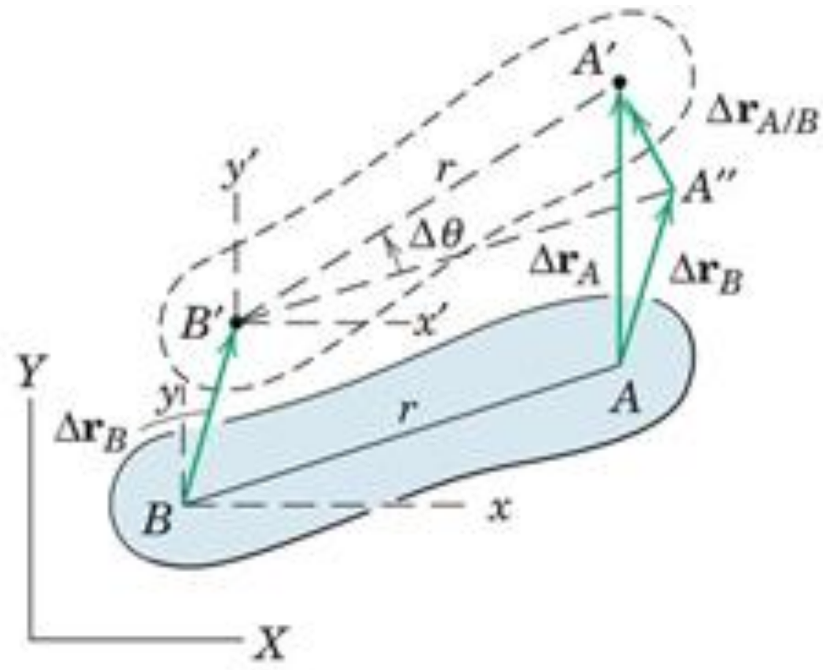
$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

Consider these two particles to be on the same rigid body. Also recall that general plane motion can be decomposed into a pure translation and a rotation.



Recall that general plane motion can be decomposed into a pure translation and a rotation.

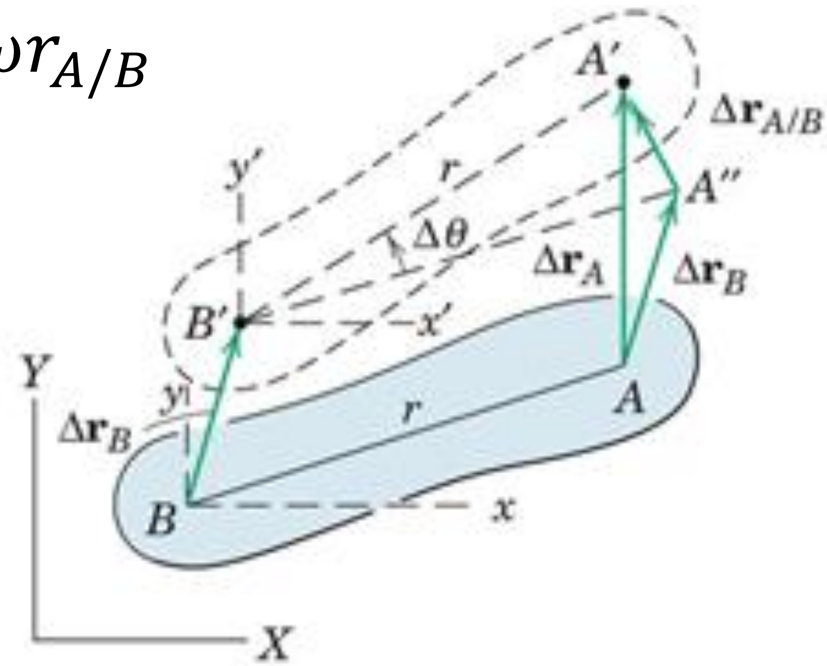
$$\Delta\vec{r}_A = \Delta\vec{r}_B + \Delta\vec{r}_{A/B}$$



Here $\Delta\vec{r}_{A/B}$ is the *rotation* of particle A around B.

Note that for a rigid body the distance between A and B which is $r_{A/B} = |\vec{r}_{A/B}|$, is constant.

$$v_{A/B} = \lim_{\Delta t \to 0} \frac{r_{A/B}\Delta\theta}{\Delta t} = \dot{\theta} r_{A/B} = \omega r_{A/B}$$



$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

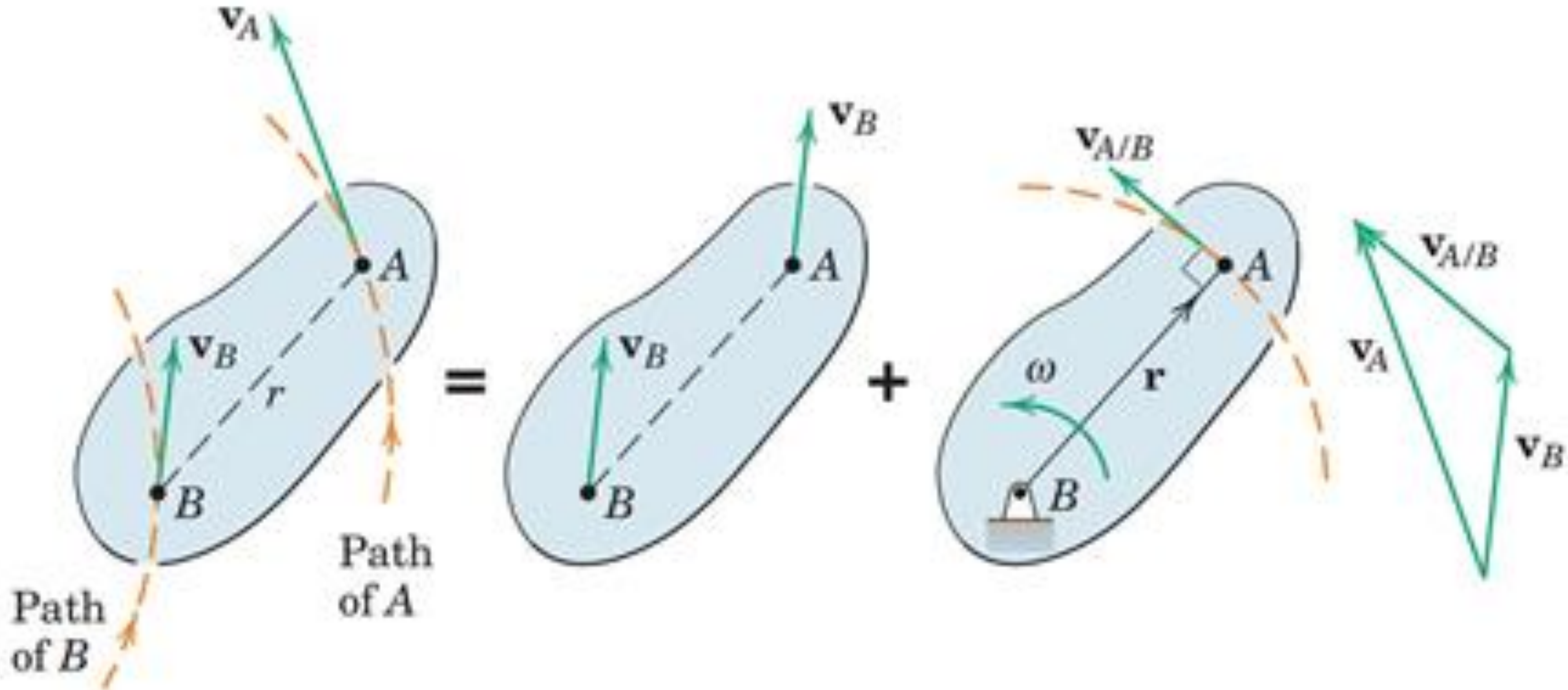
Using vector cross product:

$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}$

The relative velocity is **always perpendicular** to the relative position vector.

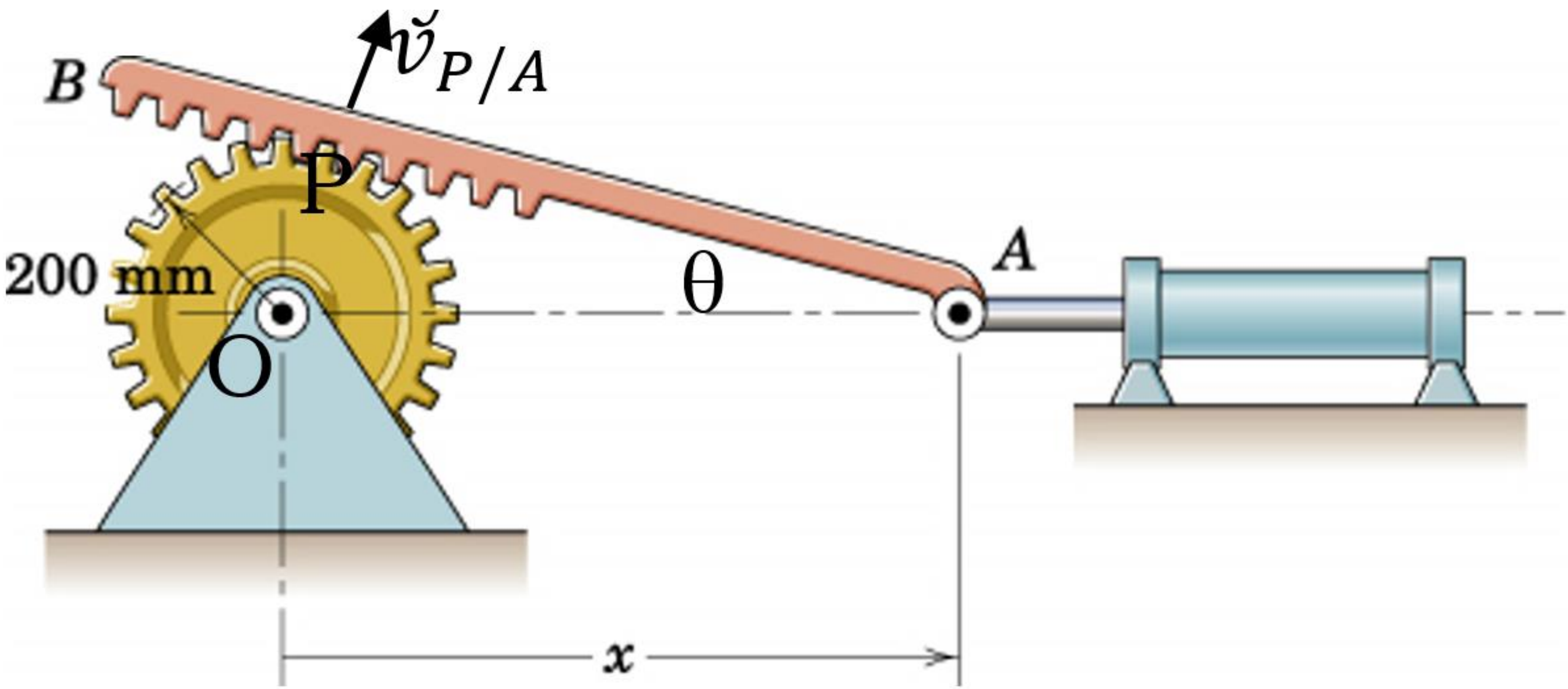
Please note that

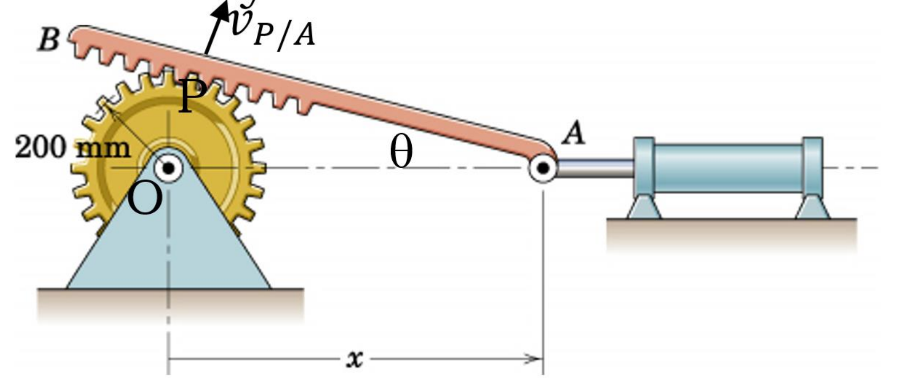
$\vec{v}_{A/B} = -\vec{v}_{B/A}$.



5/67 (4th), 5/72 (5th), 5/78 (6th), None (7th), 5/79 (8th)

The rotation of the gear is controlled by the horizontal motion of end A of the rack AB. If the piston rod has a constant velocity 300 mm/s during a short interval of motion, determine the angular velocity ω_{AB} of AB at the instant when x = 800 mm.





$$\vec{v}_P = \vec{v}_A + \vec{v}_{P/A}$$
$$\vec{v}_A = 0.3\hat{\imath}\, m/s$$
$$\vec{v}_P = v_P(cos\theta\hat{\imath} - sin\theta\hat{\jmath}), \perp OP$$
$$\vec{v}_{P/A} = v_{P/A}(sin\theta\hat{\imath} + cos\theta\hat{\jmath}), \perp AP$$
$$sin\theta = \frac{200}{800}, \theta = 14.48°$$

Substitution yields:

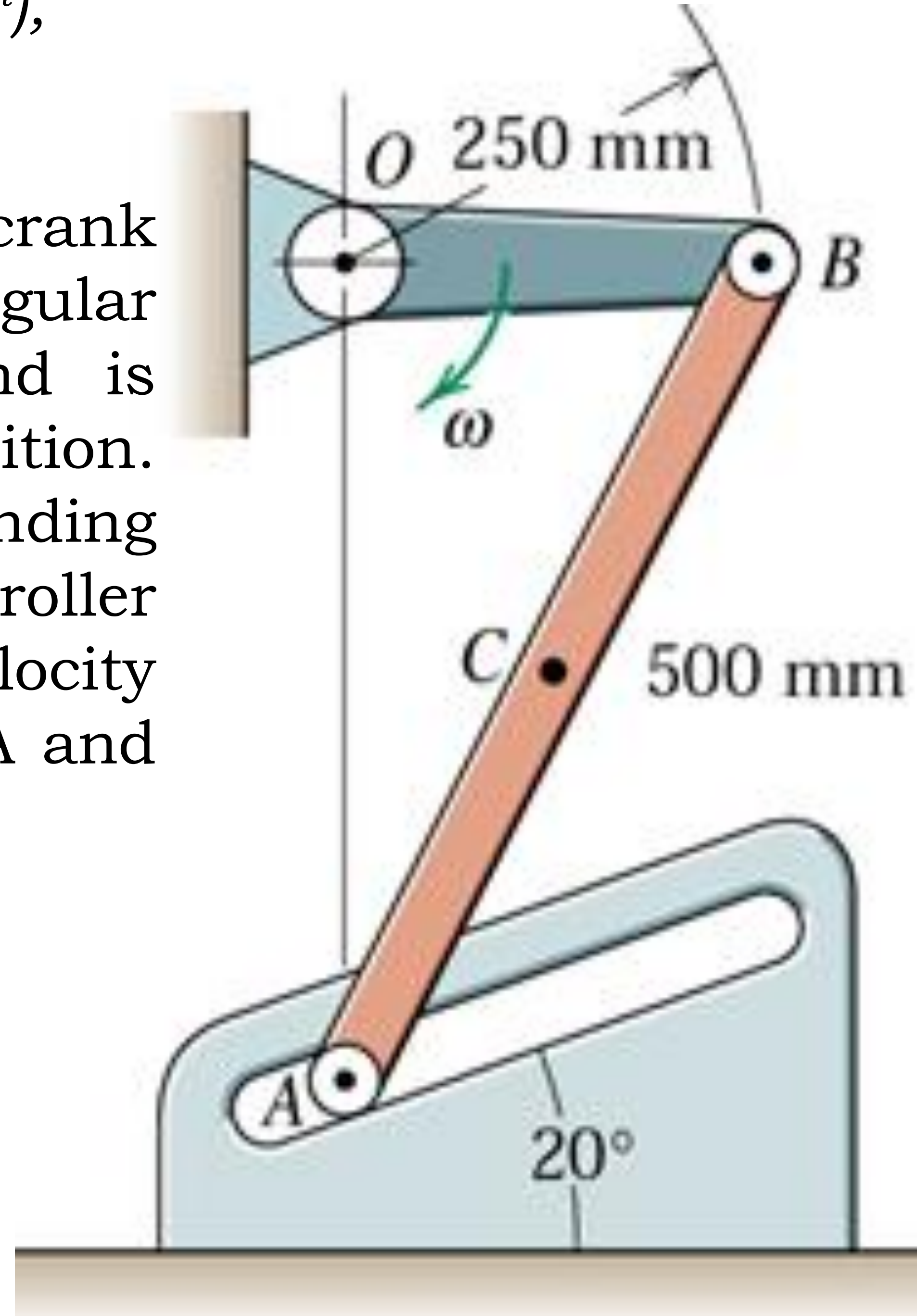
$$v_P(0.968\hat{\imath} - 0.250\hat{\jmath}) = 0.3\hat{\imath} + v_{P/A}(0.250\hat{\imath} + 0.968\hat{\jmath})$$

Solution yields:

$$v_P = 0.290 m/s$$
$$v_{P/A} = -0.0750\, m/s$$
$$\omega_{AB} = \frac{v_{P/A}}{r_{P/A}} = \frac{0.290}{\sqrt{0.8^2 - 0.2^2}} = 0.0968\, rad/s, CCW$$

5/69 (4th), 5/76 (5th), None (6th), 5/80 (7th), None (8th)

For the instant represented, crank OB has a clockwise angular velocity ω = 0.8 rad/s and is passing the horizontal position. Determine the corresponding angular velocity of the guide roller A in the 20° slot and the velocity of point C midway between A and B.



$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$\vec{v}_A = v_A(cos20°\hat{\imath} + sin20°\hat{\jmath})$

$\vec{v}_B = \omega_{OB} \times \vec{r}_{B/O} = -0.8\hat{k} \times 0.25\hat{\imath} = -0.200\hat{\jmath}\ m/s$

$\vec{v}_{A/B} = v_{A/B}(sin60°\hat{\imath} - cos60°\hat{\jmath}), \perp AB$

$v_A(cos20°\hat{\imath} + sin20°\hat{\jmath}) = -0.200\hat{\jmath} + v_{A/B}(sin60\hat{\imath} - cos60\hat{\jmath})$

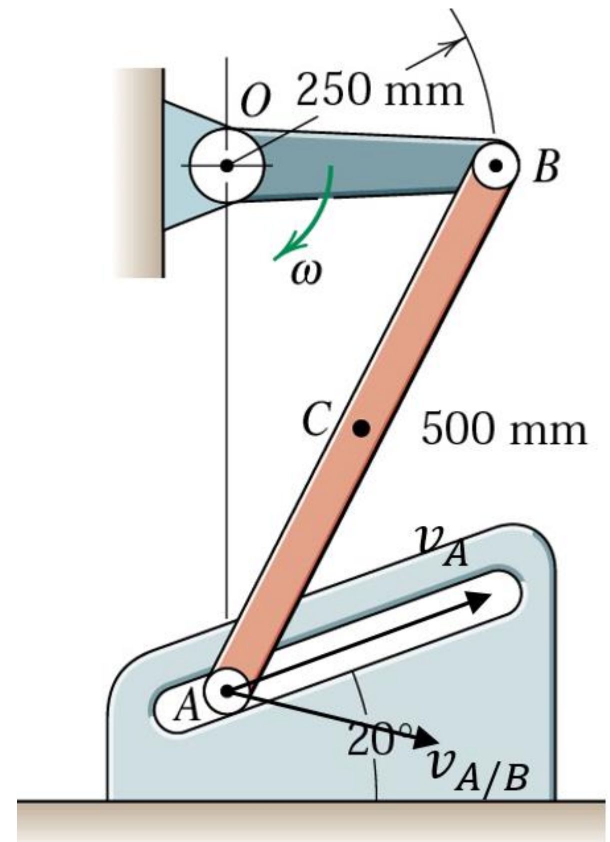
$\hat{\imath}$: $v_A cos20° = v_{A/B} sin60$

$\hat{\jmath}$: $v_A sin20° = -0.200 - v_{A/B} cos60°$

Solution yields:

$v_A = -0.226 m/s$

$v_{A/B} = -0.245 m/s$



$v_A = -0.226 m/s$
$v_{A/B} = -0.245 m/s$

$$\vec{v}_{A/B} = \vec{\omega}_{AB} \times \vec{r}_{A/B} = \omega_{AB}\hat{k} \times \left(-0.250\hat{\imath} - \sqrt{0.5^2 - 0.25^2}\hat{\jmath}\right)$$
$$= -0.245(sin60°\hat{\imath} - cos60°\hat{\jmath})$$

$\hat{\imath}$: $-0.245 sin60° = \omega_{AB}\sqrt{0.5^2 - 0.25^2}$

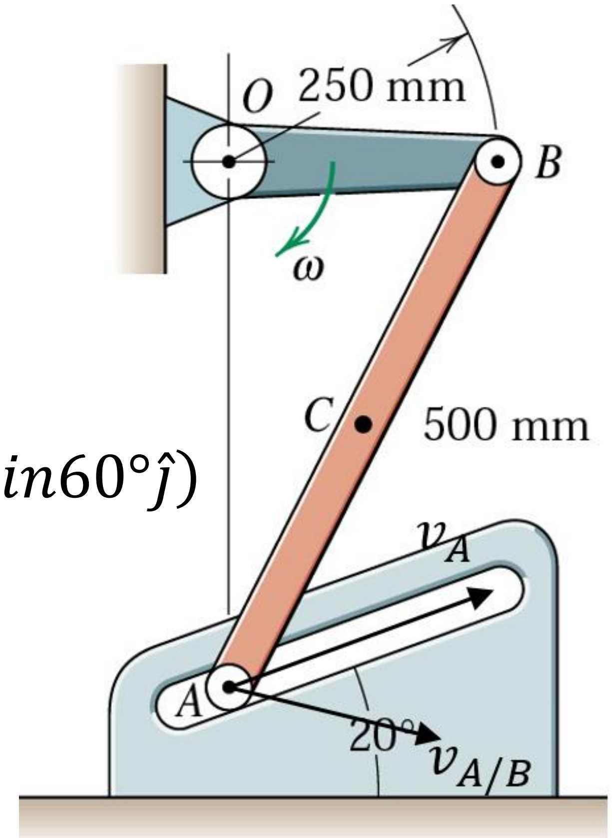
$\hat{\jmath}$: $0.245 cos60° = -\omega_{AB} 0.250$

Both equations yield identical answer:

$\omega_{AB} = -0.490\, rad/s, \vec{\omega}_{AB} = -0.490\hat{k}\, rad/s$

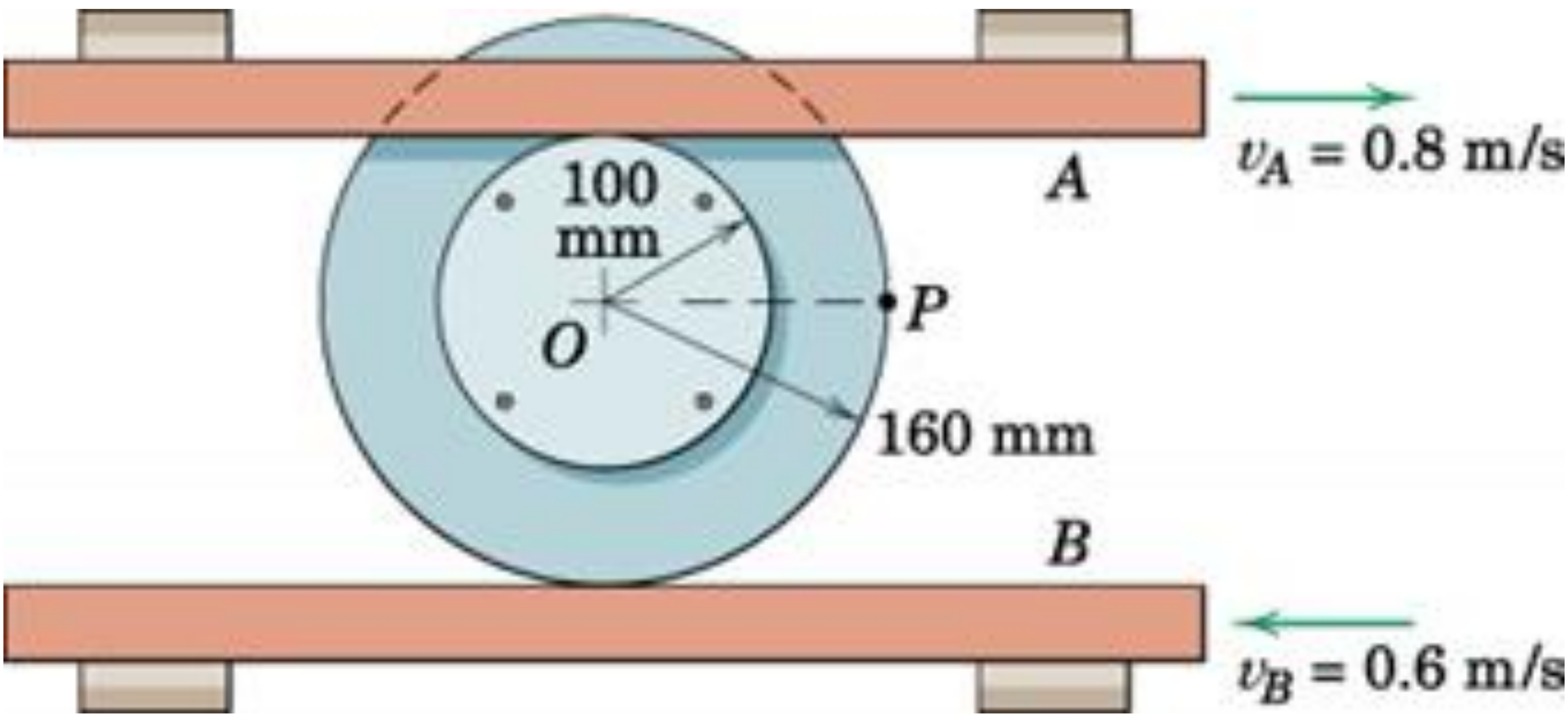
$$\vec{v}_C = \begin{cases} \vec{v}_A + \vec{v}_{C/A} = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{C/A} \\ \vec{v}_B + \vec{v}_{C/B} = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{C/B} \end{cases}$$

$$\vec{v}_C = -0.200\hat{\jmath} - 0.490\hat{k} \times 0.250(-cos60°\hat{\imath} - sin60°\hat{\jmath})$$
$$= (-0.1061\hat{\imath} - 0.1388\hat{\jmath})\, m/s$$



5/72 (4th), None (5th), 5/75 (6th), 5/75 (7th), None (8th)

Each of the sliding bars A and B engages its respective rim of the two riveted wheels without slipping. Determine the magnitude of velocity of point P for the position shown.

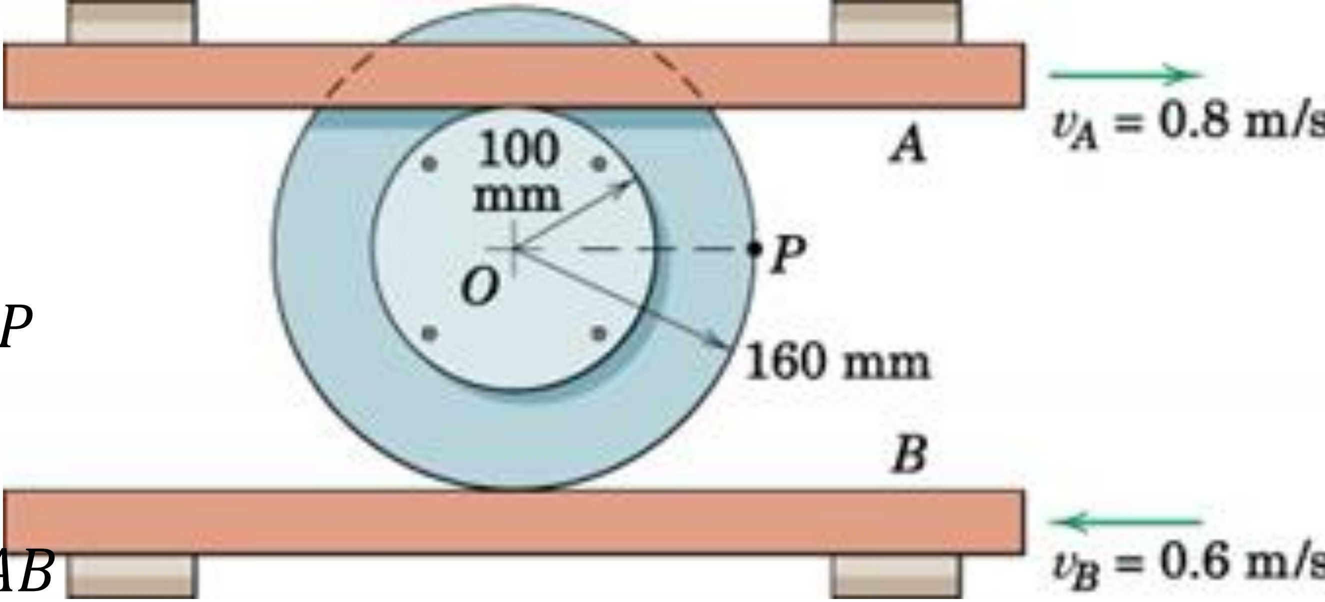


$\vec{v}_P = \vec{v}_A + \vec{v}_{P/A}$

$\vec{v}_A = 0.8\hat{\imath}\, m/s$

$\vec{v}_P = v_P??$

$\vec{v}_{P/A} = \vec{\omega} \times \vec{r}_{P/A}, \perp AP$



$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\vec{v}_B = -0.6\hat{\imath}\, m/s$

$\vec{v}_{B/A} = \vec{\omega} \times, \vec{r}_{B/A} \;\perp AB$

$-0.6\hat{\imath} = 0.8\hat{\imath} + \omega\hat{k} \times, -0.26\hat{\jmath}$

$\hat{\imath}$: $-0.6 = 0.8 + 0.26\omega, \omega = -5.38\, rad/s$

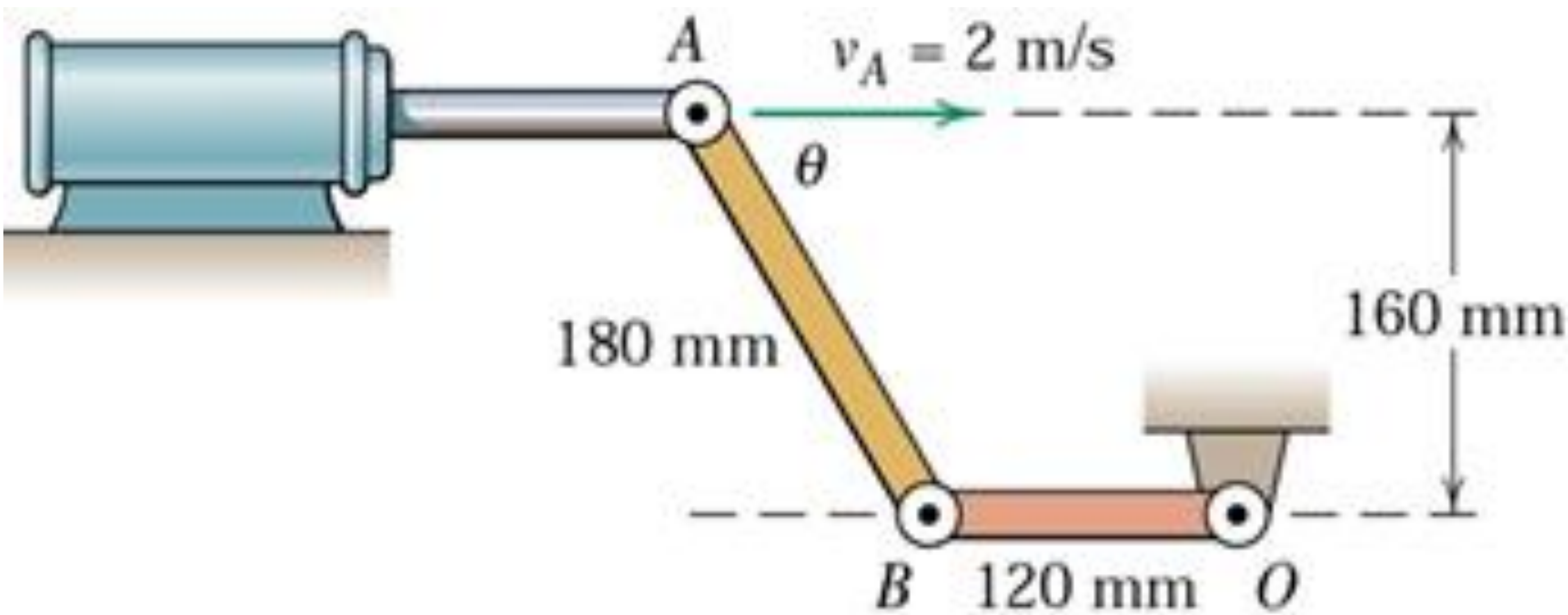
$\hat{\jmath}: 0 = 0$

$\vec{r}_{P/A} = (0.16\hat{\imath} - 0.1\hat{\jmath})$

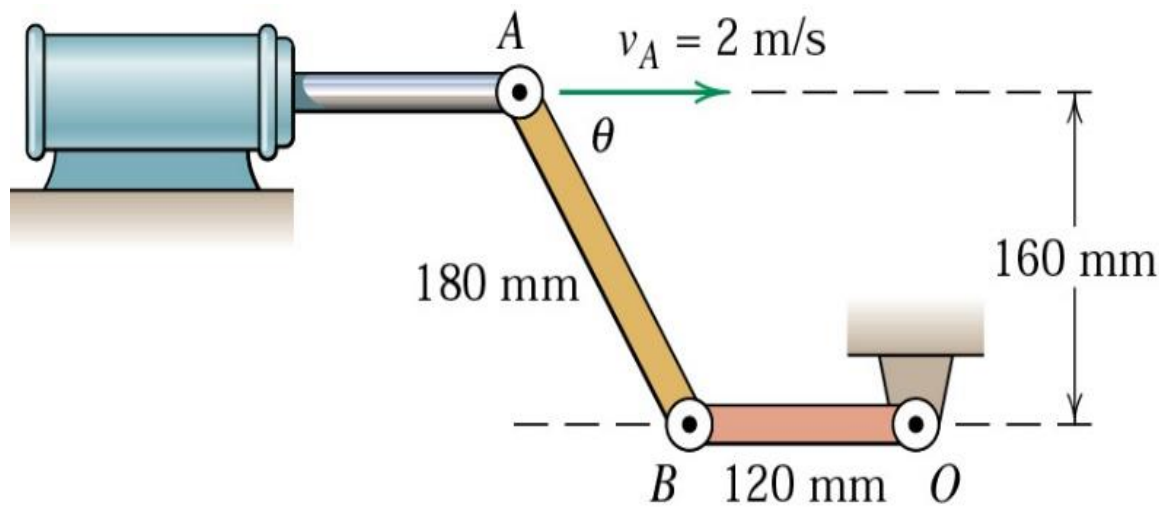
$\vec{v}_P = 0.8\hat{\imath} - 5.35\hat{k} \times (0.16\hat{\imath} - 0.1\hat{\jmath}) = (0.265\hat{\imath} - 0.856\hat{\jmath})\, m/s$

5/76 (4th), 5/79 (5th), None (6th), 5/83 (7th), None (8th)

Horizontal motion of the piston rod of the hydraulic cylinder controls the rotation of link OB about O. For the instant represented $v_A = 2$ m/s and OB is horizontal. Determine the angular velocity ω of OB for this instant.

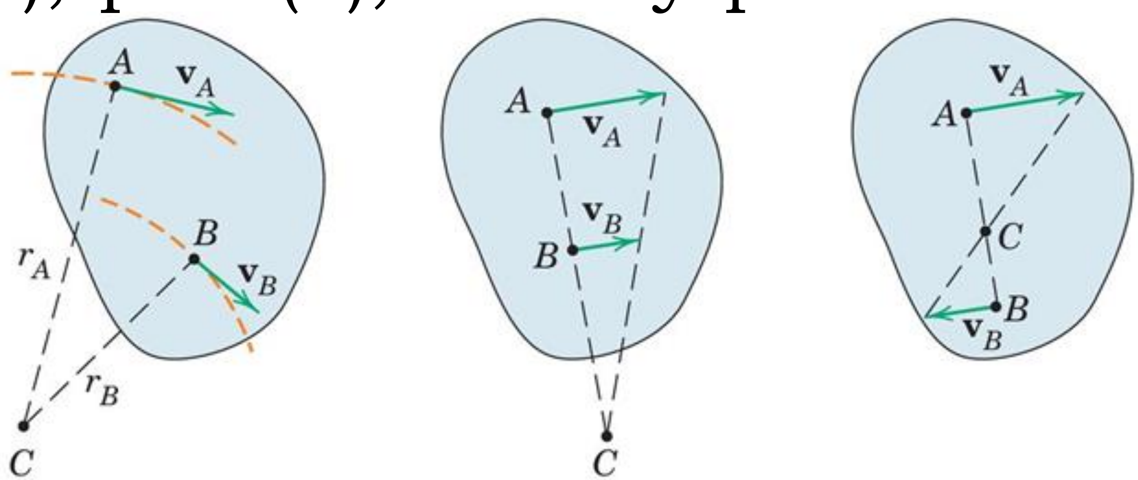


$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$
$$\vec{v}_B = \vec{\omega}_{OB} \times \vec{r}_{B/O} = \omega_{OB}\hat{k} \times (-0.1200\hat{\imath}) = -0.12\omega_{OB}\hat{\jmath}$$
$$\vec{v}_A = 2\hat{\imath}\ m/s$$
$$\vec{v}_{B/A} = \vec{\omega}_{AB} \times \vec{r}_{B/A} = \omega_{AB}\hat{k} \times \left(\sqrt{0.18^2 - 0.16^2}\hat{\imath} - 0.16\hat{\jmath}\right)$$
$$-0.12\omega_{OB}\hat{\jmath} = 2\hat{\imath} + \omega_{AB}\hat{k} \times \left(\sqrt{0.18^2 - 0.16^2}\hat{\imath} - 0.16\hat{\jmath}\right)$$
$$\hat{\imath}: 0 = 2 + 0.16\omega_{AB}$$
$$\omega_{AB} = -12.5\ rad/s$$
$$\hat{\jmath}: -0.12\omega_{OB} = -12.5\sqrt{0.18^2 - 0.16^2}$$
$$\omega_{OB} = 8.59\ rad/s$$



Analysis of General Plane Motion
Relative Motion Analysis
5/5 Instantaneous Center of Zero Velocity

If velocity vectors of two points on a rigid body are known then it is possible to find a point on the body which *instantly* has zero velocity. This point is known as instantaneous center of zero velocity (ICZV), pole (P), velocity pole or centrode (C).

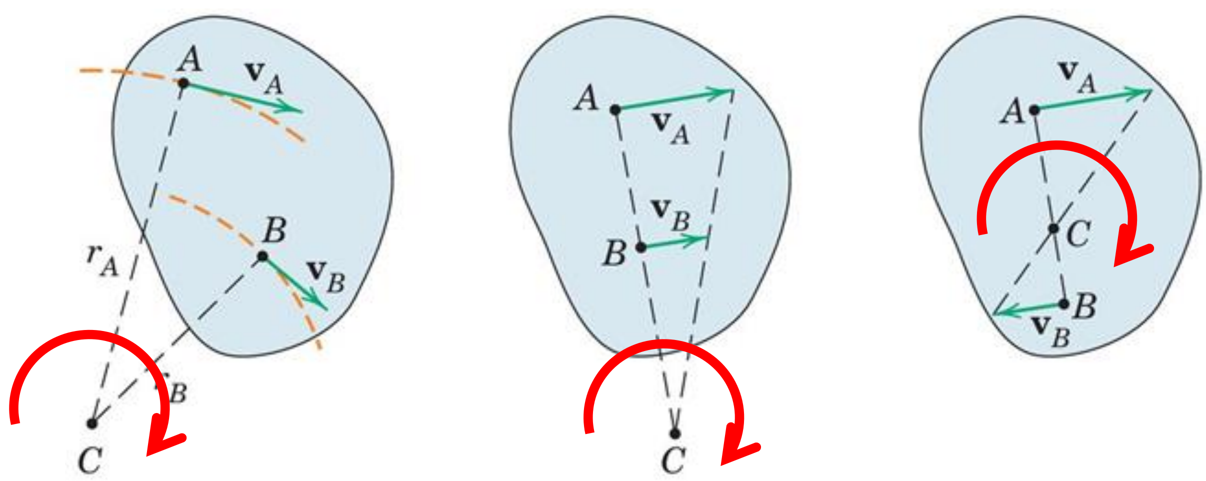


Please note that for general plane motion the location of this point changes in time and as we will see later, in general ICZV has a non-zero acceleration.

$$\omega = \frac{v_A}{r_{A/ICZV}} = \frac{v_B}{r_{B/ICZV}} = \frac{v_C}{r_{C/ICZV}} = \cdots = \frac{v_.}{r_{./ICZV}}$$

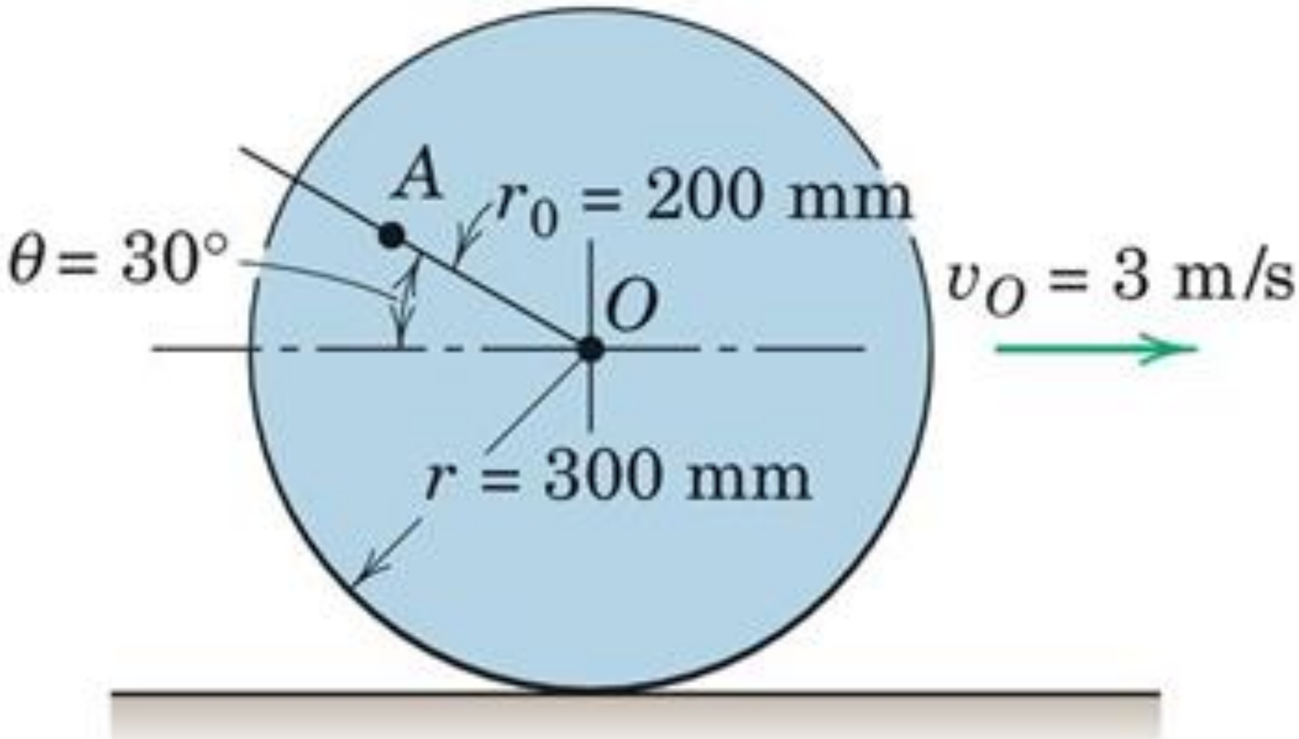
Direction of ω is determined by inspection since mostly the velocity analysis by instant centers is performed graphically. However one may utilize the vector expression as well:

$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/ICZV}$$



Sample Problem 5/11

The wheel rolls to the right without slipping, with its center O having a velocity $v_O = 3$ m/s. Locate the instantaneous center of zero velocity and use it to find the velocity of point A for the position indicated.



Since it is rolling without slipping point C which is momentarily in contact with ground is ICZV. Please recognize that location of contact point changes as the wheel rolls and *recall it has non-zero acceleration from Sample Problem 5/4.*

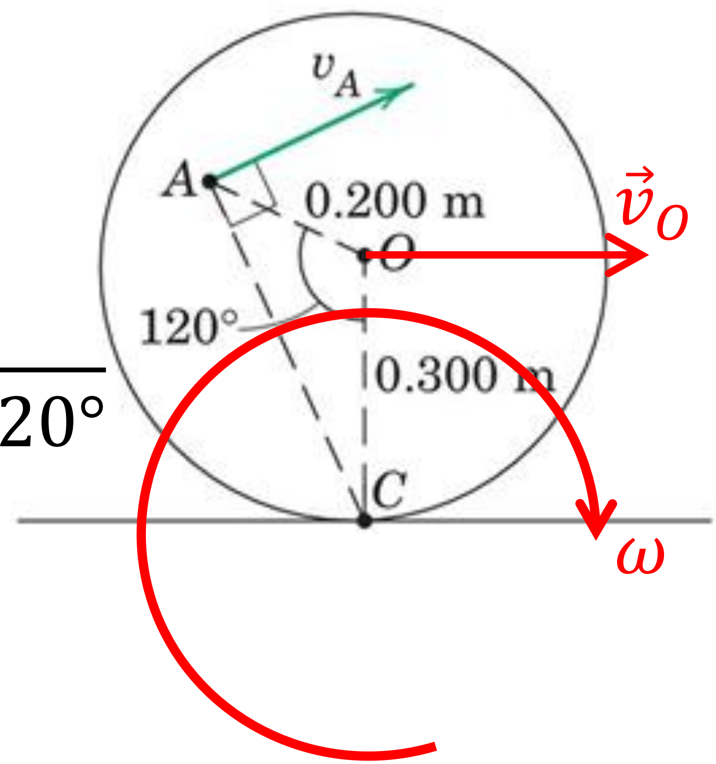
Geometric Approach (as in book)

$$\omega = \frac{v_O}{r_{O/C}} = \frac{3}{0.3} = 10\,rad/s\,(CW)$$

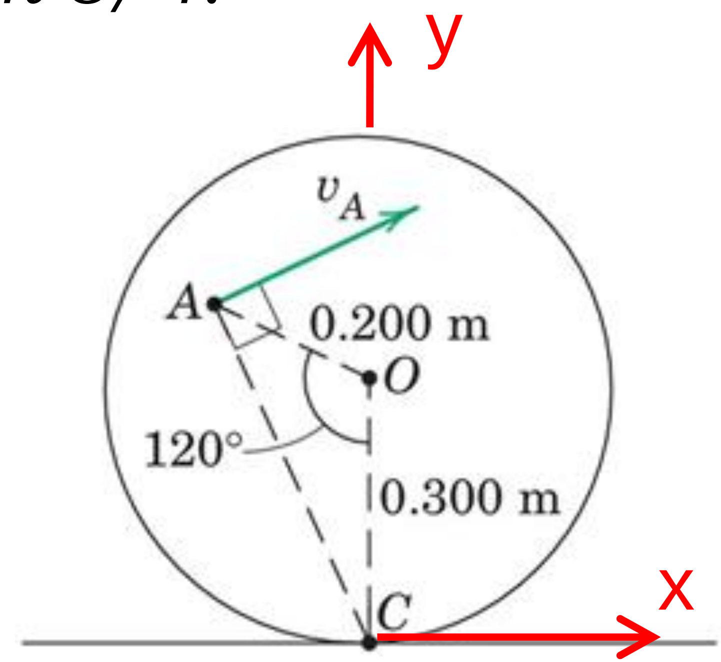
$$v_A = |AC|\omega$$

$$= \sqrt{0.3^2 + 0.2^2 - 2 * 0.3 * 0.2 * cos120°}$$

$$* 10 = 4.36\,m/s \perp AC$$



Since it is rolling without slipping point C which is momentarily in contact with ground is ICZV. Please recognize that location of contact point changes as the wheel rolls and *recall it has non-zero acceleration from Sample Problem 5/4.*



Vectorial Approach (alternative)

$$\vec{v}_O = \vec{\omega} \times \vec{r}_{O/ICZV}$$

$$3\hat{\imath} = \omega\hat{k} \times 0.3\hat{\jmath}$$

$$\omega = -10\, rad/s$$

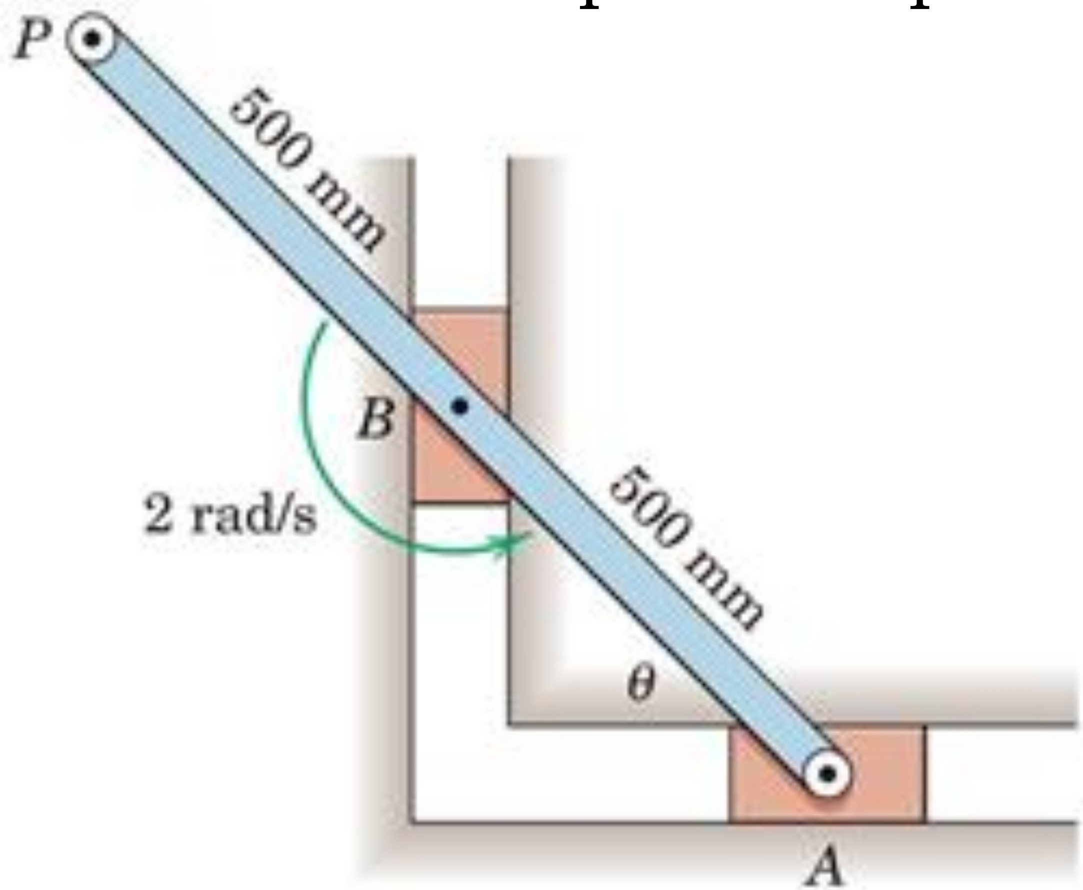
$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/ICZV}$$

$$= -10\hat{k} \times [-0.2cos30°\hat{\imath} + (0.3 + 0.2sin30°)\hat{\jmath}]$$

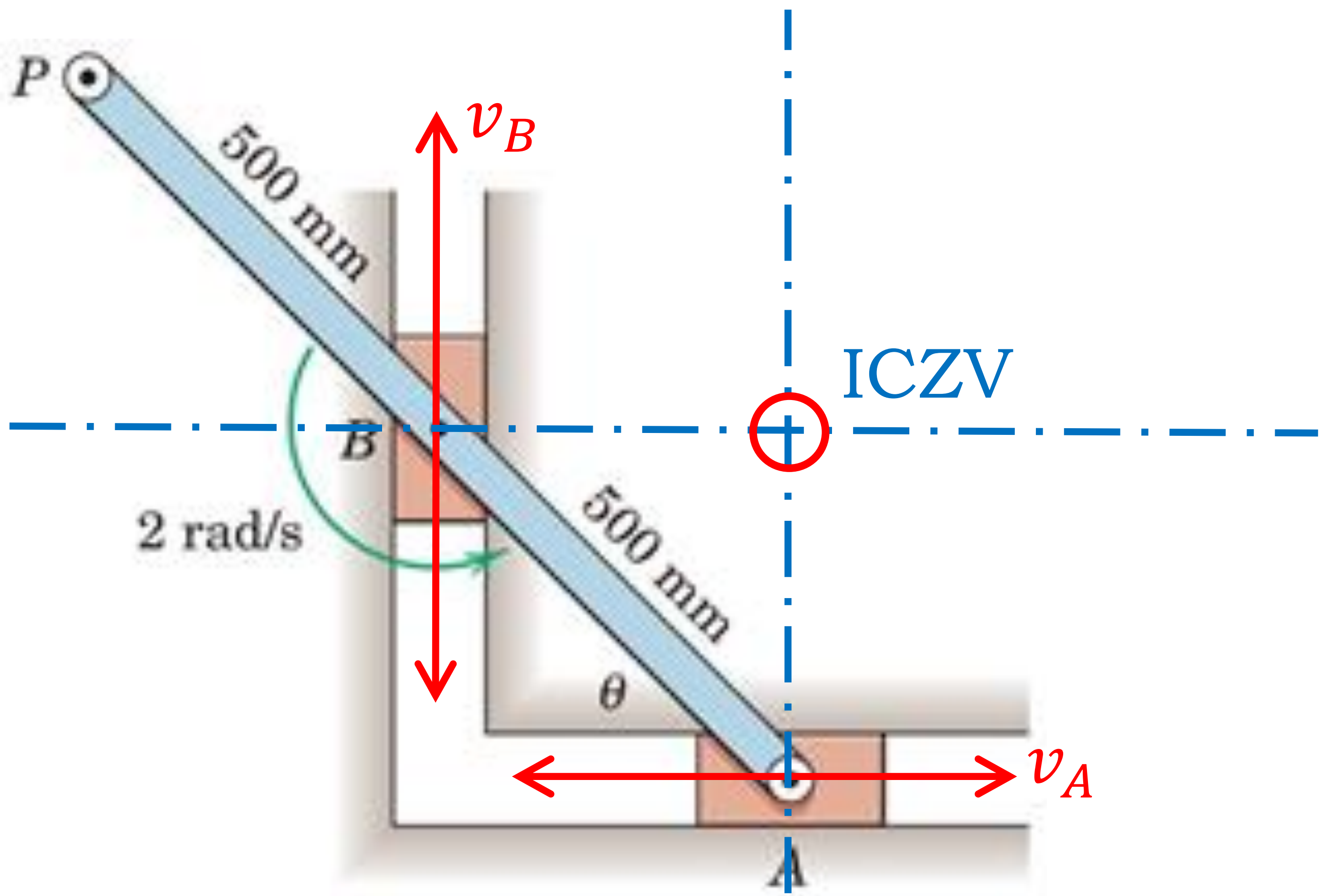
$$= (4\hat{\imath} + 1.732\hat{\jmath})\, m/s$$

Example 5/93 (4th), 5/95 (5th), 5/100 (6th), 5/100 (7th), 5/103 (8th)

Motion of the bar is controlled by the constrained paths of A and B. If the angular velocity of the bar is 2 rad/s counterclockwise as the position $\theta = 45°$ is passed, determine the speeds of points A, B and P.



v_A is along the horizontal slider,
v_B is along the vertical slider.
ICZV is on a line $\perp$ to v_A drawn through A
ICZV is on a line $\perp$ to v_B drawn through B
ICZV is at the intersection of these two lines!



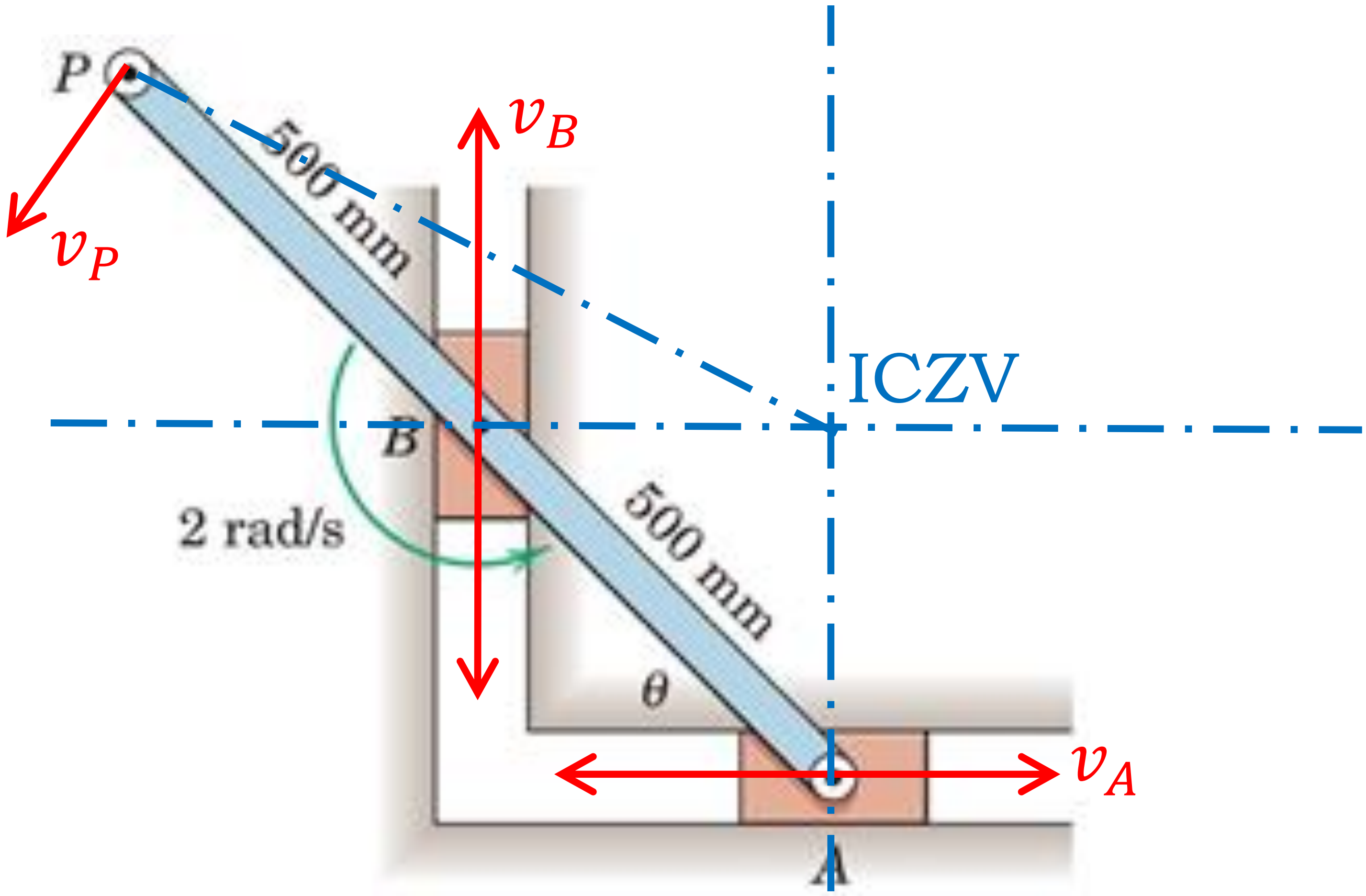
$$v_A = |A\ ICZV|\omega = \frac{0.5}{\sqrt{2}}2 = 0.707m/s \rightarrow$$

$$v_B = |B\ ICZV|\omega = \frac{0.5}{\sqrt{2}}2 = 0.707\ m/s\ \downarrow$$

$$v_P = |P\ ICZV|\omega$$

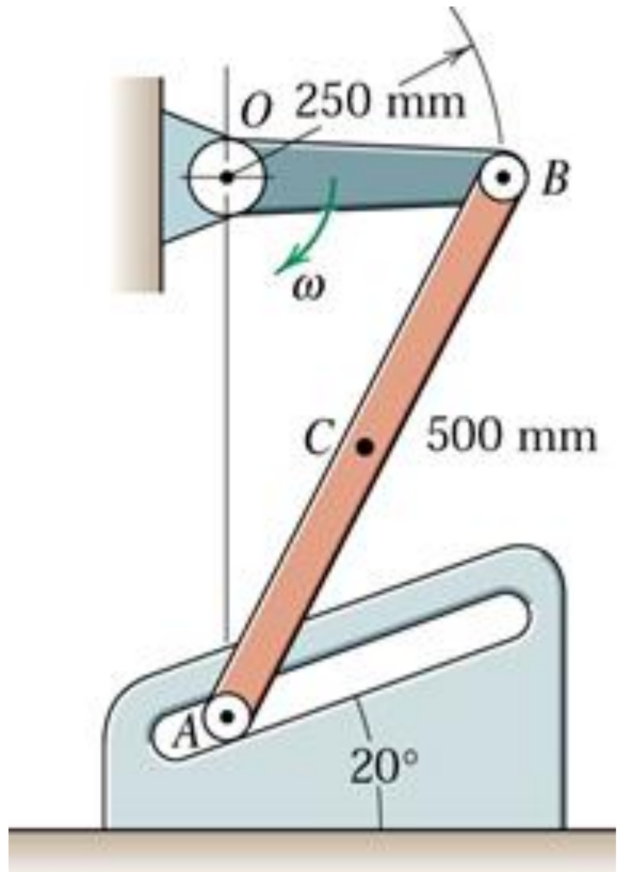
$$|P\ ICZV| = \sqrt{0.353^2 + 0.5^2 - 2 * 0.5 * 0.353cos135°} = 0.790\ m$$

$$v_P = 0.790 * 2 = 1.580\ m/s\ \perp\ |P\ ICZV| \swarrow$$

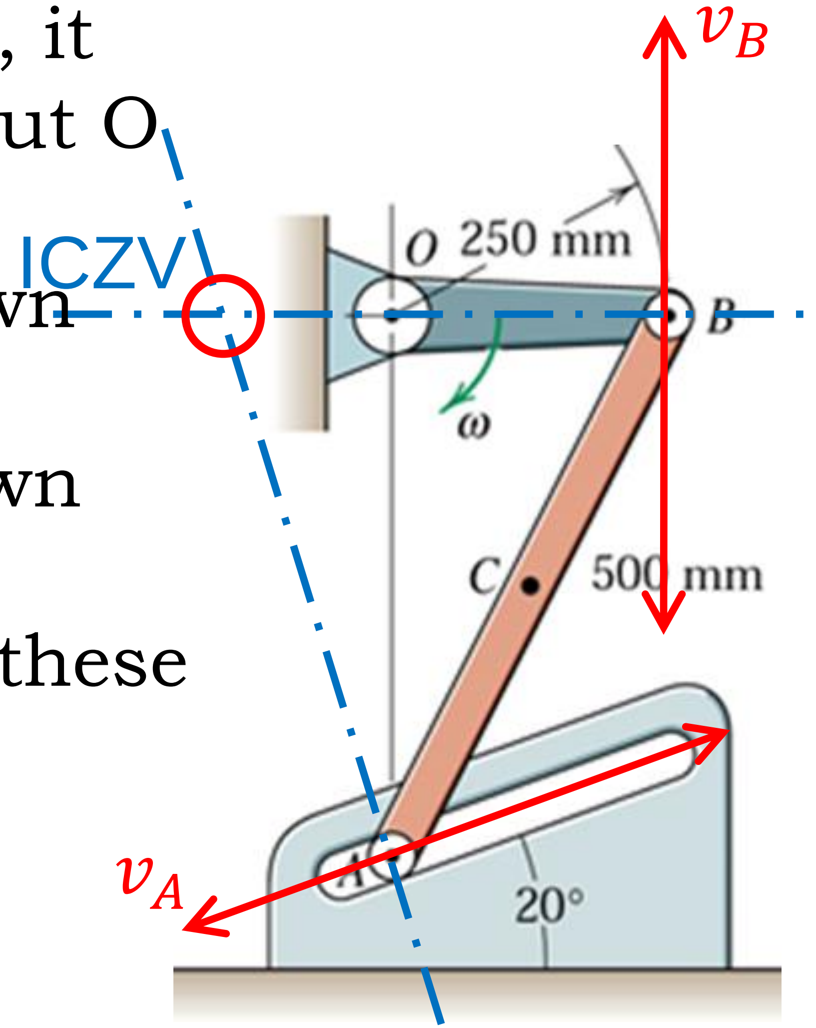


Example 5/95 (4th), 5/97 (5th), None (6th), 5/99 (7th), 5/99 (8th)

At the instant represented, crank OB has a clockwise angular velocity ω = 0.8 rad/s and is passing the horizontal position. By using instantaneous center of zero velocity determine the corresponding velocity of the guide roller A in the 20° slot and the velocity of point C midway between A and B.



v_A is along the slot,
Consider point B on body OB, it makes fixed axis rotation about O so v_B is $\perp$ to OB.
ICZV is on a line $\perp$ to v_A drawn through A
ICZV is on a line $\perp$ to v_B drawn through B
ICZV is at the intersection of these two lines!



$$|A\ ICZV| = \frac{0.5cos30°}{cos20°} = 0.461\,m$$

$$|B\ ICZV| = 0.461sin20° + 0.25 = 0.408\,m$$

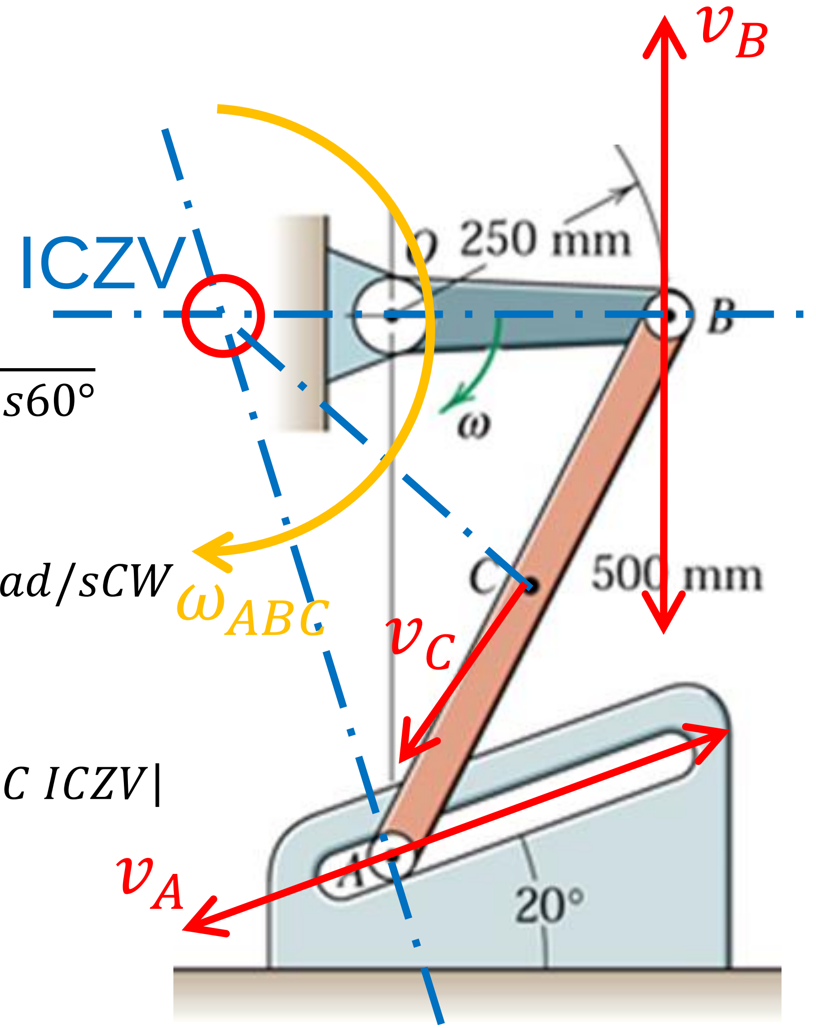
$$|C\ ICZV| = \sqrt{|B\ ICZV|^2 + |BC|^2 - 2|B\ ICZV||BC|cos60°}$$

$$= 0.356\,m$$

$$\omega_{ABC} = \frac{v_B}{|B\ ICZV|} = \frac{\omega|OB|}{|B\ ICZV|} = \frac{0.8 * 0.25}{0.408} = 0.490rad/sCW$$

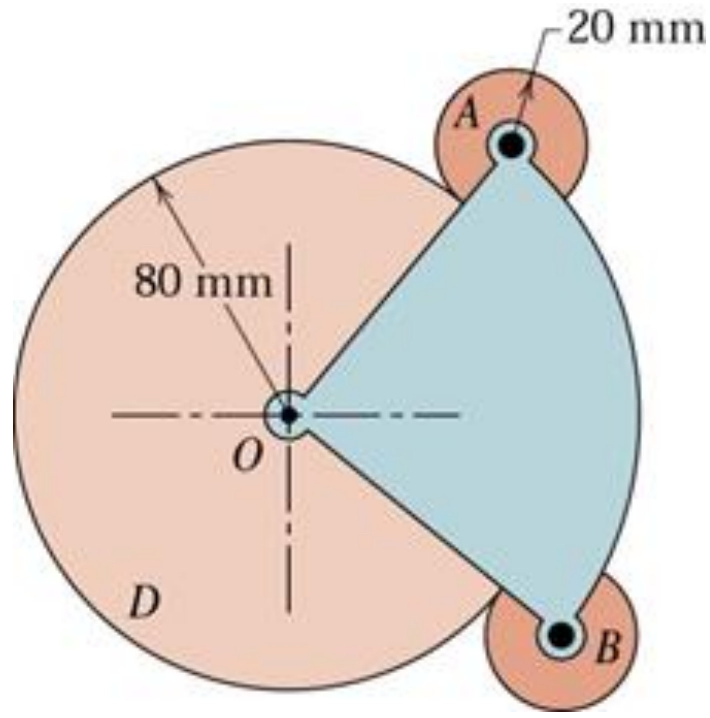
$$v_A = |A\ ICZV|\omega = 0.461 * 0.490 = 0.226\,m/s \swarrow$$

$$v_C = |C\ ICZV|\omega = 0.356 * 0.490 = 0.1747\,m/s \perp\ |C\ ICZV|$$



Example 5/101 (4th), 5/108 (5th), 5/116 (6th), None (7th), None (8th)

The gear D (teeth not shown) rotates clockwise about O with a constant angular velocity of 4 rad/s. The 90° sector AOB is mounted on an independent shaft at O, and each of the small gears at A and B meshes with gear D. If the sector has a counterclockwise angular velocity of 3 rad/s at the instant represented, determine the corresponding angular velocity ω of each of the small gears.

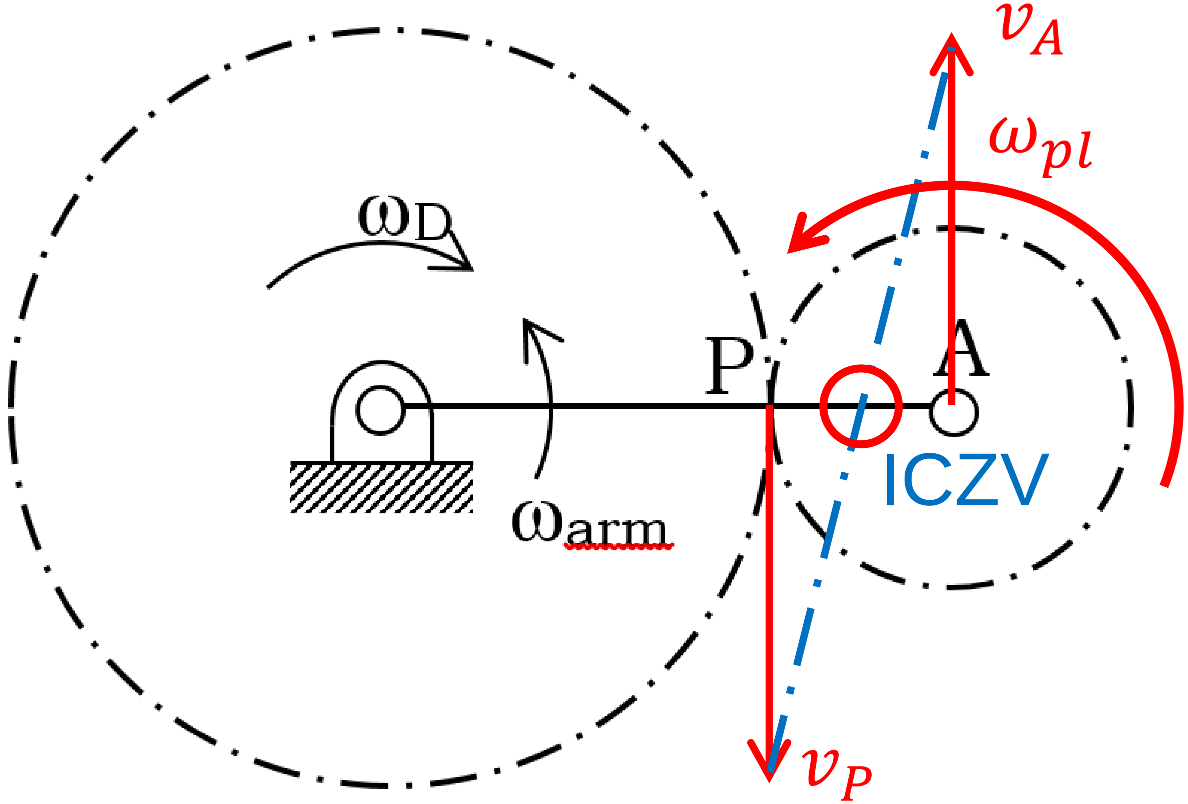


This is a planetary gear drive. At the center we have the sun gear, around we have three planets and the arm (blue sector in the question) is behind the planets connecting the centers of the planets to the center of the sun gear by pin joints. What is extra here is the so called *ring gear* which has internal teeth. Just watch how the planets move during different motions of the sun and the ring gears (i.e. how their angular velocities change).

$$v_P = |OP|\omega_D = 0.08 * 4 = 0.320\, m/s \downarrow$$

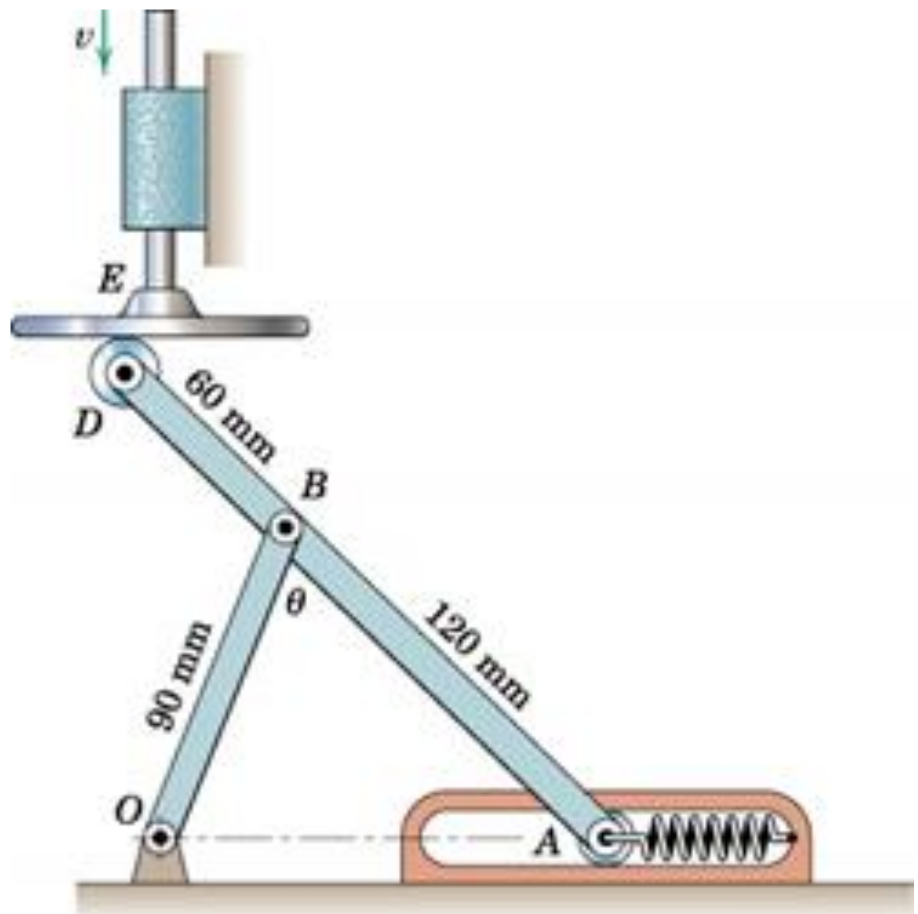
$$v_A = |OA|\omega_{arm} = 0.1 * 3 = 0.3\, m/s \uparrow$$

$$\omega_{pl} = \frac{v_P - v_A}{|PA|} = \frac{0.32 - (-0.3)}{0.02} = 31\, rad/s\ CCW$$



Example 5/112 (4^{th}), 5/114 (5^{th}), 5/119 (6^{th}), 5/116 (7^{th}), 5/118 (8^{th})

Motion of the roller A against its restraining spring is controlled by the downward motion of the plunger E. For an interval of motion the velocity of E is v = 0.2 m/s. Determine the velocity of A when θ becomes 90°.



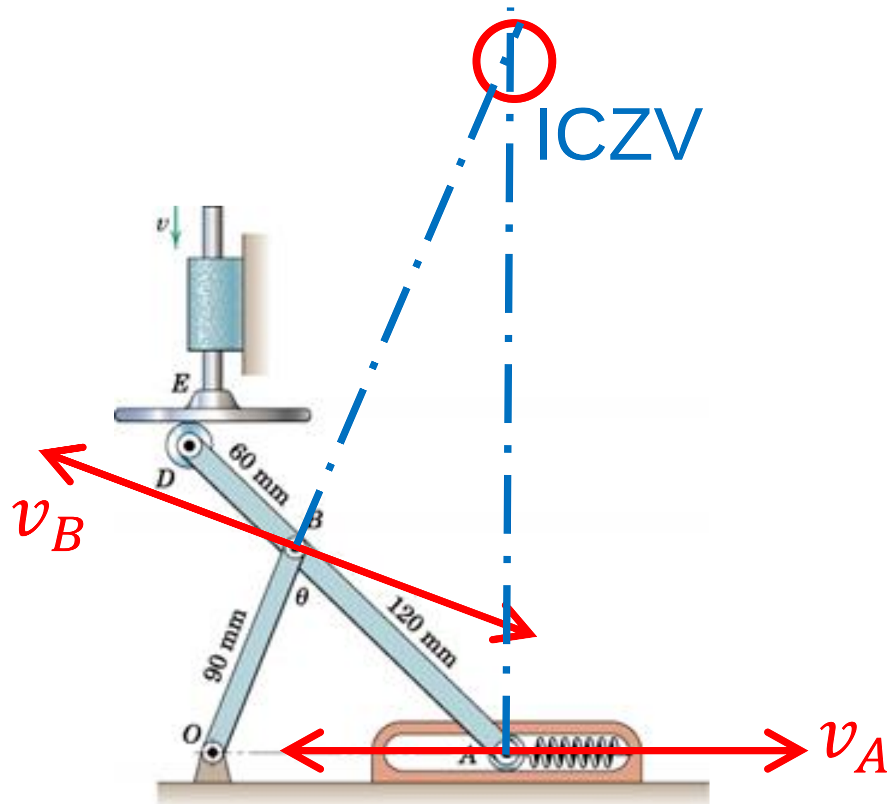
v_A is along the slot,

Consider point B on body OB, it makes fixed axis rotation about O so v_B is $\perp$ to OB.

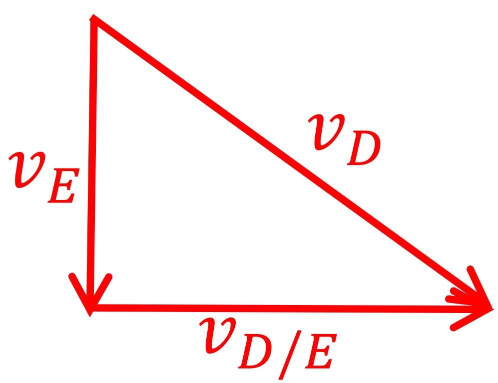
ICZV is on a line $\perp$ to v_A drawn through A

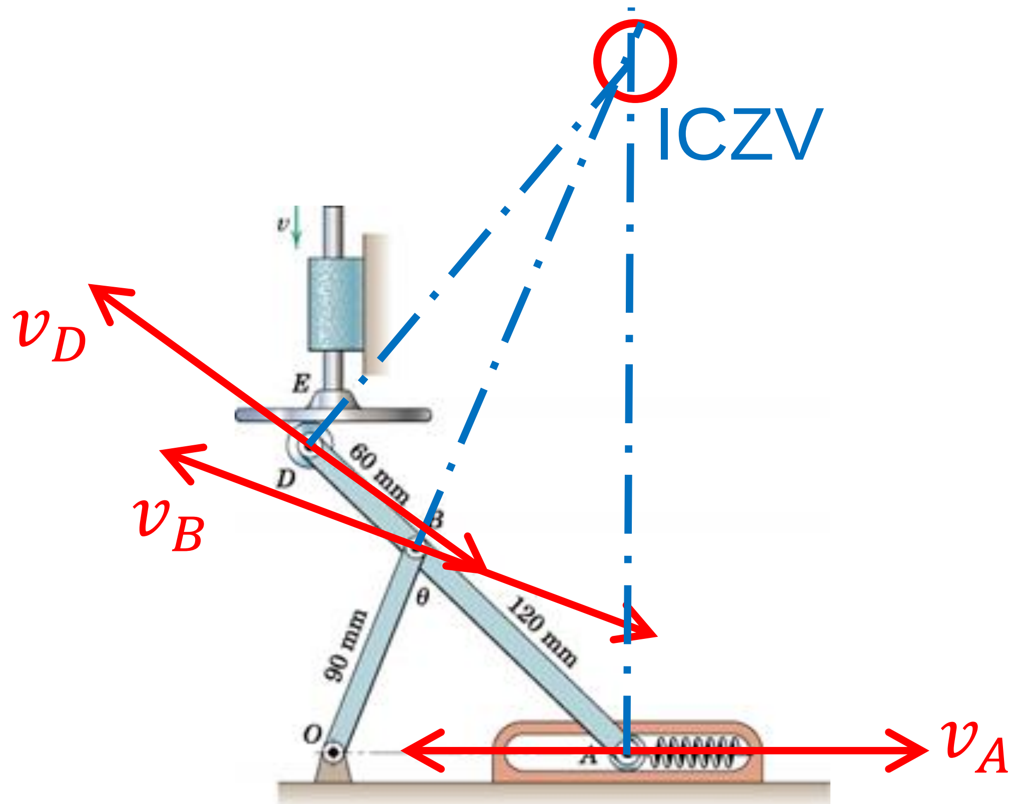
ICZV is on a line $\perp$ to v_B drawn through B

ICZV is at the intersection of these two lines!



v_D is $\perp$ to D ICZV line,
v_E is downwards
therefore the vertical component of v_D should be equal to v_E and there is a relative motion of D with respect to E which is sliding by the roller which is horizontal!





Using trigonometry:

$$|A\ ICZV| = \frac{5}{3} * 120 = 200\ mm$$

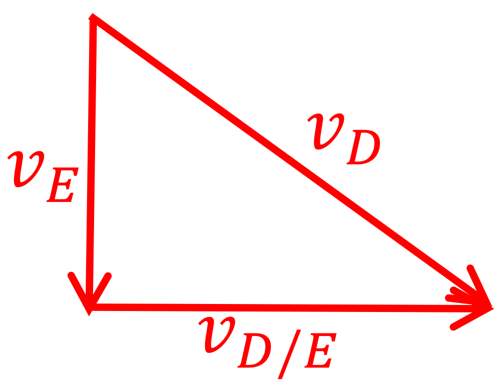
$$|B\ ICZV| = 160\ mm$$

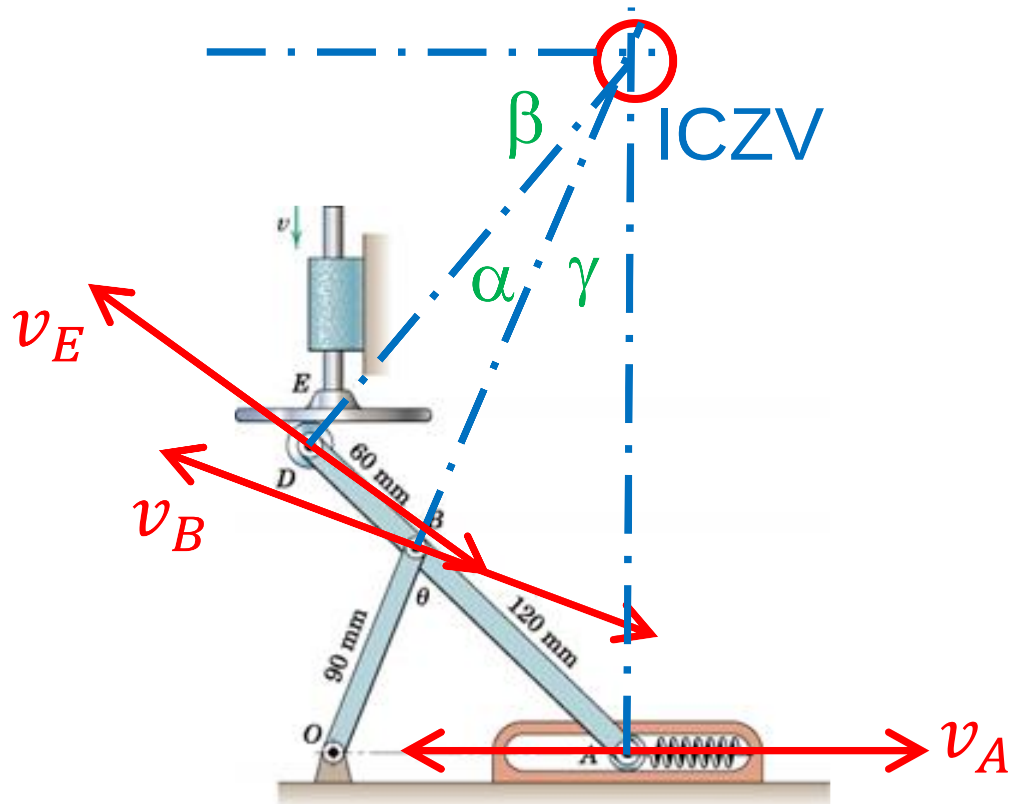
$$|D\ ICZV| = \sqrt{60^2 * 160^2} = 170.9\ mm$$

$$\gamma = sin^{-1}\frac{120}{200} = 36.9°$$

$$\alpha = tan^{-1}\frac{60}{160} = 20.6°$$

$$\beta = 90° - \alpha - \gamma = 32.6°$$





$$v_D = \frac{v}{cos\beta} = 0.327\ m/s$$

$$\omega_{ABD} = \frac{v_D}{|D\ ICZV|} = \frac{v_A}{|A\ ICZV|}\ (CCW)$$

$$v_A = 0.278\ m/s \rightarrow$$