ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

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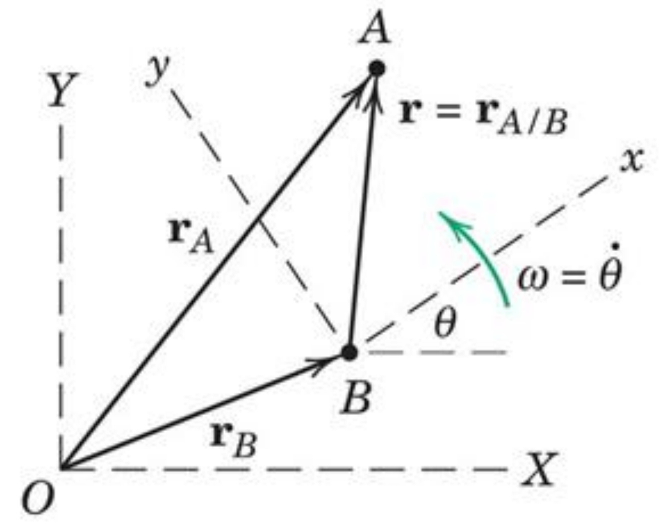
Relative Velocity

$$\vec{r}_A = \vec{r}_B + (x\hat{\imath} + y\hat{\jmath})$$

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + (\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}) + \boxed{(x\dot{\hat{\imath}} + y\dot{\hat{\jmath}})}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + (x\dot{\hat{\imath}} + y\dot{\hat{\jmath}})$$



$$\dot{\hat{\imath}} = \vec{\omega} \times \hat{\imath}$$

$$\dot{\hat{\jmath}} = \vec{\omega} \times \hat{\jmath}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + (x\vec{\omega} \times \hat{\imath} + y\vec{\omega} \times \hat{\jmath})$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + \vec{\omega} \times (x\hat{\imath} + y\hat{\jmath})$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{rel}$$

$$\boxed{\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}}$$

$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$

Interpretation of Terms

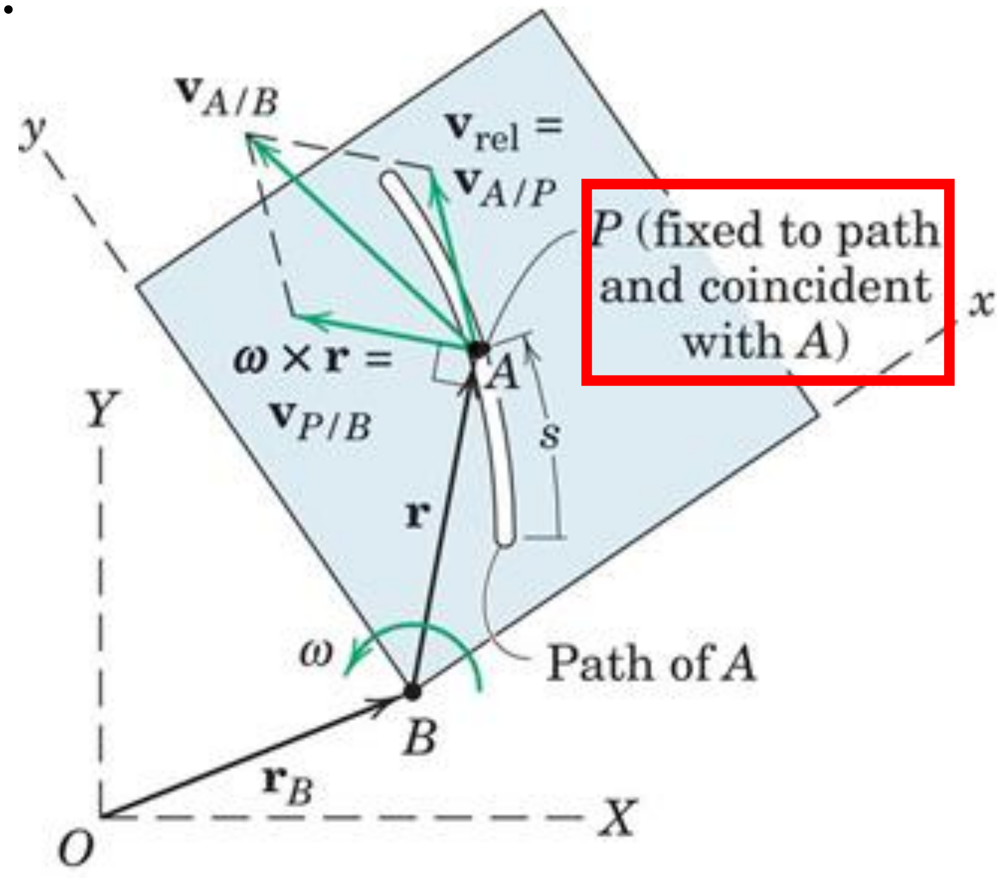
Assume the moving coordinate system x-y be formed of a plate with a slot in it where particle A can move in relative to the moving coordinate system.

Assume a point P, *instantly coincident* with particle A fixed on the moving plane.

$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$

$\vec{v}_A = \underbrace{\vec{v}_B + \vec{v}_{P/B}} + \vec{v}_{A/P}$

$\vec{v}_A = \vec{v}_P + \vec{v}_{A/P}$



Transformation of a Time Derivative (Transport or Coriolis Theorem)

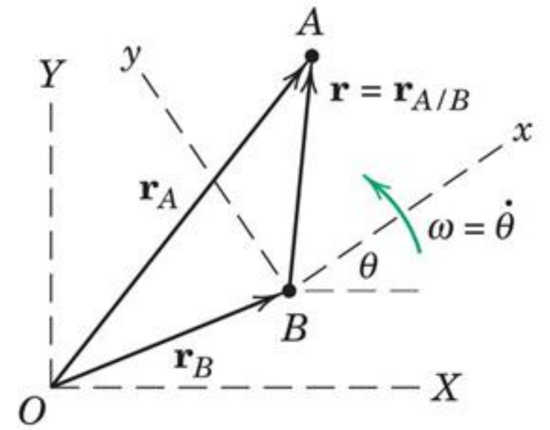
Let $\vec{V}$ be any vector quantity

$$\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}$$

The time derivative of this quantity in X-Y coordinates is

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \underbrace{\left(\dot{V}_x\hat{\imath} + \dot{V}_y\hat{\jmath}\right)} + \underbrace{\left(V_x\dot{\hat{\imath}} + V_y\dot{\hat{\jmath}}\right)}$$

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \left(\frac{d\vec{V}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{V}$$



The term $\vec{\omega} \times \vec{V}$ is the difference between the time derivative of $\vec{V}$ in the rotating (x-y) and non-rotating (X-Y) coordinate frames.

Relative Acceleration

Relative acceleration equation may be obtained either by taking time derivative of relative velocity equation

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$$

or by applying Transport/Coriolis Theorem.

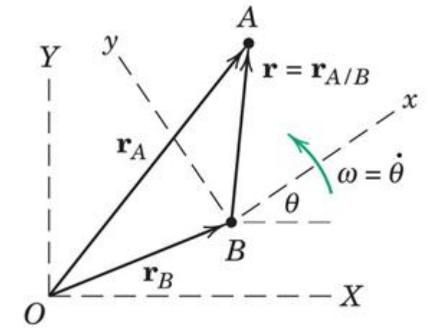
$$\frac{d}{dt}\vec{v}_A = \frac{d}{dt}\vec{v}_B + \frac{d}{dt}(\vec{\omega} \times \vec{r}_{rel}) + \frac{d}{dt}\vec{v}_{rel}$$

$$\dot{\hat{\imath}} = \vec{\omega} \times \hat{\imath},\ \dot{\hat{\jmath}} = \vec{\omega} \times \hat{\jmath}$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}} \times \vec{r}_{rel} + \vec{\omega} \times \frac{d}{dt}(x\hat{\imath} + y\hat{\jmath}) + \frac{d}{dt}(\dot{x}\hat{\imath} + \dot{y}\hat{\jmath})$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} + \vec{\omega} \times \left[(\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}) + (x\dot{\hat{\imath}} + y\dot{\hat{\jmath}})\right] + (\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}) + (\dot{x}\dot{\hat{\imath}} + \dot{y}\dot{\hat{\jmath}})$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$



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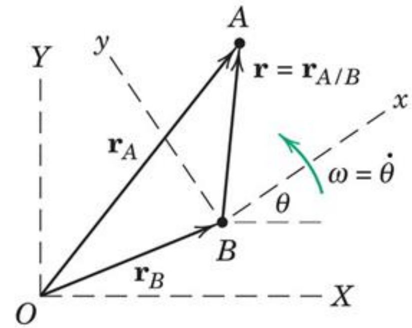
$$\frac{d}{dt}\vec{v}_A = \frac{d}{dt}\vec{v}_B + \frac{d}{dt}(\vec{\omega} \times \vec{r}_{rel}) + \frac{d}{dt}\vec{v}_{rel}$$

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \vec{\omega} \times \vec{V} + \left(\frac{d\vec{V}}{dt}\right)_{x-y}$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}} \times \vec{r}_{rel} + \vec{\omega} \times \dot{\vec{r}}_{rel} + \frac{d}{dt}\vec{v}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}) + (\vec{\omega} \times \vec{v}_{rel} + \dot{\vec{v}}_{rel})$$

$$\boxed{\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2\vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}}$$



$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

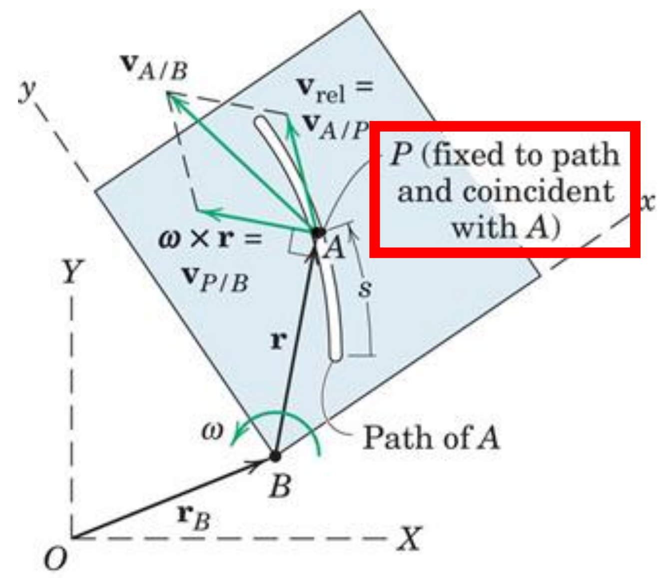
Interpretation of Terms

Assume the moving coordinate system x-y be formed of a plate with a slot in it where particle A can move in relative to the moving coordinate system.

Assume a point P, *instantly coincident* with particle A is fixed on the moving plane.

$$\vec{a}_A = \vec{a}_B + \underbrace{\vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel}}_{\vec{a}_{P/B}} + \underbrace{2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}}_{\vec{a}_{A/P}}$$

$$\vec{a}_A = \vec{a}_B + \quad \vec{a}_{P/B} \quad + \quad \vec{a}_{A/P}$$



$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

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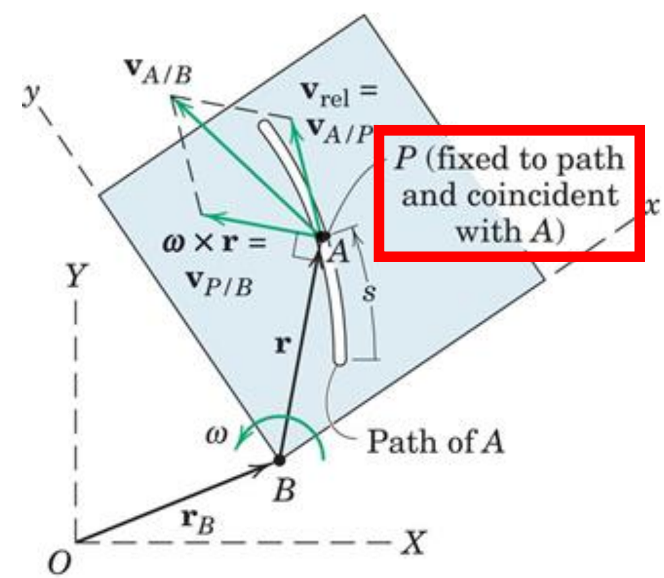
$$\vec{a}_A = \vec{a}_P + \vec{a}_{A/P}$$



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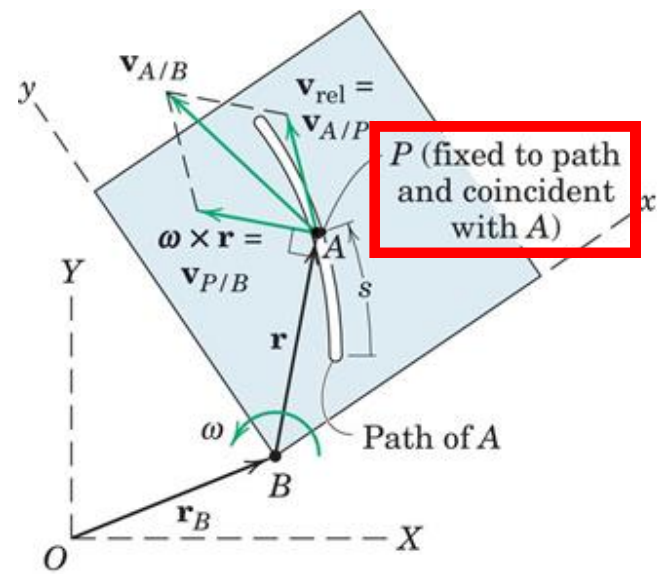


$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

Coriolis Acceleration

Non-rotating frame: $\vec{\omega} = \vec{0}$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + \mathbf{2\vec{\omega} \times \vec{v}_{rel}} + \vec{a}_{rel}$$

Coriolis Acceleration

Rotating frame: $\vec{\omega} \neq \vec{0}$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + \mathbf{2\vec{\omega} \times \vec{v}_{rel}} + \vec{a}_{rel}$$

Coriolis Acceleration

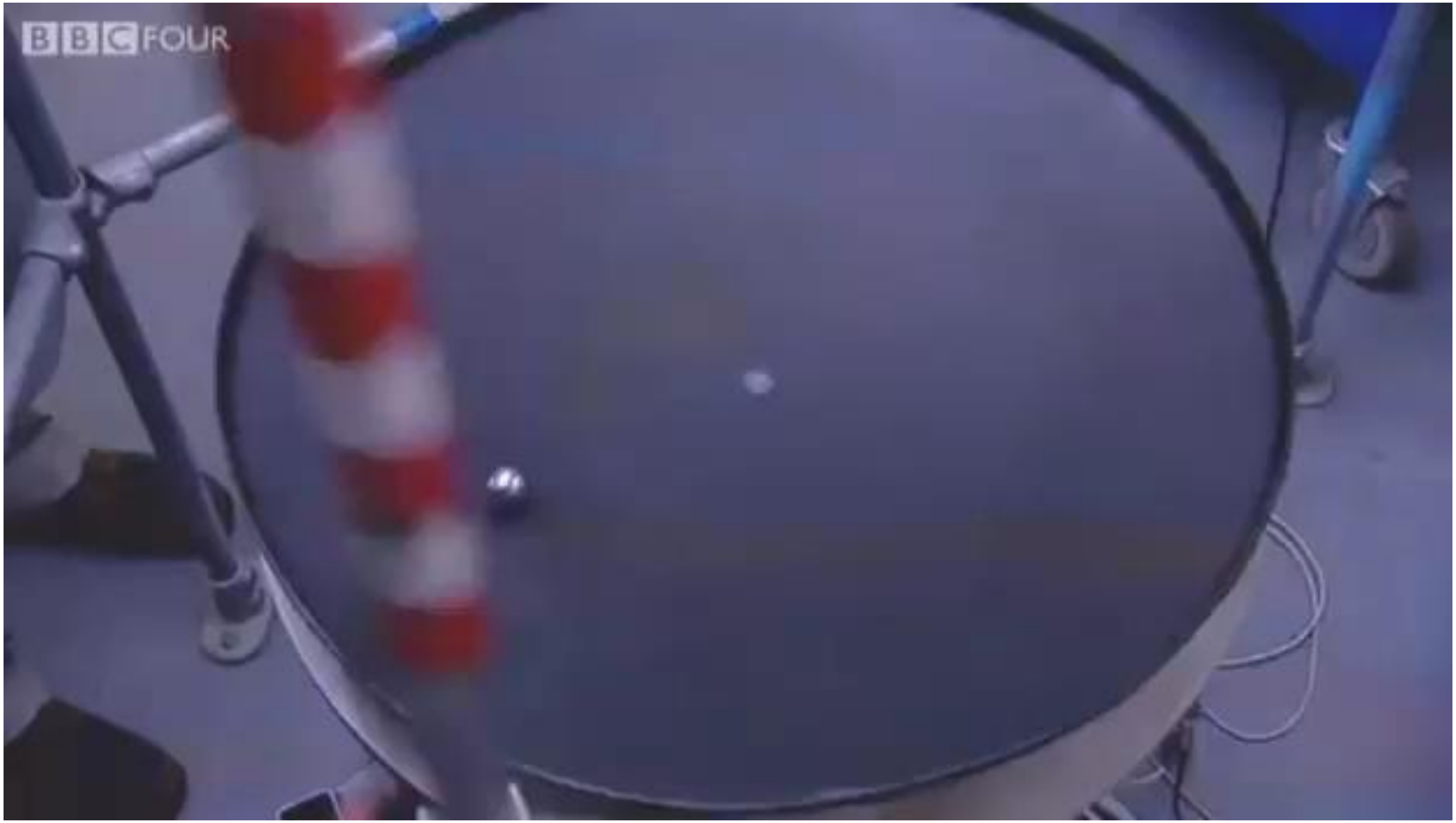
Rotating frame: $\vec{\omega} \neq \vec{0}$

https://www.youtube.com/watch?v=mPsLanVS1Q8

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + \mathbf{2\vec{\omega} \times \vec{v}_{rel}} + \vec{a}_{rel}$$

Coriolis Acceleration

Rotating frame: $\vec{\omega} \neq \vec{0}$



https://www.youtube.com/watch?v=Wda7azMvabE

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + \mathbf{2\vec{\omega} \times \vec{v}_{rel}} + \vec{a}_{rel}$$

Coriolis Acceleration

Rotating frame: $\vec{\omega} \neq \vec{0}$

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$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$$

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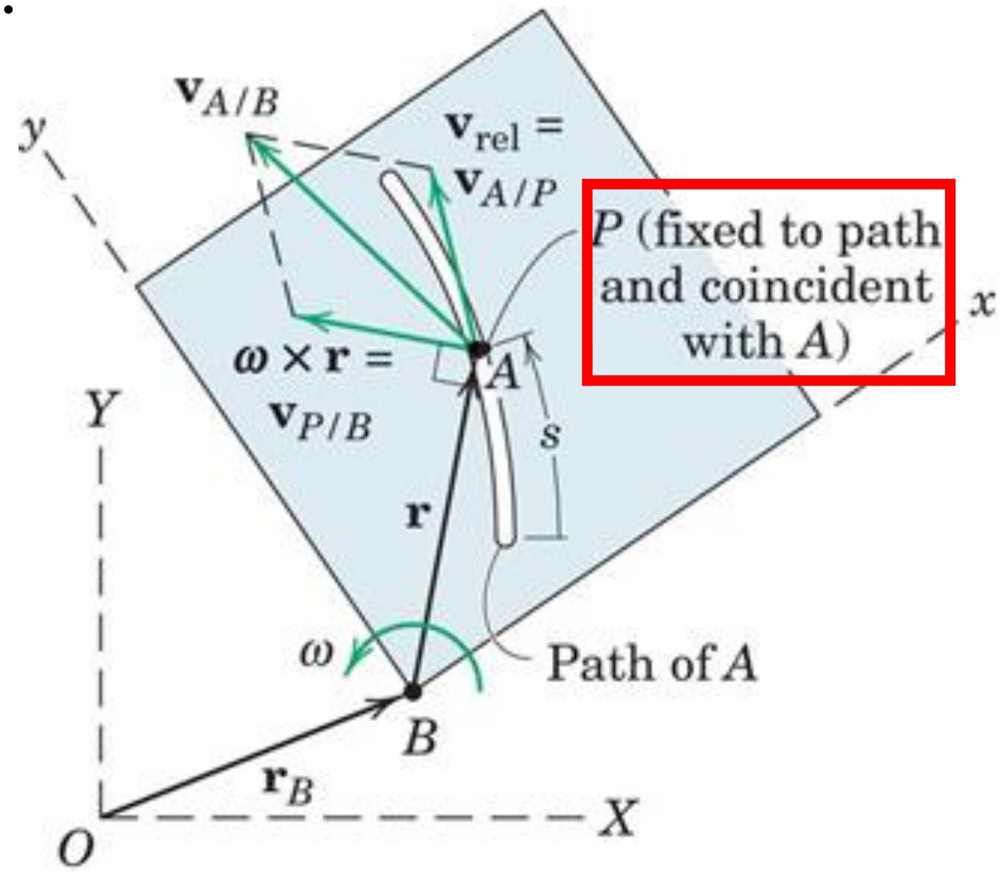
$$\vec{v}_A = \vec{v}_P + \vec{v}_{A/P}$$



$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

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$$\vec{a}_A = \vec{a}_B + \vec{a}_{P/B} + \vec{a}_{A/P}$$

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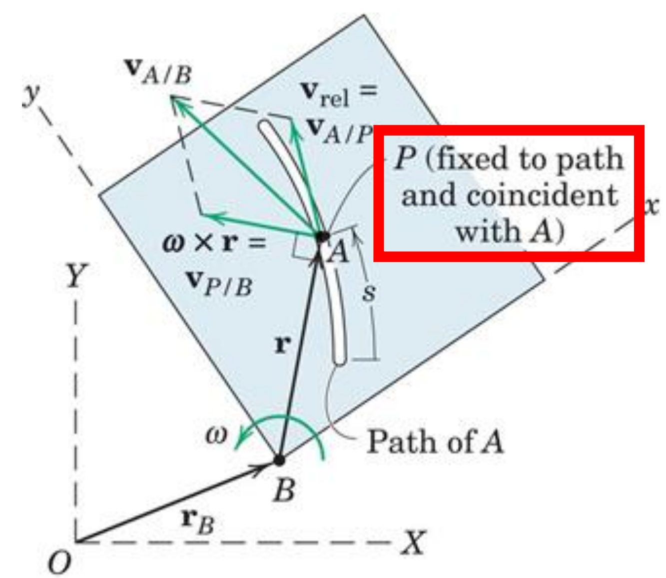
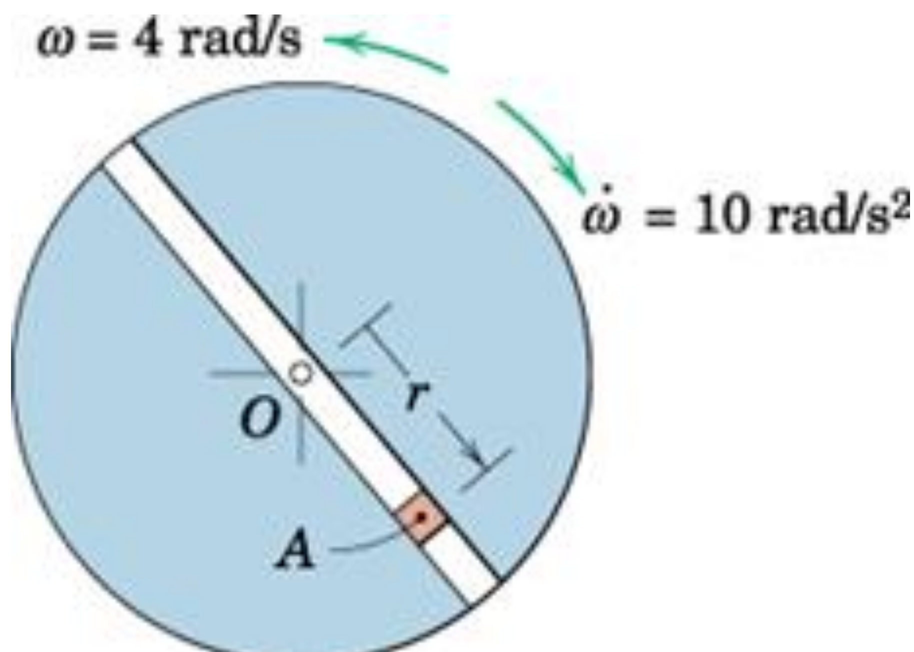
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$



Sample Problem 5/16

At the instant represented, the disk with radial slot is rotating about O with a counterclockwise angular velocity of 4 rad/s which is decreasing at the rate of 10 rad/s^2. The motion of slider A is separately controlled, and at this instant, r is 150 mm, increasing with $\dot{r}$ = 125 mm/s and $\ddot{r}$ = 2025 mm/s^2. Determine the absolute velocity and acceleration of A for this position.



In the solution presented by the textbook the formula is directly applied. However one may think point P fixed on the disk and instantly coincident with the slider A and obtain absolute velocity and acceleration of point A.

$\vec{v}_A = \vec{v}_P + \vec{v}_{A/P}$

Since point P fixed on the rotating disk it makes fixed axis rotation about O,

$\vec{v}_P = \vec{\omega} \times \vec{r}_P = 4\hat{k} \times 0.15\hat{\imath} = 0.600\hat{\jmath}\ m/s$

$\vec{v}_{A/P} = \dot{\vec{r}} = 0.125\hat{\imath}\ m/s$

$\vec{v}_A = 0.6\hat{\jmath} + 0.125\hat{\imath} = (0.1250\hat{\imath} + 0.600\hat{\jmath})\ m/s$

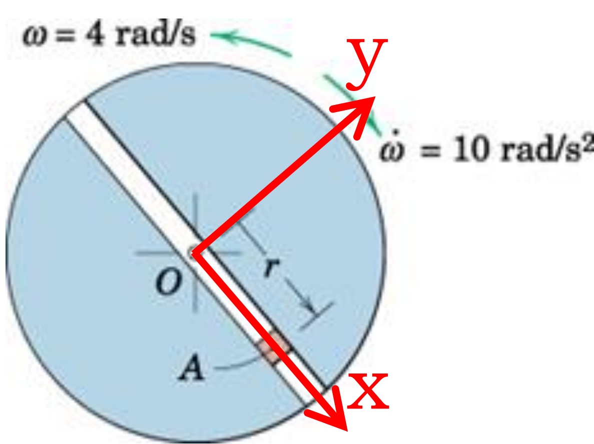
$\vec{a}_A = \vec{a}_P + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$

$\vec{a}_P = \vec{\alpha} \times \vec{r}_{rel} - \omega^2\vec{r}_{rel} = -10\hat{k} \times 0.15\hat{\imath} - 4^2 0.15\hat{\imath}$

$\vec{a}_P\ = (2.40\hat{\imath} - 1.500\hat{\jmath})\ m/s^2$

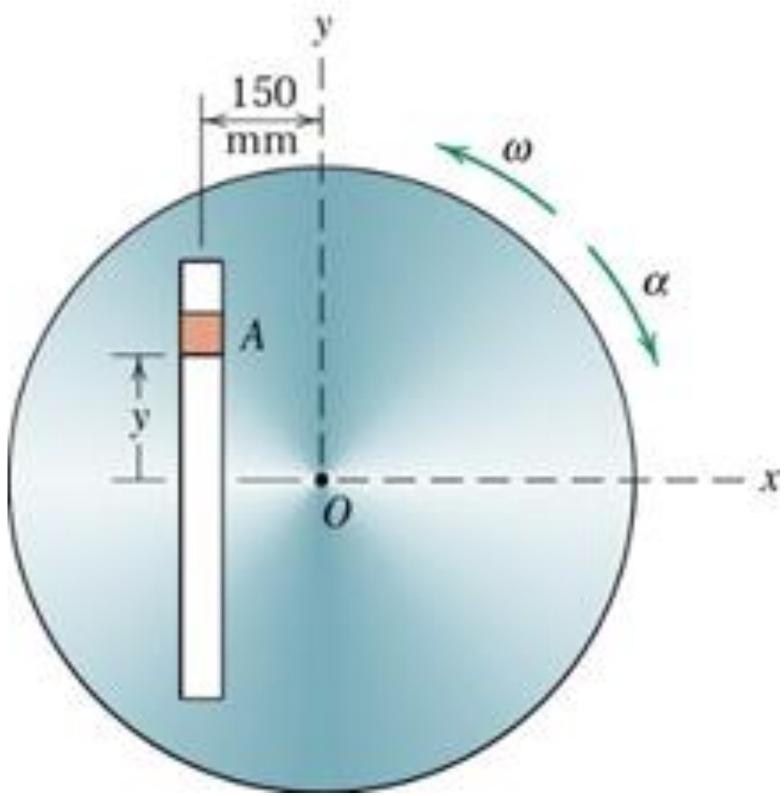
$\vec{a}_A = 2.40\hat{\imath} - 1.500\hat{\jmath} + 2 * 4\hat{k} \times 0.125\hat{\imath} + 2.025\hat{\imath}$

$\vec{a}_A\ = (-0.375\hat{\imath} - 0.500\hat{\jmath})\ m/s^2$



5/150 (4th), 5/151 (5th), None (6th), 5/159 (7th), None (8th)

The disk rotates about a fixed axis through O with angular velocity ω = 5 rad/s and angular acceleration α = 3 rad/s^2 at the instant represented, in the directions shown. The slider A moves in the straight slot. Determine the absolute velocity and acceleration of A for the same instant, when y = 250 mm, $\dot{y}$ = –600 mm/s and $\ddot{y}$ = 750 mm/s^2.



Again one may assume point P fixed on the disk, instantly coincident with A (or may apply the formula directly).
Velocity analysis is required first since acceleration equations contain velocities too.

$$\vec{v}_A = \vec{v}_P + \vec{v}_{A/P}$$

Since point P on the disk makes fixed axis rotation about O,

$$\vec{v}_P = \vec{\omega} \times \vec{r}_P = 5\hat{k} \times (-0.15\hat{\imath} + 0.25\hat{\jmath}) = (-1.25\hat{\imath} - 0.75\hat{\jmath})\ m/s$$

$$\vec{v}_{A/P} = \dot{y}\hat{\jmath} = -0.6\hat{\jmath}\ m/s$$

$$\vec{v}_A = -1.25\hat{\imath} - 0.75\hat{\jmath} - 0.6\hat{\jmath} = (-1.250\hat{\imath} - 1.350\hat{\jmath})\ m/s$$

$$\vec{a}_A = \vec{a}_P + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

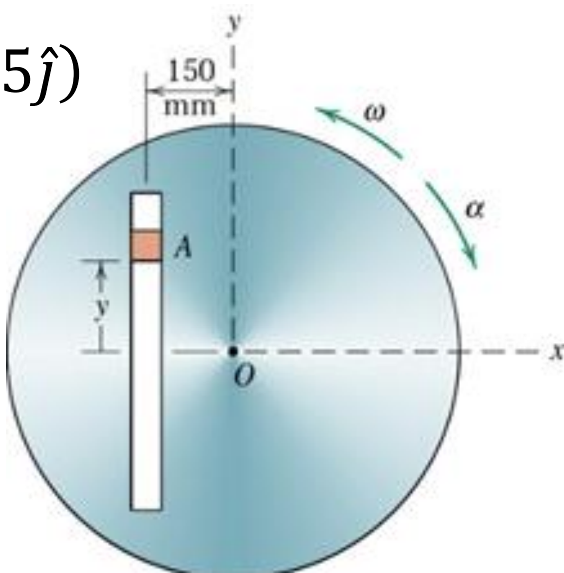
$$\vec{a}_P = \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel}$$

$$\vec{a}_P = -3\hat{k} \times (-0.15\hat{\imath} + 0.25\hat{\jmath}) - 5^2(-0.15\hat{\imath} + 0.25\hat{\jmath})$$

$$\vec{a}_P = (4.50\hat{\imath} - 5.80\hat{\jmath})\ m/s^2$$

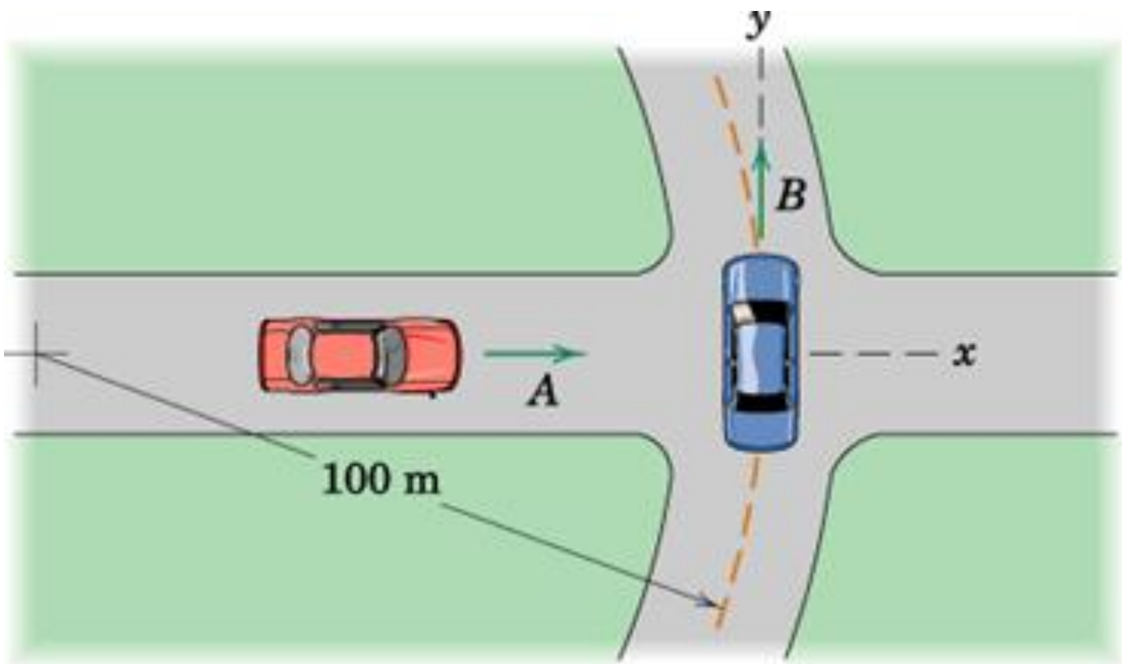
$$\vec{a}_A = 4.5\hat{\imath} - 5.8\hat{\jmath} + 2 * 5\hat{k} \times -0.6\hat{\jmath} + 0.75\hat{\jmath}$$

$$\vec{a}_A = (-10.50\hat{\imath} - 5.05\hat{\jmath})\ m/s^2$$



5/155 (4th), 5/156 (5th), 5/165 (6th), None (7th), 5/163 (8th)

Car B is rounding the curve with a constant speed of 54 km/h, and car A is approaching car B in the intersection with a constant speed of 72 km/h. Determine the velocity which car A appears to have to an observer riding and **turning** with car B. The x-y axes are attached to car B. Is this apparent velocity the negative of velocity that B appears to have to a nonrotating observer in car A? The distance separating two cars at the instant depicted is 40 m.



Here use of a point P fixed in the moving coordinate system and instantly coincident with A is hard to visualize and direct application of formula is straight forward.

$v_A = 72\ km/h \equiv 20\ m/s$

$v_B = 54\ km/h \equiv 15\ m/s$

When motion is observed through B using x-y coordinates it is **rotating** about the center of curvature of the road therefore

$$\omega = \frac{v_B}{r} = \frac{15}{100} = 0.1500\ rad/s\ (CCW)$$

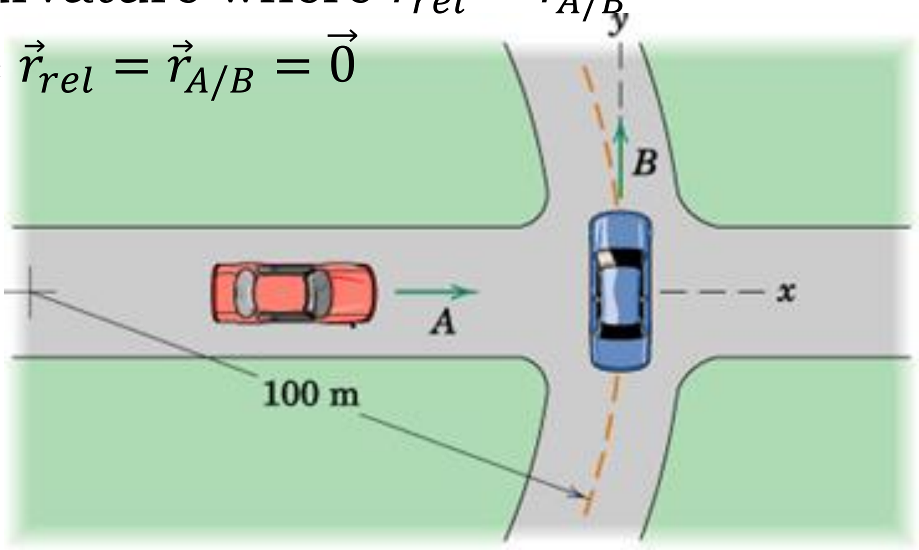
$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$

$\vec{r}_{rel} = \vec{r}_{A/B} = -40\hat{\imath}\ m$

$20\hat{\imath} = 15\hat{\jmath} + 0.15\hat{k} \times -40\hat{\imath} + \vec{v}_{rel}$

$\vec{v}_{rel} = (20.0\hat{\imath} - 9.00\hat{\jmath})\ m/s$

Also consider car A at the center of curvature where $\vec{r}_{rel} = \vec{r}_{A/B} = -100\hat{\imath}\ m$ and about to hit car B where $\vec{r}_{rel} = \vec{r}_{A/B} = \vec{0}$



For center of curvature:

$20\hat{\imath} = 15\hat{\jmath} + 0.15\hat{k} \times -100\hat{\imath} + \vec{v}_{rel}$

$\vec{v}_{rel} = (20.0\hat{\imath} - 0.00\hat{\jmath})\ m/s$

At the instant of hit:

$20\hat{\imath} = 15\hat{\jmath} + 0.15\hat{k} \times \vec{0} + \vec{v}_{rel}$

$\vec{v}_{rel} = (20.0\hat{\imath} - 15.00\hat{\jmath})\ m/s$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

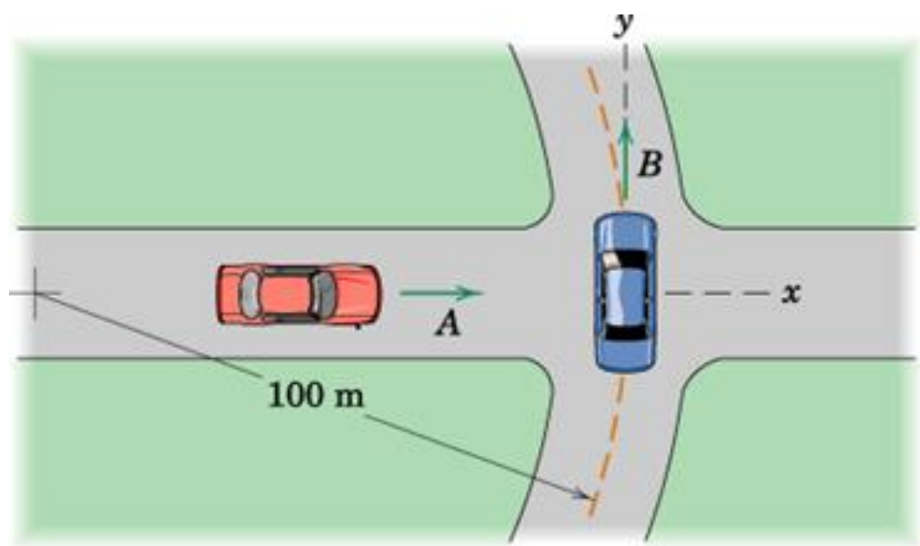
$$\vec{a}_A = \vec{0}$$

$$\vec{a}_B = \omega^2 \vec{r} = -0.15^2 * 100\hat{\imath} = -2.25\hat{\imath}\ m/s^2$$

$$\vec{\alpha} = \vec{0}, \vec{\omega} = \overrightarrow{const}$$

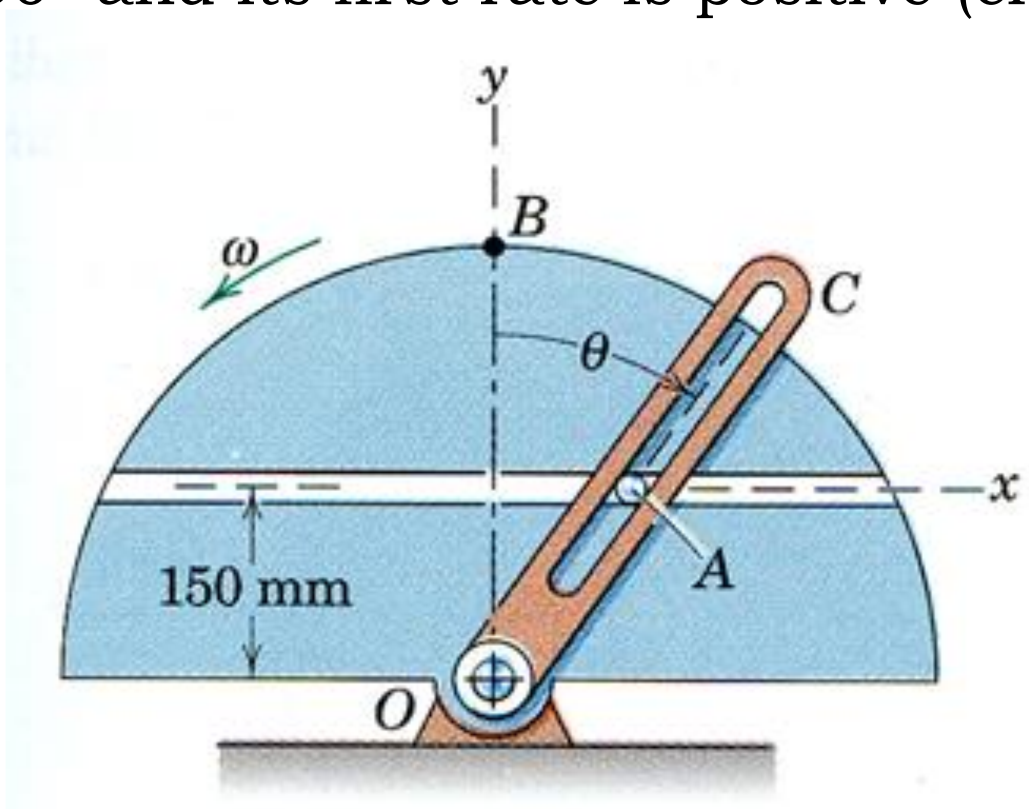
$$\vec{0} = -2.25\hat{\imath} + \vec{0} - 0.15^2 * -40\hat{\imath} + 2 * 0.15\hat{k} \times (20.0\hat{\imath} - 9.00\hat{\jmath}) + \vec{a}_{rel}$$

$$\vec{a}_{rel} = (1.350\hat{\imath} - 6.00\hat{\jmath})\ m/s^2$$



5/162 (4^{th}), None (5^{th}), None (6^{th}), None (7^{th}), None (8^{th})

The slotted disk sector rotates with a constant counterclockwise angular velocity $\omega = 3$ rad/s. Simultaneously the slotted arm OC oscillates about line OB (fixed to the disk) so that θ changes at a constant rate of 2 rad/s except at the extremities of the oscillation during reversal of direction. Determine the total acceleration of the pin when $\theta = 30°$ and its first rate is positive (clockwise).



One may assume a point P fixed on the disk and instantly coincident with A

$$\vec{a}_A = \vec{a}_P + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

$$\vec{r}_{rel} = (0.15tan\theta\hat{\imath} + 0.15\hat{\jmath}) = (0.0866\hat{\imath} + 0.15\hat{\jmath})m$$

$$\vec{a}_P = \vec{\alpha}_{disk} \times \vec{r}_{rel} - \omega_{disk}{}^2\vec{r}_{rel}$$

$$\vec{a}_P = \vec{0} \times (0.0866\hat{\imath} + 0.15\hat{\jmath}) - 3^2(0.0866\hat{\imath} + 0.15\hat{\jmath})$$

$$\vec{a}_P = (-0.779\hat{\imath} - 1.350\hat{\jmath})\ m/s^2$$

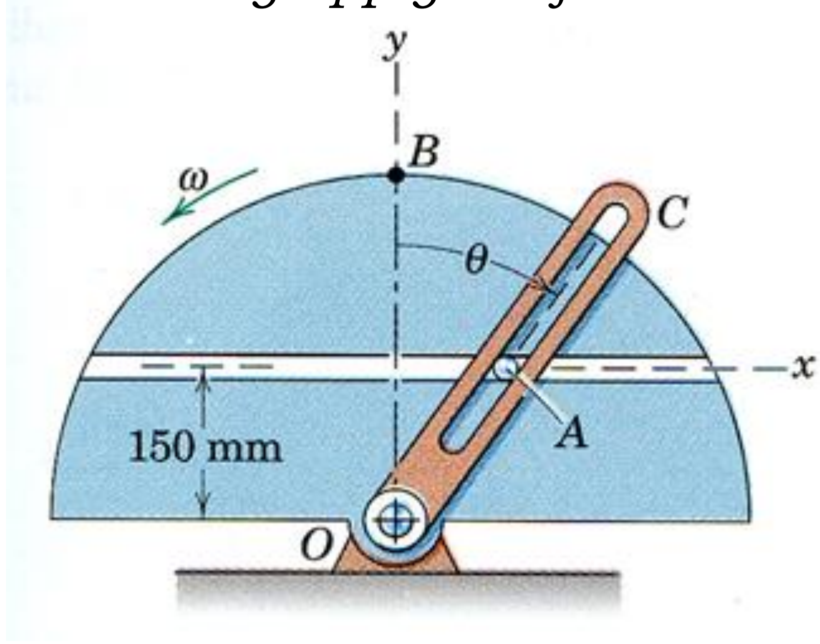
$$\vec{v}_{rel} = \dot{\vec{r}}_{rel} = \dot{x}\hat{\imath} = \frac{d}{dt}(0.15tan\theta\hat{\imath} + 0.15\hat{\jmath})$$

$$\vec{v}_{rel} = 0.15\dot{\theta}sec^2\theta\hat{\imath} = 0.15 * 2 * sec^2 30°\hat{\imath} = 0.400\hat{\imath}\ m/s$$

$$\vec{a}_{rel} = \dot{\vec{v}}_{rel} = \ddot{x}\hat{\imath} = \frac{d}{dt}\left(0.15\dot{\theta}sec^2\theta\hat{\imath}\right) = 2 * 0.15\dot{\theta}sec^2\theta tan\theta = 0.923\hat{\imath}\ m/s \text{ for } \ddot{\theta} = 0$$

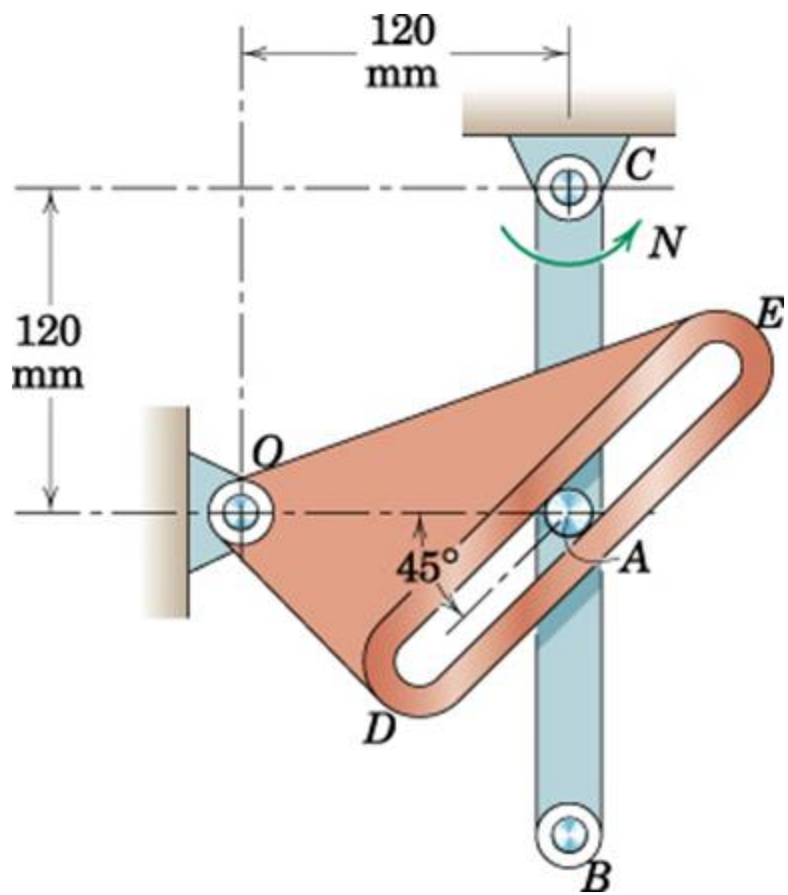
$$\vec{a}_A = (-0.779\hat{\imath} - 1.350\hat{\jmath}) + 2\left(-2\hat{k}\right) \times 0.4\hat{\imath} + 0.923\hat{\imath} = (0.1230\hat{\imath} - 1.050\hat{\jmath})\ m/s^2$$

Alternatively you could directly apply the formula with the same terms.



5/163 (4th), 5/168 (5th), 5/178 (6th), 5/176 (7th), 5/179 (8th)

For the instant represented, link CB is rotating counterclockwise at a constant rate N = 4 rad/s, and its pin A causes a clockwise rotation of the slotted member ODE. Determine the angular velocity ω and angular acceleration α of ODE for this instant.



Assume a point P fixed on body ODE instantly coincident with A.

$$\vec{v}_P = \vec{v}_A + \vec{v}_{P/A}$$

$$\vec{v}_P = \vec{\omega}_{ODE} \times \vec{r}_{P/O} = \omega_{ODE}\hat{k} \times 0.12\hat{\imath} = 0.12\omega_{ODE}\hat{\jmath}$$

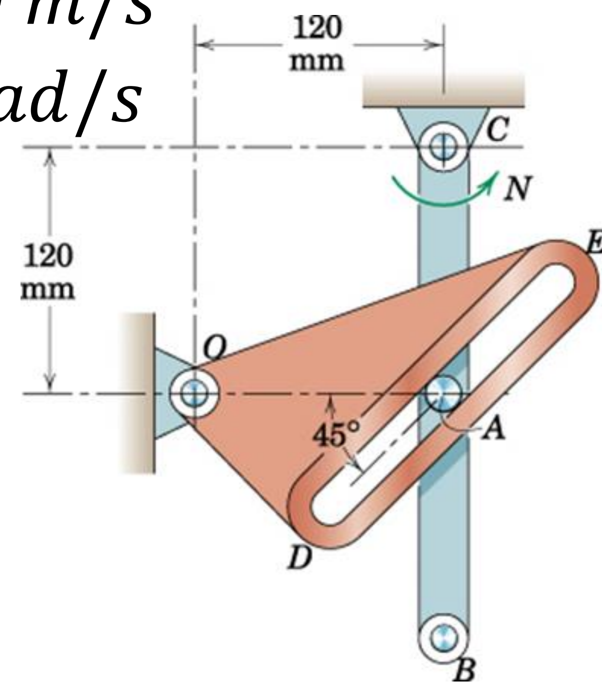
$$\vec{v}_A = N\hat{k} \times \vec{r}_{A/C} = 4\hat{k} \times -0.12\hat{\jmath} = 0.48\hat{\imath}\, m/s$$

$$\vec{v}_{P/A} = v_{P/A}(cos45°\hat{\imath} + sin45°\hat{\jmath})$$

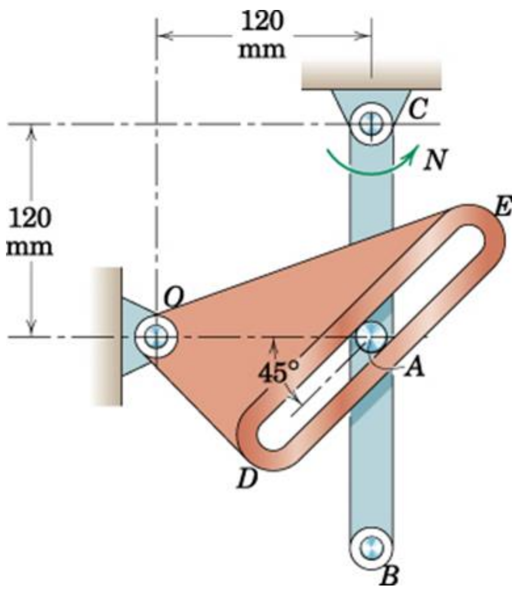
$$0.12\omega_{ODE}\hat{\jmath} = 0.48\hat{\imath} + v_{P/A}(cos45°\hat{\imath} + sin45°\hat{\jmath})$$

$$\hat{\imath}:\ 0 = 0.48 + v_{P/A}cos45°, v_{P/A} = -0.679\, m/s$$

$$\hat{\jmath}:\ 0.12\omega_{ODE} = v_{P/A}sin45°, \omega_{ODE} = -4\, rad/s$$



Assume a point P fixed on body ODE instantly coincident with A.



$$\vec{a}_P = \vec{a}_A + \vec{a}_{P/A}$$

$$\vec{a}_P = \vec{a}_{P_t} + \vec{a}_{P_n}$$

$$\vec{a}_P = \vec{\alpha}_{ODE} \times \vec{r}_{P/O} - {\omega_{ODE}}^2 \vec{r}_{P/O}$$

$$\vec{a}_P = \alpha_{ODE}\hat{k} \times 0.12\hat{\imath} - 4^2 * 0.12\hat{\imath}$$

$$\vec{a}_P = (-1.920\hat{\imath} + 0.12\alpha_{ODE}\hat{\jmath})\ m/s^2$$

$$\vec{a}_A = \vec{a}_{A_t} + \vec{a}_{A_n} = \vec{\alpha}_{CA} \times \vec{r}_{A/C} - N^2 \vec{r}_{A/C}$$

$$\vec{a}_A = \vec{0} \times -0.12\hat{\jmath} - 4^2 * -0.12\hat{\jmath} = 1.920\hat{\jmath}\ m/s^2$$

$$\vec{a}_{P/A} = 2\vec{\omega}_{ODE} \times \vec{v}_{P/A} + \vec{a}_{rel}$$

$$\vec{a}_{P/A} = 2 * -4\hat{k} \times -0.679(cos45°\hat{\imath} + sin45°\hat{\jmath}) + a_{rel}(cos45°\hat{\imath} + sin45°\hat{\jmath})$$

$$\vec{a}_{P/A} = 3.84(-\hat{\imath} + \hat{\jmath}) + a_{rel}(cos45°\hat{\imath} + sin45°\hat{\jmath})$$

$$(-1.92\hat{\imath} + 0.12\alpha_{ODE}\hat{\jmath}) = 1.92\hat{\jmath} + 3.84(-\hat{\imath} + \hat{\jmath}) + a_{rel}(cos45°\hat{\imath} + sin45°\hat{\jmath})$$

$$\hat{\imath}:\ -1.92 = -3.84 + a_{rel}cos45°$$

$$a_{rel} = 2.72\ m/s^2$$

$$\hat{\jmath}:\ 0.12\alpha_{ODE} = 1.92 + 3.84 + 2.72sin45°$$

$$\alpha_{ODE} = 64.0\ rad/s^2$$